CBE 34487

6/18/20

TRANSPORT PHENOMENA

DETAILED ANALYSIS OF

HEAT, MASS + MOMENTUM TRANSPORT
BASED ON THE FUNDAMENTAL
DIFFERENTIAL EQUATIONS
DERIVED FROM CONSERVATION
OF ENERGY, MASS + MOMENTUM

SOME OF MY BOOKS..

TABLE I. SCHEMATIC DIAGRAM OF THE ORGANIZATION OF TRANSPORT PHENOMENA

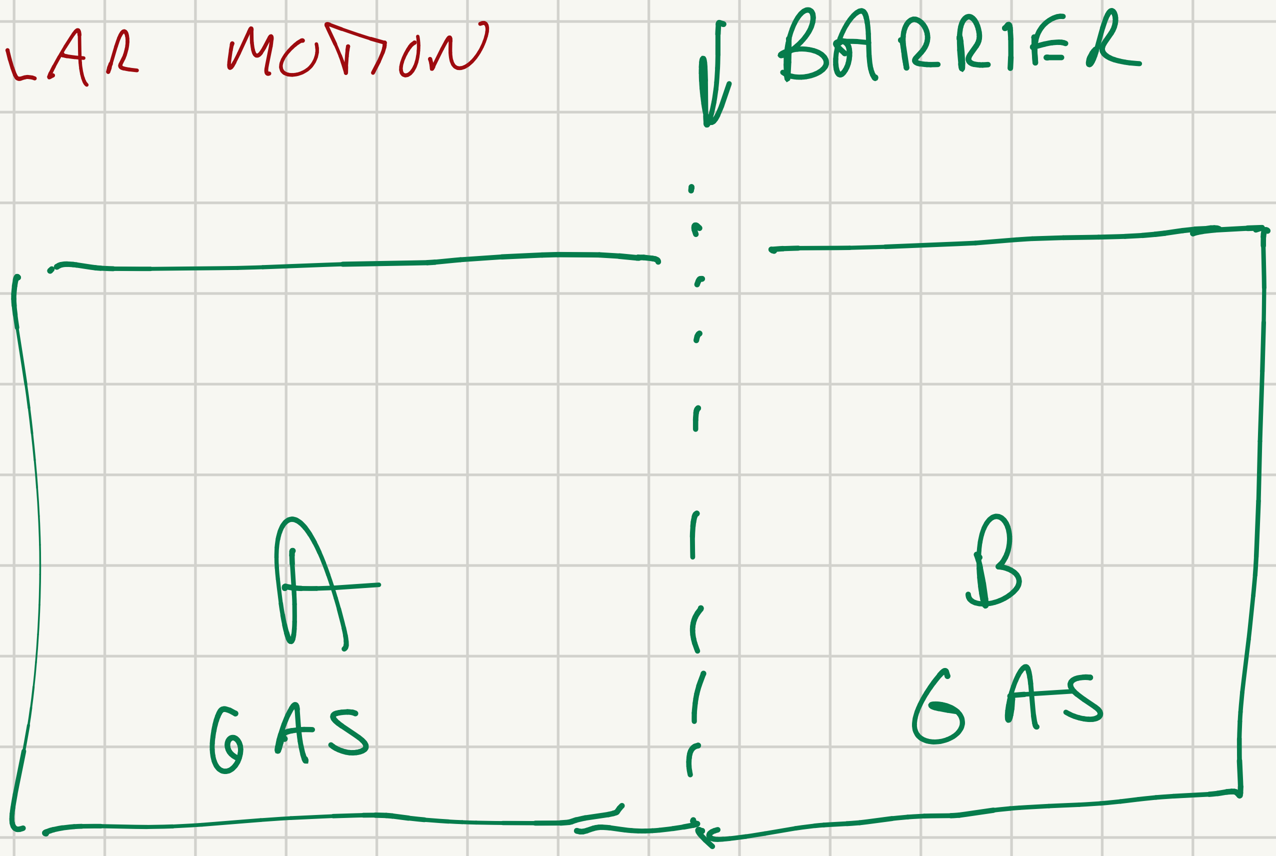
Entity Being Transported → / Type of Transport ↓	Momentum	Energy	Mass
TRANSPORT BY MOLECULAR MOTION	1 VISCOSITY μ Newton's law of viscosity Temperature, pressure, and composition dependence of μ Kinetic theory of μ	8 THERMAL CONDUCTIVITY k Fourier's law of heat conduction Temperature, pressure, and composition dependence of k Kinetic theory of k	16 DIFFUSIVITY $\mathfrak{D}_{AB}$ Fick's law of diffusion Temperature, pressure, and composition dependence of $\mathfrak{D}_{AB}$ Kinetic theory of $\mathfrak{D}_{AB}$
TRANSPORT IN LAMINAR FLOW OR IN SOLIDS, IN ONE DIMENSION	2 SHELL MOMENTUM BALANCES Velocity profiles Average velocity Momentum flux at surfaces	9 SHELL ENERGY BALANCES Temperature profiles Average temperature Energy flux at surfaces	17 SHELL MASS BALANCES Concentration profiles Average concentration Mass flux at surfaces
TRANSPORT IN AN ARBITRARY CONTINUUM	3 EQUATIONS OF CHANGE (ISOTHERMAL) Equation of continuity Equation of motion Equation of energy (isothermal)	10 EQUATIONS OF CHANGE (NONISOTHERMAL) Equation of continuity Equation of motion for forced and free convection Equation of energy (nonisothermal)	18 EQUATIONS OF CHANGE (MULTICOMPONENT) Equations of continuity for each species Equation of motion for forced and free convection Equation of energy (multicomponent)
TRANSPORT IN LAMINAR FLOW OR IN SOLIDS, WITH TWO INDEPENDENT VARIABLES	4 MOMENTUM TRANSPORT WITH TWO INDEPENDENT VARIABLES Unsteady viscous flow Two-dimensional viscous flow Ideal two-dimensional flow Boundary-layer momentum transport	11 ENERGY TRANSPORT WITH TWO INDEPENDENT VARIABLES Unsteady heat conduction Heat conduction in viscous flow Two-dimensional heat conduction in solids Boundary-layer energy transport	19 MASS TRANSPORT WITH TWO INDEPENDENT VARIABLES Unsteady diffusion Diffusion in viscous flow Two-dimensional diffusion in solids Boundary-layer mass transport
TRANSPORT IN TURBULENT FLOW	5 TURBULENT MOMENTUM TRANSPORT Time-smoothing of equations of change Eddy viscosity Turbulent velocity profiles	12 TURBULENT ENERGY TRANSPORT Time-smoothing of equations of change Eddy thermal conductivity Turbulent temperature profiles	20 TURBULENT MASS TRANSPORT Time-smoothing of equations of change Eddy diffusivity Turbulent concentration profiles
TRANSPORT BETWEEN TWO PHASES	6 INTERPHASE MOMENTUM TRANSPORT Friction factor f Dimensionless correlations	13 INTERPHASE ENERGY TRANSPORT Heat-transfer coefficient h Dimensionless correlations (forced and free convection)	21 INTERPHASE MASS TRANSPORT Mass-transfer coefficient k_x Dimensionless correlations (forced and free convection)
TRANSPORT BY RADIATION	*Numbers refer to the chapters in this book*	14 RADIANT ENERGY TRANSPORT Planck's radiation law Stefan-Boltzmann law Geometrical problems Radiation through absorbing media	*This book may be studied either by "columns" or by "rows"*
TRANSPORT IN LARGE FLOW SYSTEMS	7 MACROSCOPIC BALANCES (ISOTHERMAL) Mass balance Momentum balance Mechanical energy balance (Bernoulli equation)	15 MACROSCOPIC BALANCES (NONISOTHERMAL) Mass balance Momentum balance Mechanical and total energy balance	22 MACROSCOPIC BALANCES (MULTICOMPONENT) Mass balances for each species Momentum balance Mechanical and total energy balance

MECHANISMS OF TRANSPORT

MASS DIFFUSION

RANDOM MOLECULAR MOTION MIXES GASES

BARRIER



AT SOME TIME, REMOVE BARRIER ...

UNIFORM COMPOSITION

NOTATION

FOR MASS FLUX $\Rightarrow$ $\bar{N}_A$

FOR "A"

MECHANISMS OF TRANSPORT

HEAT
~~DIFFUSION~~ CONDUCTION

INSULATING BARRIER

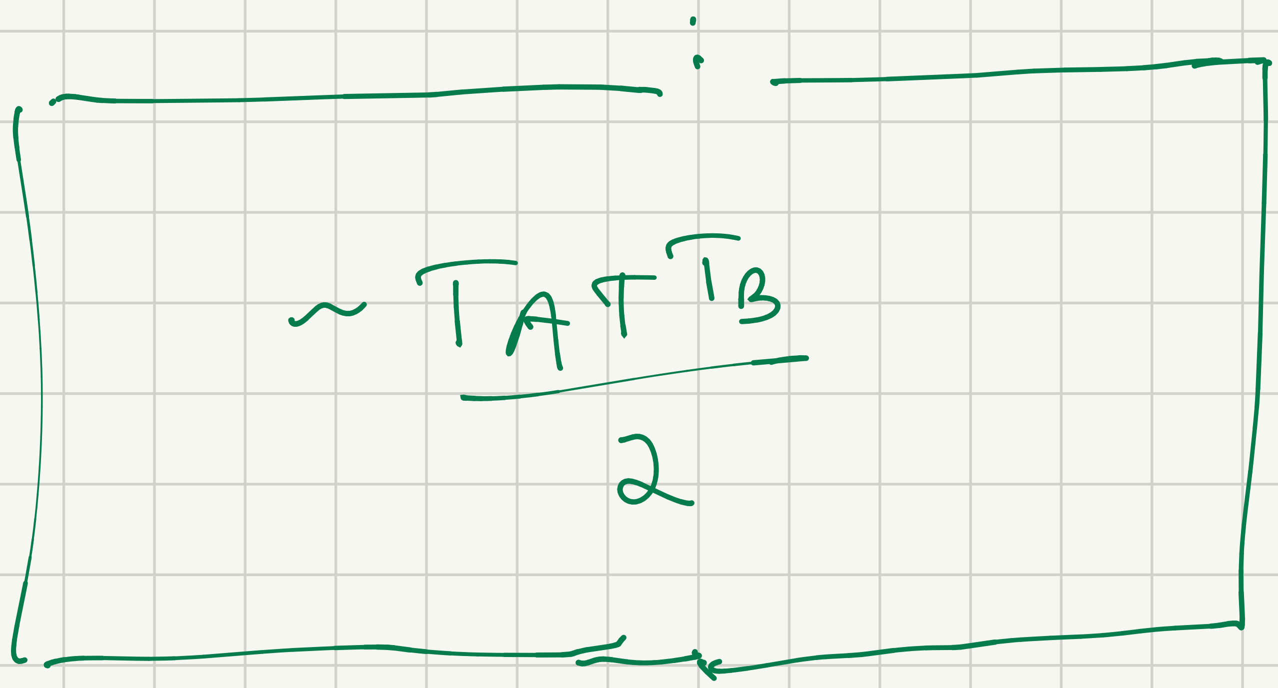
RANDOM MOLECULAR MOTION, OR VIBRATION IN SOLID (OR FREE ELECTRONS..)

IF GAS OR LIQUID

ASSUME NO GRAVITY

AT SOME TIME, REMOVE BARRIER ...

EVENS OUT TEMP



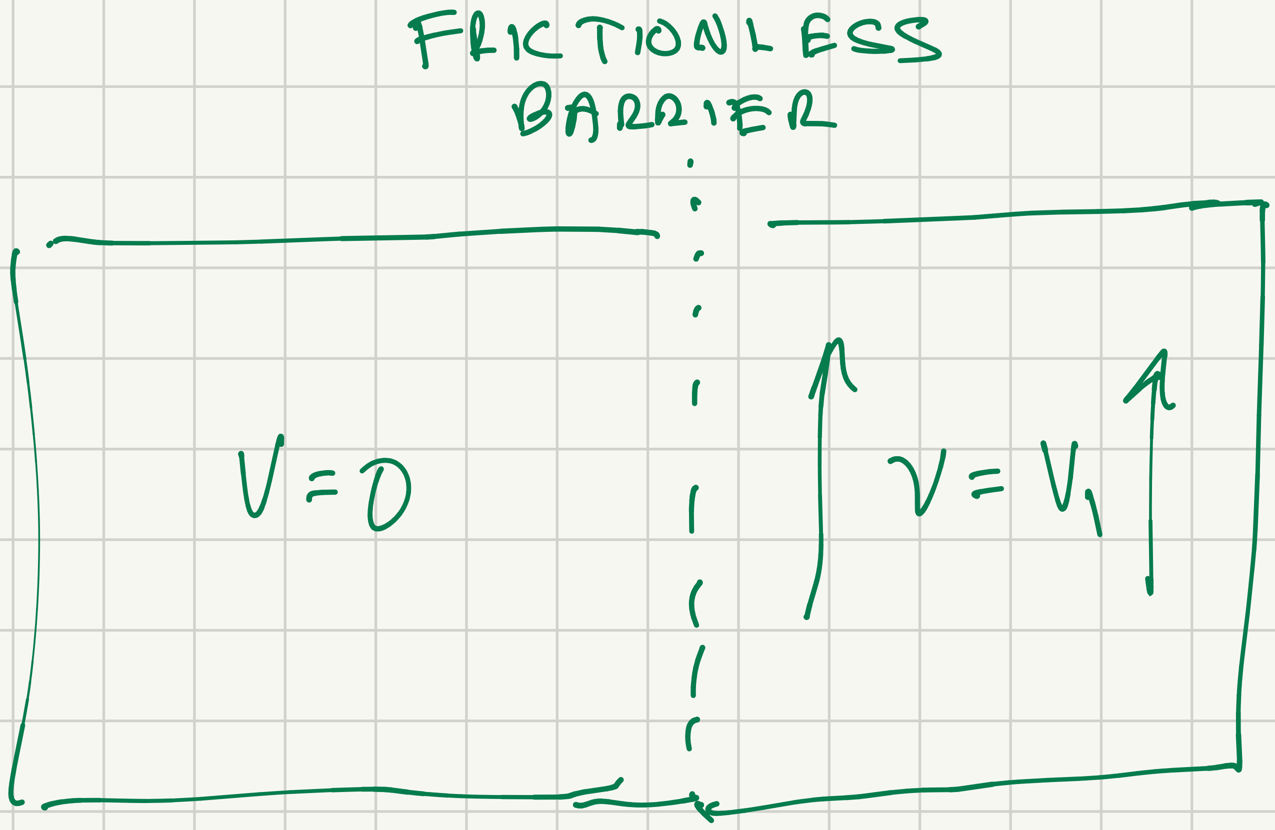
UNIFORM TEMPERATURE

NOTATION FOR HEAT FLUX $\Rightarrow$ $\bar{q}_A$

MECHANISMS OF TRANSPORT

<u>MOMENTUM "DIFFUSION"</u> → SHEAR & NORMAL "STRESSES"

RANDOM MOLECULAR MOTION EVENS OUT VELOCITY

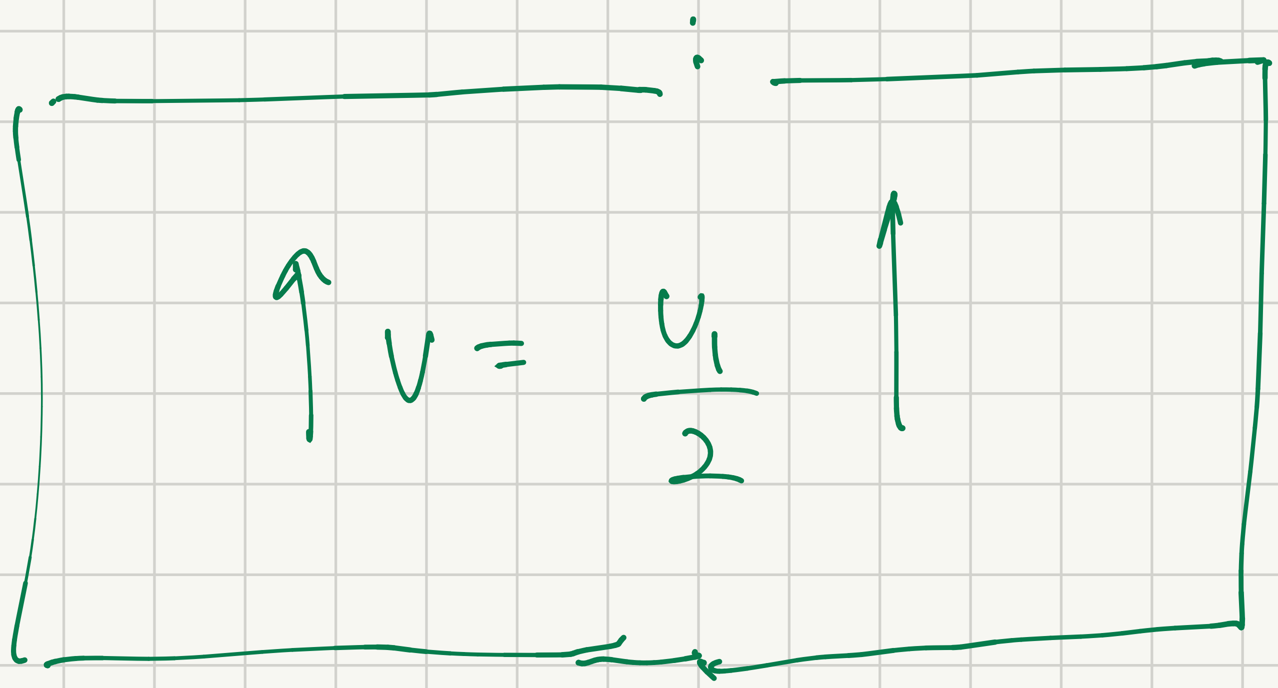


IF GAS OR LIQUID

ASSUME NO GRAVITY

A LITTLE HARDER TO CONSTRUCT THIS ONE

AT SOME TIME, REMOVE BARRIER . . .



UNIFORM TEMPERATURE

NOTATION

FOR MOMENTUM FLUX ⇒ $\bar{\bar{\tau}}$

MECHANISMS OF TRANSPORT

1) MOLECULAR MOTION

MASS → DIFFUSION

HEAT → CONDUCTION

MOMENTUM → DIFFUSION

2) "CONVECTION"

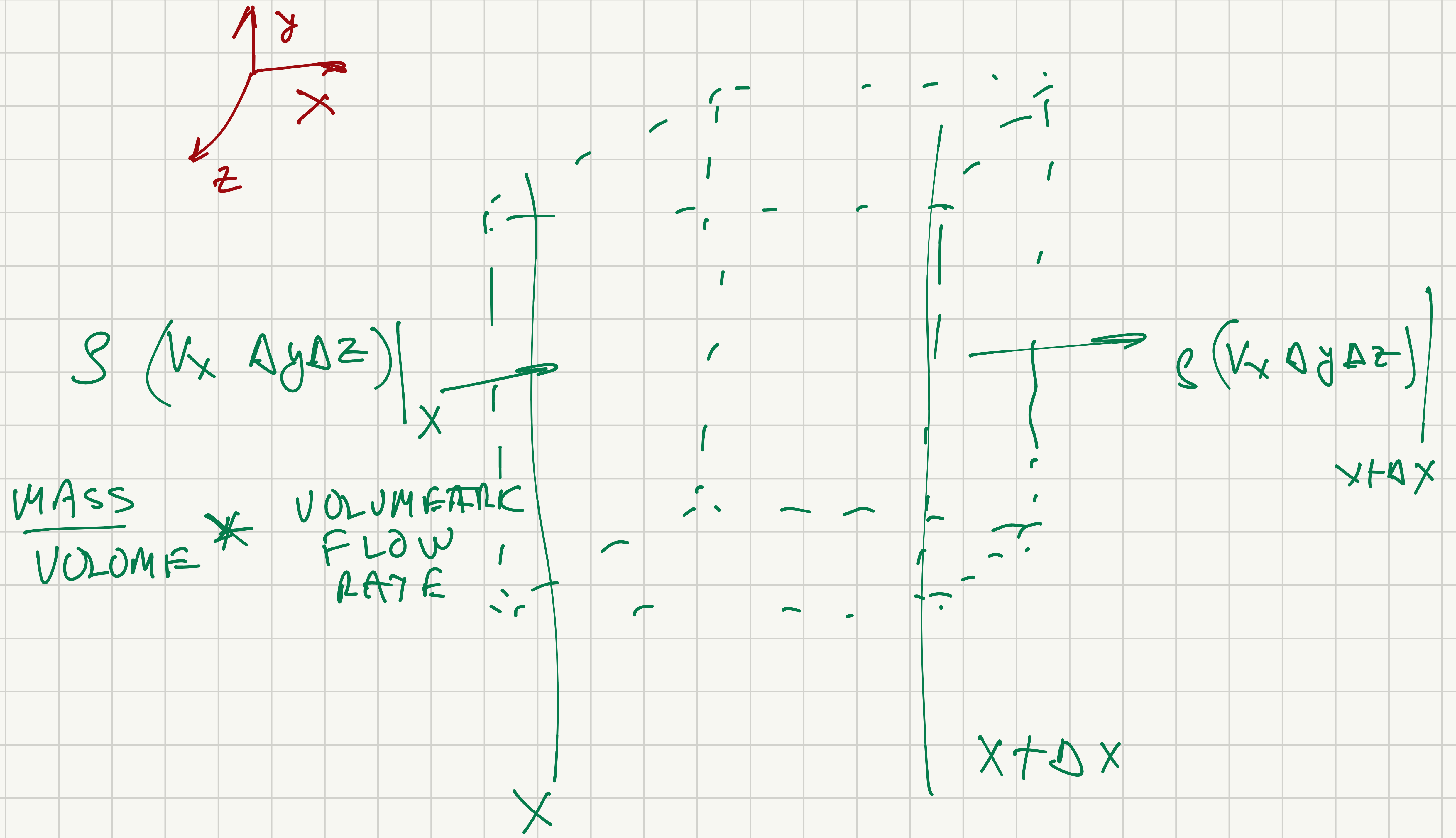
MOTION OF FLUID CARRIES QUANTITY

a) MASS

b) ENERGY

c) MOMENTUM

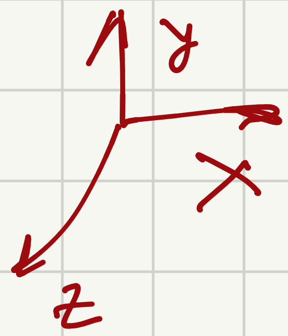
RATE OF CHANGE OF MASS IN DIFFERENTIAL CUBE



MASS ENTERING A SPECIFIC FACE OF CONCEPTUAL CUBE, $\Delta y \Delta z$

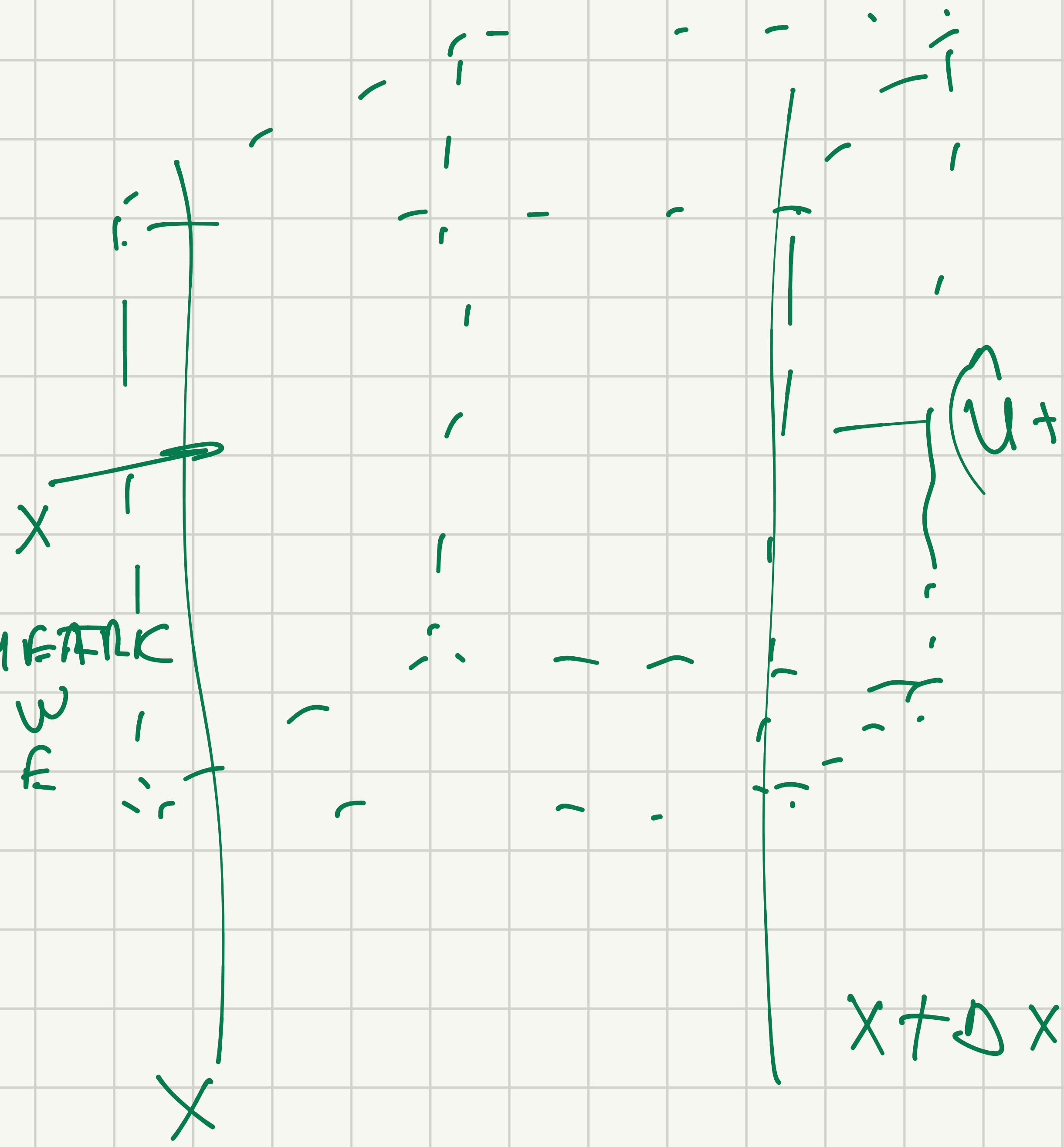
MASS LEAVING A SPECIFIC FACE OF CONCEPTUAL CUBE, $\Delta y \Delta z$

RATE OF CHANGE OF ENERGY IN DIFFERENTIAL CUBE



$$\left(\hat{U}+\frac{v^2}{2}\right)\rho\,(V_x\,\Delta y\,\Delta z)\Big|_{x}$$

$\frac{\text{ENERGY}}{\text{MASS}} \times \frac{\text{MASS}}{\text{VOLUME}} *$ VOLUMETRIC FLOW RATE

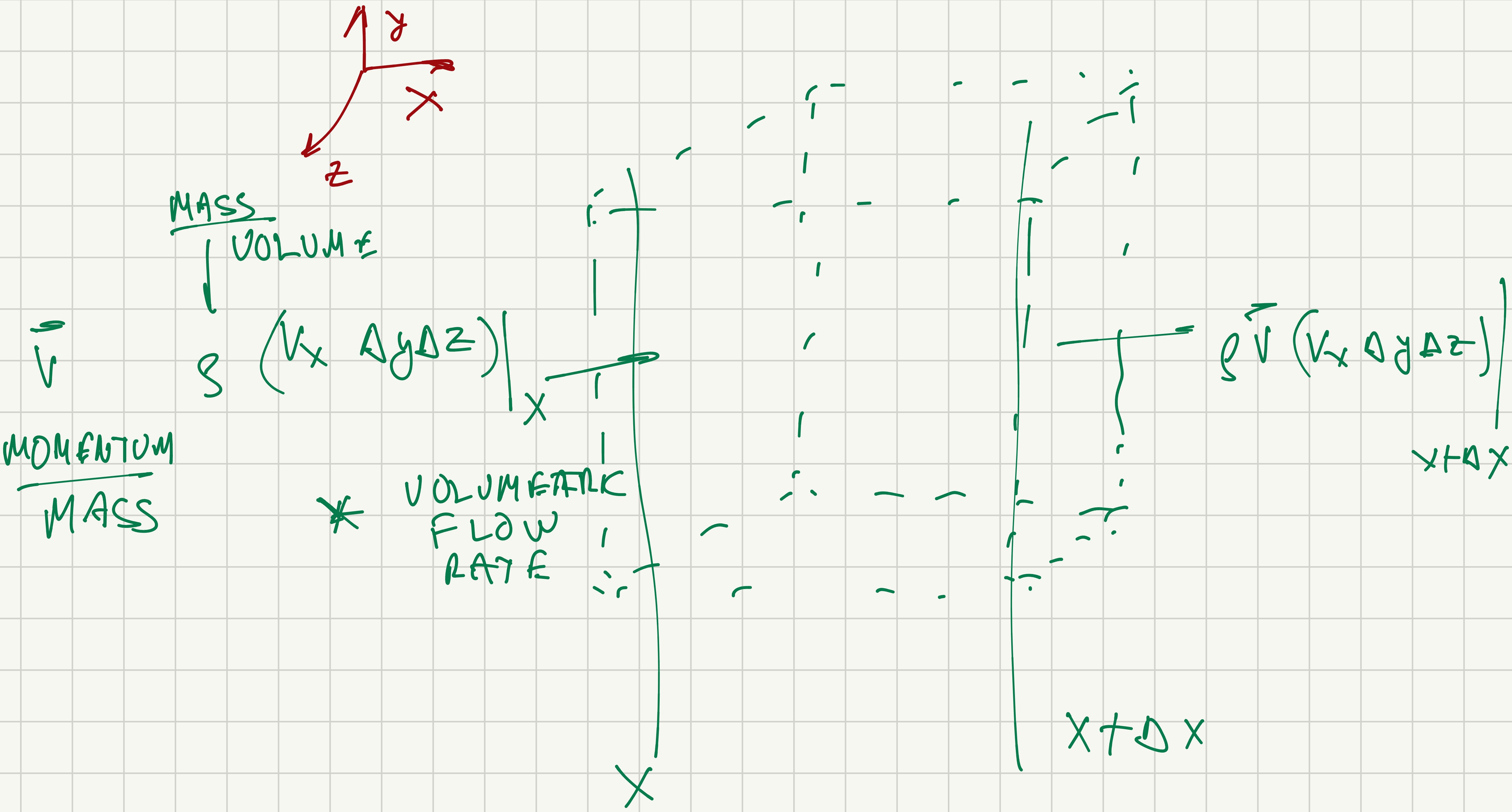


$$\left(\hat{U}+\frac{v^2}{2}\right)\rho\,(V_x\,\Delta y\,\Delta z)\Big|_{x+\Delta x}$$

ENERGY ENTERING A SPECIFIC FACE OF CONCEPTUAL CUBE, $\Delta y \Delta z$

ENERGY LEAVING A SPECIFIC FACE OF CONCEPTUAL CUBE, $\Delta y \Delta z$

RATE OF CHANGE OF MOMENTUM IN DIFFERENTIAL CUBE



MOMENTUM ENTERING A SPECIFIC FACE OF CONCEPTUAL CUBE, $\Delta y \Delta z$

MOMENTUM LEAVING A SPECIFIC FACE OF CONCEPTUAL CUBE, $\Delta y \Delta z$

MECHANISMS OF TRANSPORT

1) MOLECULAR MOTION

MASS → DIFFUSION $\frac{\partial N_x}{\partial x}$

HEAT → CONDUCTION $\frac{\partial q_x}{\partial x}$

MOMENTUM → DIFFUSION $\frac{\partial \tau_{xy}}{\partial x}$

2) "CONVECTION"

MOTION OF FLUID CARRIES QUANTITY

a) MASS $\frac{\partial}{\partial x} \rho V_x (1)$ $\frac{\text{MASS}}{\text{MASS}}$

b) ENERGY $\frac{\partial}{\partial x} \rho V_x \left(\hat{U} + \frac{V^2}{2}\right)$ $\frac{\text{ENERGY}}{\text{MASS}}$

c) MOMENTUM $\frac{\partial}{\partial x} \rho V_x (\vec{V})$ $\frac{\text{MOMENTUM}}{\text{MASS}}$

FINISH DERIVATION AND WE HAVE

MASS:

$$\frac{\partial C_A}{\partial t} + \frac{\partial}{\partial x} N_A + \frac{\partial}{\partial y} N_A + \frac{\partial}{\partial z} N_A = 0$$

ENERGY:

TABLE 10.2–2

THE EQUATION OF ENERGY IN TERMS OF ENERGY AND MOMENTUM FLUXES

(Eq. 10.1–19)

Rectangular coordinates:

$$\rho \hat{C}_v \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = -\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right]$$
$$- T \left(\frac{\partial p}{\partial T} \right)_\rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) - \left\{ \tau_{xx} \frac{\partial v_x}{\partial x} + \tau_{yy} \frac{\partial v_y}{\partial y} + \tau_{zz} \frac{\partial v_z}{\partial z} \right\}$$
$$- \left\{ \tau_{xy} \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) + \tau_{xz} \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) + \tau_{yz} \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right) \right\} \quad \text{(A}$$

FINISH DERIVATION AND WE HAVE

MOMENTUM:

TABLE 3.4–2

THE EQUATION OF MOTION IN RECTANGULAR COORDINATES (x, y, z)

In terms of $\boldsymbol{\tau}$:

x-component

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) + \rho g_x \quad (A)$$

y-component

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} - \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right) + \rho g_y \quad (B)$$

z-component

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z \quad (C)$$

...rms of velocity gradients for a Newtonian fluid with constant ρ and μ:

...componen...

EVEN WITH ALL OF THIS
WE CAN SOLVE ANYTHING

WE NEED "CONSTITUTIVE"*
EQUATIONS FOR THE FLUXES

$$\begin{cases} J_{Ax} = N_A - x_A (N_A + N_B) \\ J_{Ax} = -D \dfrac{\partial C}{\partial x} \end{cases}$$

$$q_{Ax} = -k \frac{\partial T}{\partial x}$$

$$\tau_{xx} = -\mu \frac{\partial v_x}{\partial x}$$

* EQUATIONS, NOT FROM 1ST PRINCIPLES
THAT MATCH BEHAVIOR AS VERIFIED
BY EXPERIMENTS ...

MASS

$$\frac{\partial C_A}{\partial t} + \underbrace{V_x \frac{\partial C_A}{\partial x} + V_y \frac{\partial C_A}{\partial y} + V_z \frac{\partial C_A}{\partial z}}_{} = \underbrace{D_{AB}\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right)}_{}$$

ENERGY

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} + V_z \frac{\partial T}{\partial z} \right) = k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right]$$

MOMENTUM:

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_z}{\partial z} \right) = -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 U_x}{\partial y^2} + \frac{\partial^2 V_x}{\partial z^2} \right)$$

DIFFUSION

$$\rho \left(\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \vec{\nabla} V \right) = -\vec{\nabla} P + \mu \nabla^2 \vec{V} + \rho \vec{g}$$

CONVECTION

THE MATHEMATICAL FORM OF THE THREE DIFFERENT (SETS OF) EQUATIONS IS SAME REFLECTING THE SAME BASIC MECHANISMS OF TRANSPORT.

HENCE: MATHEMATICAL PROCEDURES WILL BE THE SAME OR SIMILAR.

EXAMPLE SITUATIONS

Fluidized-solids reactors with continuous solids feed—II

Conversion for overflow and carryover particles

SAKAE YAGI and DAIZO KUNII
Department of Chemical Engineering, University of Tokyo, Tokyo, Japan

(*Received 6 April 1960; in revised form 4 January 1961*)

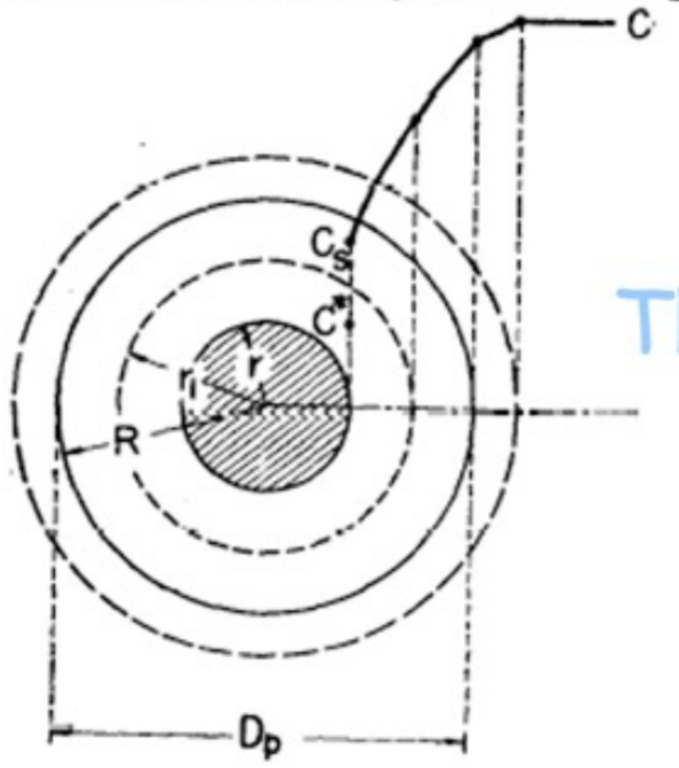


FIG. 1. Model of single particle, in which solid phase remains around the unreacted core. $D_p = x$.

The Shrinking Core!

ON THE APPLICATION OF THE SHRINKING CORE MODEL TO LIQUID–SOLID REACTIONS

NILS LINDMAN and DANIEL SIMONSSON
Department of Chemical Technology, Royal Institute of Technology, S-100 44 Stockholm 70, Sweden

(*Received 4 December 1977 accepted 2 May 1978*)

The basic equation in the shrinking core model for a spherical particle is derived from a differential mass balance for the fluid reactant diffusing through the ash layer

$$\epsilon \frac{\partial c}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D_{eff}\frac{\partial c}{\partial r} - r^2 vc\right) \quad r_c < r < R \qquad (1)$$

Theoretical Analysis of Antibody Targeting of Tumor Spheroids: Importance of Dosage for Penetration, and Affinity for Retention[1]

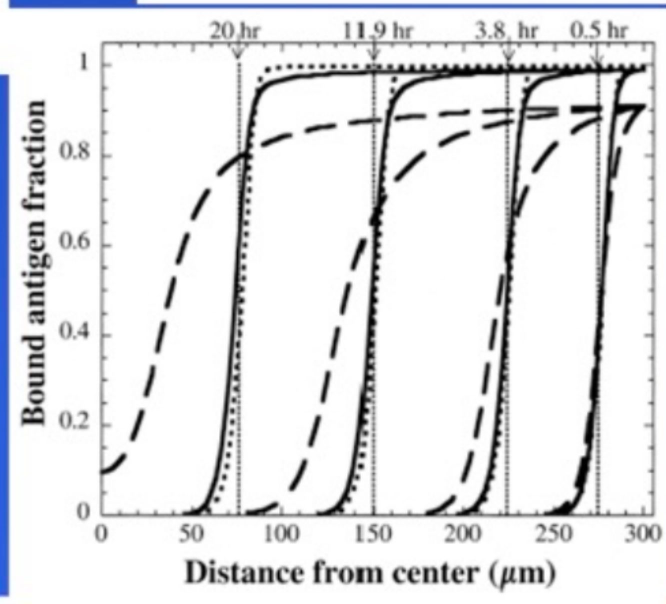
Christilyn P. Graff and K. Dane Wittrup[2]

The moving reaction front observed in these simulations is analogous to one described in the classic chemical reaction engineering literature. Combustion of carbon deposits in catalyst particles is observed to produce such moving fronts with outer shells and inner cores, and a simplified analytical theory termed the SCM[3] was derived to describe these phenomena (27, 28). The central assumption of the SCM is that diffusion from the surface of the sphere to the internal reaction front is significantly slower than consumption of the reactant at the reaction front at a critical radius r_c. The antibody spheroid

$$\frac{\partial Ab}{\partial t} = D\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial Ab}{\partial r}\right) - \frac{k_{on}}{\varepsilon}AbAg + k_{off}B$$

$$\frac{\partial B}{\partial t} = \frac{k_{on}}{\varepsilon}AbAg - k_{off}B - k_eB$$

$$\frac{\partial Ag}{\partial t} = R_s - \frac{k_{on}}{\varepsilon}AbAg + k_{off}B - k_eAg$$



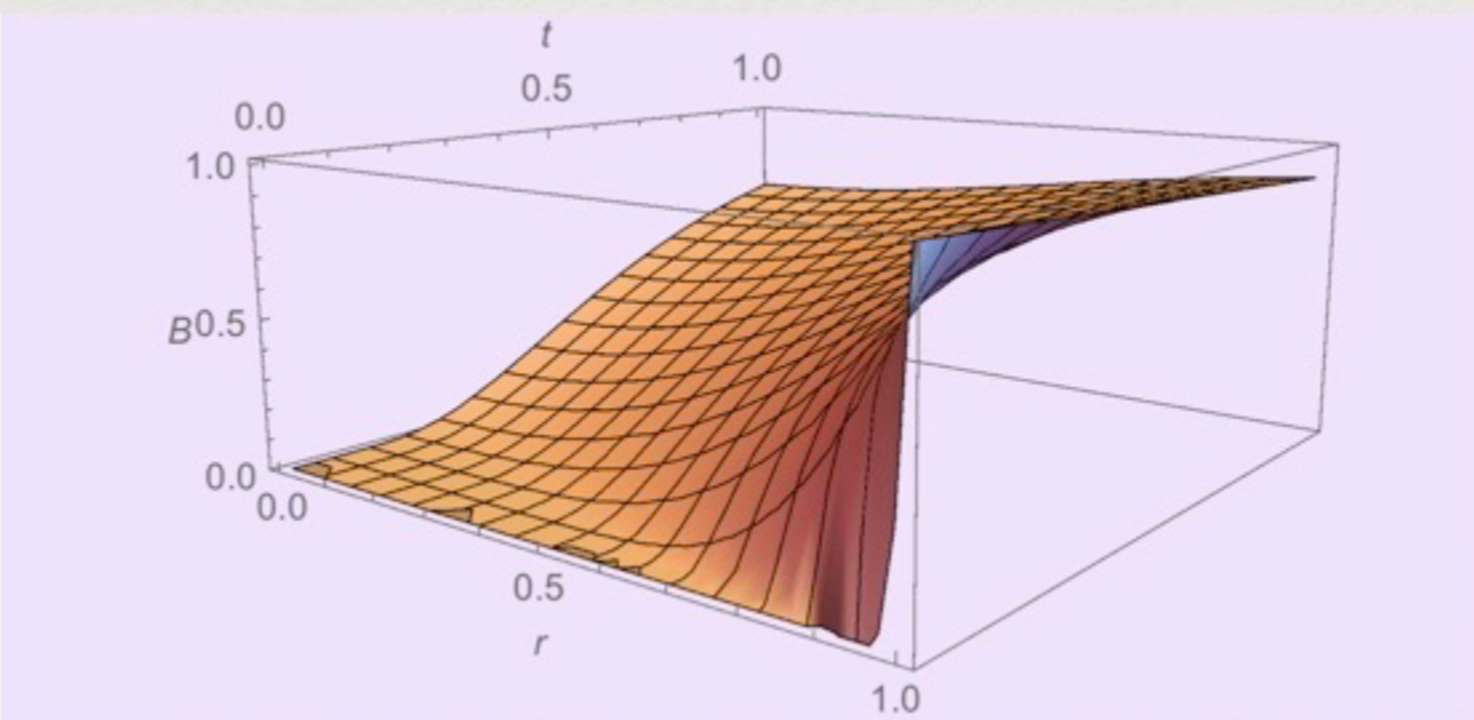
From a paper in the journal "Cancer Research", 2003 by two Chemical Engineers

Cancer: Shrinking core

```
In[67]:= eqs = {D[Ab[r, t], t] == α1/r^2 D[r^2 D[Ab[r, t], r], r] - kon/ϵ Ab[r, t] Ag[r, t] + koff B[r, t],
    D[Ac[r, t], t] == α2/r^2 D[r^2 D[Ac[r, t], r], r],
    D[B [r, t], t] ==   kon/ϵ Ab[r, t] Ag[r, t]   - koff B[r, t] - kdeath B[r, t],
    D[Ag[r, t], t] == rs - kon/ϵ Ab[r, t] Ag[r, t]   + koff B[r, t] - ke Ag[r, t]}

In[71]:= sol = NDSolve[{eqs, inits, bcs}, {Ab, Ac, B, Ag}, {r, r0, R}, {t, 0, t1}, MaxSteps → 50000]
```

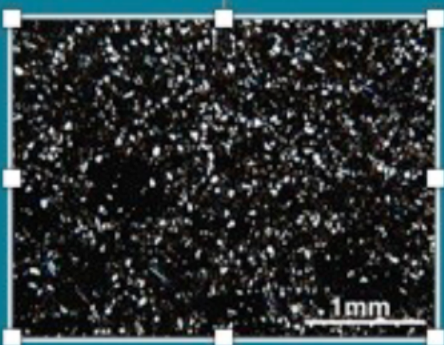
Out[20]=



- **Flow of oil in sandstone**
  - Governing equation

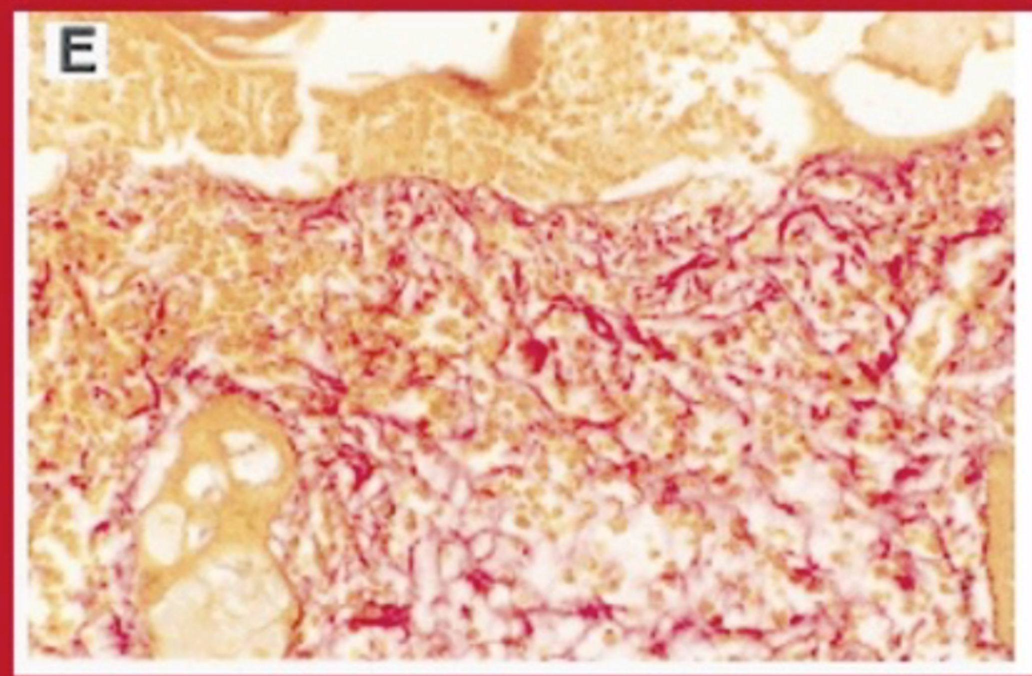
$$\frac{\partial P}{\partial t} - K_e \frac{\partial^2 P}{\partial x^2} = 0$$

  - P is the local pressure causing flow
  - K_e is an effective hydraulic "conductivity" the response of fluid flow to the change in pressure

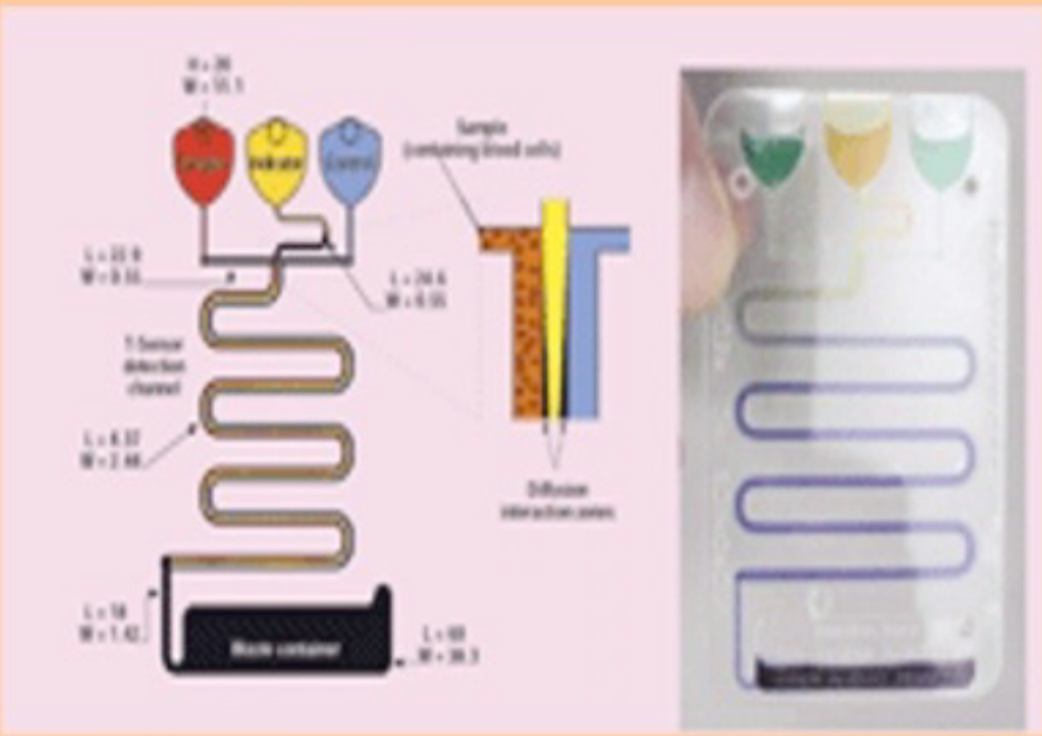


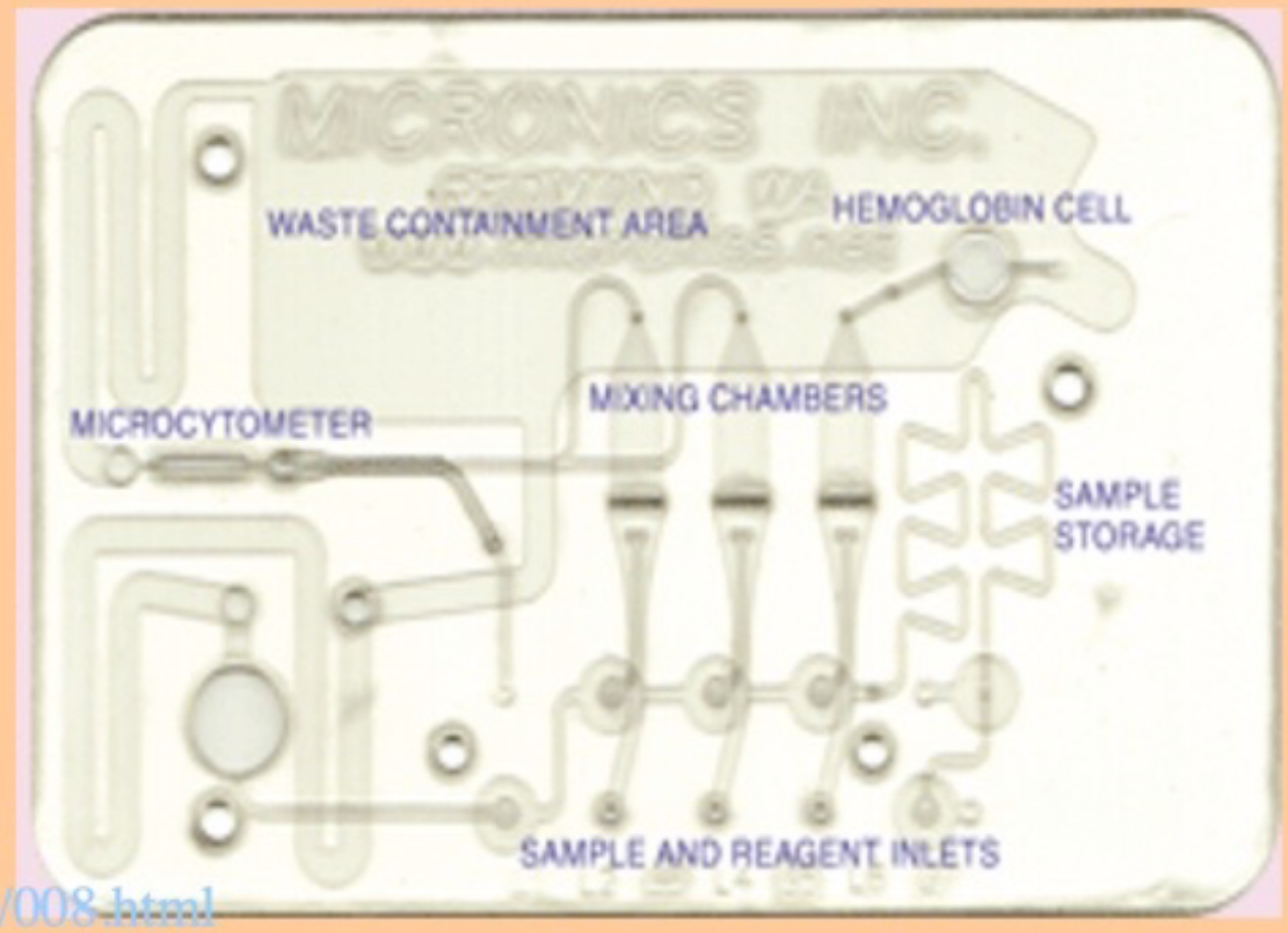
- **Interstitial lymph fluid flow**
  - Governing equation

$$\frac{1}{(2\mu + \lambda)} \frac{\partial P^*}{\partial t} - K \frac{\partial^2 P^*}{\partial x^2} + \beta P^* = 0$$



- **Flow in microfluidic devices**
  - **When things shrink, qualitative differences occur.**
  - **For example, a miniature propeller would not pump fluid!**





http://www.devicelink.com/ivdt/archive/00/11/008.html

HOW FAR

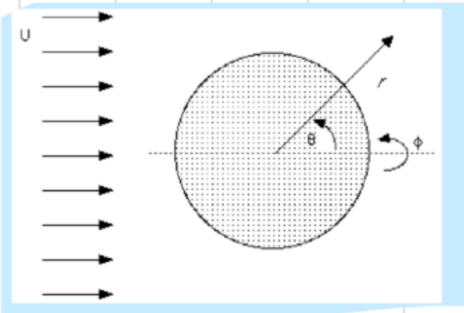
$R_e \ll 1$

FOLLOW AIR

HOW FAST WILL THESE FALL

$$v = \frac{2}{9}\frac{(\rho_P - \rho_A)}{\mu} g R^2$$

$$\Rightarrow \quad \vec{\nabla} P = \mu \nabla^2 \vec{v}$$



$$\left\{ \begin{array}{l} \vec{\nabla} \cdot \vec{v} = 0 \\ \vec{\nabla} p = \mu \nabla^2 \vec{v} \end{array} \right\}$$

USE SPHERICAL COORDINATES

- CONTINUITY EQ.
- r- DIRECTION N.S. EQ
- θ - DIRECTION N.S. EQ

$\left.\begin{array}{l} r\text{-DIRECTION N.S. EQ} \\ \theta\text{-DIRECTION N.S. EQ} \end{array}\right\} Re \to 0$

SEQUENTIALLY CONSIDER 3 PDE'S USING AN ASSUMED FORM OF SOLUTION

$$\left.\begin{array}{l} V_r \sim f(r)\cos\theta \\ V_\theta \sim g(r)\sin\theta \end{array}\right\} \text{FROM B.C.'S}$$

THESE FORMS FOR V_r & V_θ WILL CAUSE TRIG FUNCTIONS TO CANCEL

EULER EQ:

NOTE: SAME SAME SAME SAME

$$r^{4} f^{iv}(r) + 8r^{3} f'''(r) + 8r^{2} f''(r) - 8r^{1} f'(r) = 0$$

$\therefore f(r) \sim r^{\alpha}$ SUBS AND GET A POLYNOMIAL

$$f(r) = \frac{C_1}{3r^2} + \frac{C_2}{r} + \frac{1}{2} r^2 C_3 + C_4$$

TABLE 3.4

(Continued)

Spherical coordinates

r direction

$$\rho\left[\cancel{\frac{\partial v_r}{\partial t}} + v_r\frac{\partial v_r}{\partial t} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\cancel{\frac{\partial v_r}{\partial \phi}} - \frac{v_\theta^2 + \cancel{v_\phi^2}}{r}\right] = -\frac{\partial p}{\partial r} + \cancel{\rho g_r}$$

Typo ∂r

$$+\,\mu\left[\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_r}{\partial r}\right)\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_r}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\cancel{\frac{\partial^2 v_r}{\partial\phi^2}} - 2\frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta} - \frac{2}{r^2}v_\theta\cot\theta - \frac{2}{r^2\sin^2\theta}\cancel{\frac{\partial v_\phi}{\partial\phi}}\right] \quad (3.3.28a)$$

θ direction

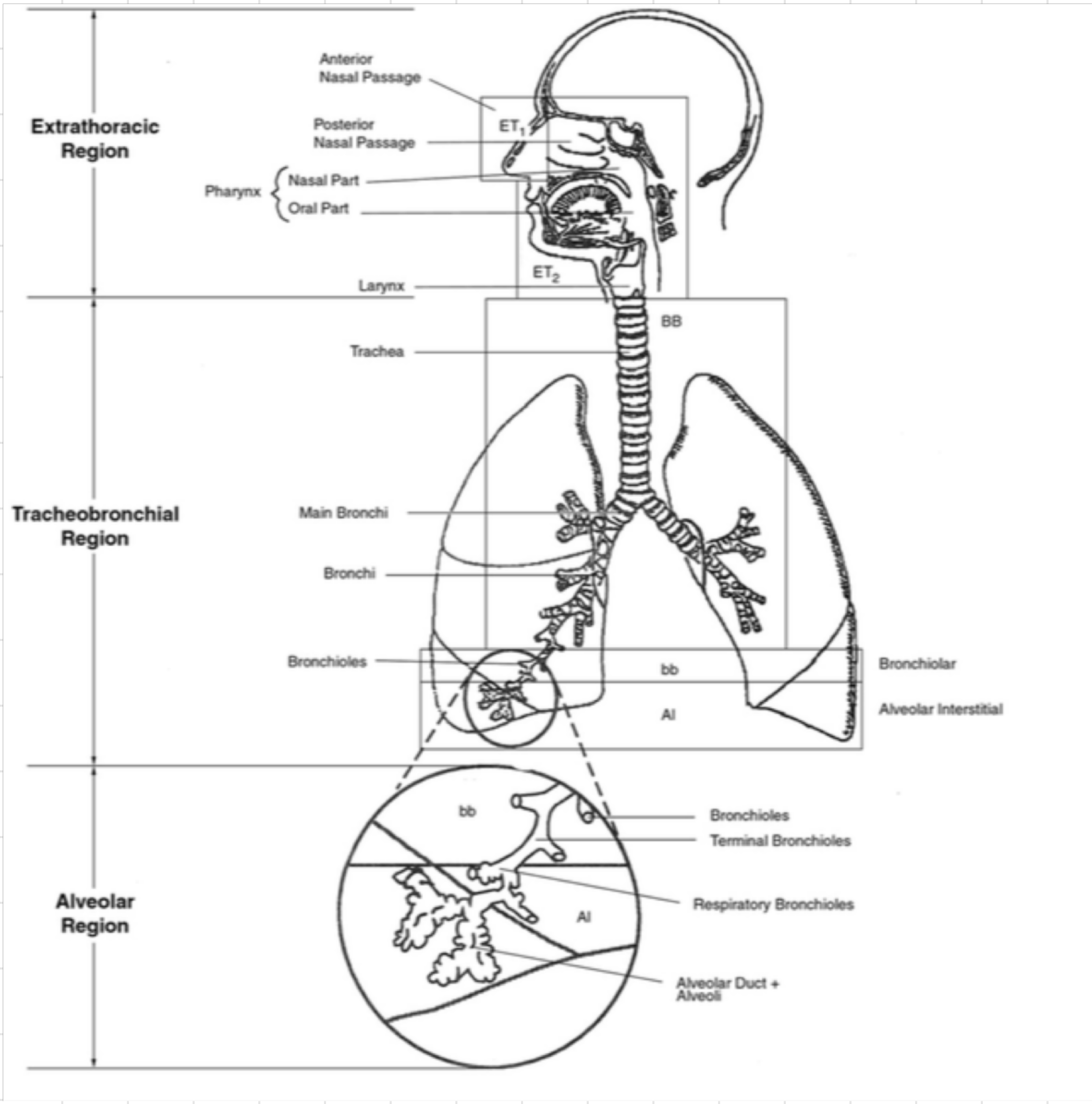
$$\rho\left(\cancel{\frac{\partial v_\theta}{\partial t}} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_\phi}{r\sin\theta}\cancel{\frac{\partial v_\theta}{\partial\phi}} + \frac{v_\theta v_r}{r} - \cancel{\frac{v_\phi^2\cot\theta}{r}}\right) = -\frac{1}{r}\frac{\partial p}{\partial\theta} + \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right)\right.$$

$$\left. + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_\theta}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\cancel{\frac{\partial^2 v_\theta}{\partial\phi^2}} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta} - \frac{v_\theta}{r^2\sin^2\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\cancel{\frac{\partial v_\phi}{\partial\phi}}\right] + \cancel{\rho g_\theta} \quad (3.3.28b)$$

ϕ direction

$$\rho\left(\cancel{\frac{\partial v_\phi}{\partial t}} + v_r\cancel{\frac{\partial v_\phi}{\partial r}} + \frac{v_\theta}{r}\cancel{\frac{\partial v_\phi}{\partial\theta}} + \frac{v_\phi}{r\sin\theta}\cancel{\frac{\partial v_\phi}{\partial\phi}} + \cancel{\frac{v_\phi v_r}{r}} + \cancel{\frac{v_\theta v_\phi}{r}}\cot\theta\right) = -\frac{1}{r\sin\theta}\overset{=0}{\frac{\partial p}{\partial\phi}} + \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cancel{\frac{\partial v_\phi}{\partial r}}\right)\right.$$

$$\left. + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\cancel{\frac{\partial v_\phi}{\partial\theta}}\right) + \frac{1}{r^2\sin^2\theta}\cancel{\frac{\partial^2 v_\phi}{\partial\phi^2}} + \frac{2}{r^2\sin\theta}\cancel{\frac{\partial v_r}{\partial\phi}} - \cancel{\frac{v_\phi}{r^2\sin^2\theta}} + \frac{2\cos\theta}{r^2\sin^2\theta}\cancel{\frac{\partial v_\theta}{\partial\phi}}\right] + \cancel{\rho g_\phi} \quad (3.3.28c)$$

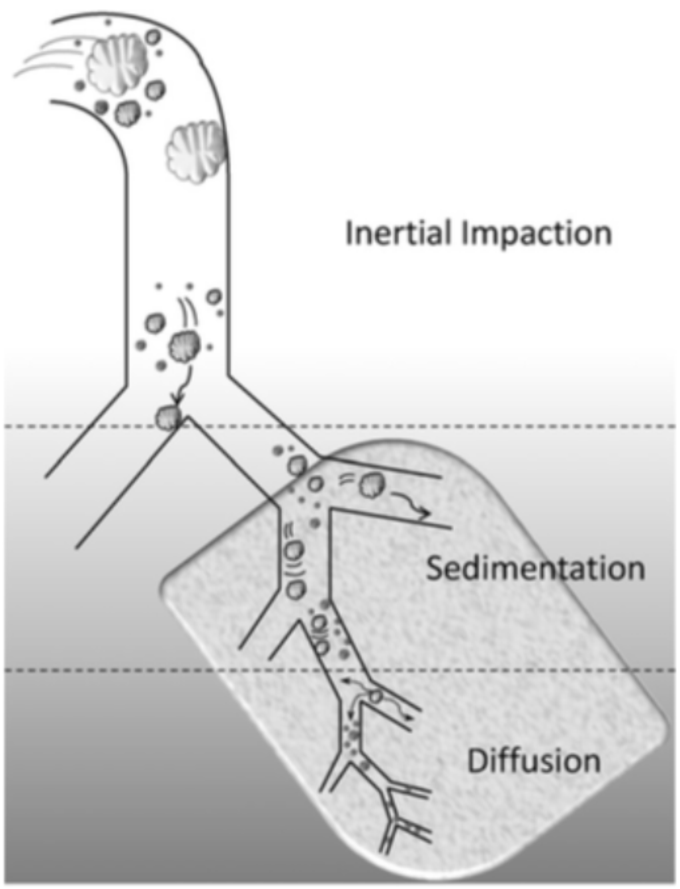


TORTUOUS PATH, PARTICLES STICK TO WALL: CLEARED BY CILIA.

LONG PATH: ONLY LAST ~5 BRANCHES ABSORB

OR, COULD BREATH BACK OUT.

PARTICLE CLEARING MECHANISMS



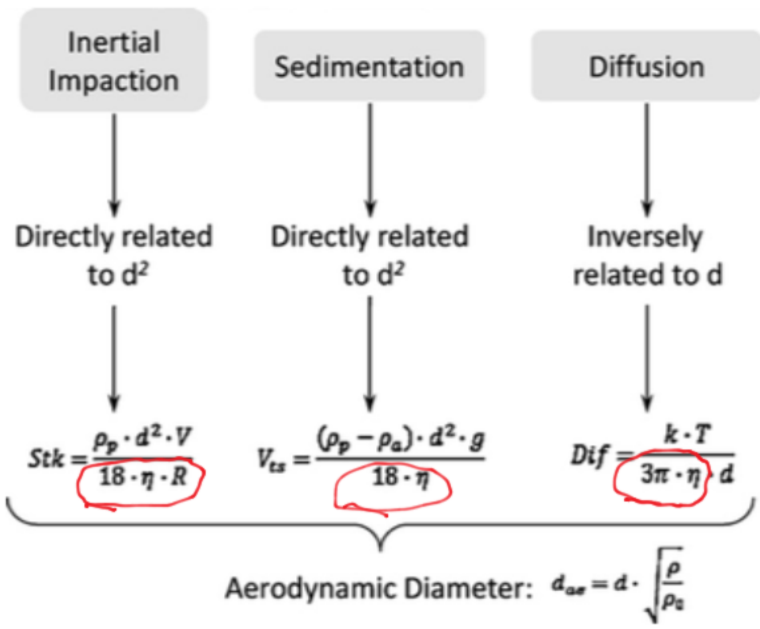


Fig. 2. The influence of particle size on deposition. *d*: particle diameter; *Stk*: Stokes number; ρ_p: particle density; *V*: air velocity; η: air viscosity; *R*: airway radius; V_{ts}: terminal settling velocity; ρ_a: air density; *g*: gravitational acceleration; *Dif*: diffusion coefficient; *k*: Boltzmann's constant; *T*: absolute temperature; d_{ae}: aerodynamic diameter; ρ_0: unity density.

B, mass, *m*, and velocity, *v*, according to Eq. (1) (Gonda, 2004):

WHERE DO EQUATIONS COME FROM ?

SOLUTION TO NAVIER-STOKES EQUATIONS FOR FLOW PAST A SPHERE : $Re \Rightarrow 0$

VISCOSITY OF A DILUTE ($\phi < .1$) SUSPENSION OF SPHERES

SUSPENSION

$$\frac{\mu_s}{\mu_f} = 1 + \phi\left(\frac{\mu_f + \frac{5}{2}\mu_p}{\mu_f + \mu_p}\right)$$

FLUID

VOLUME FRACTION OF PARTICLES

VISCOSITY OF PARTICLES

$\mu_p \to \infty$ FOR SOLID

$$\frac{\mu_s}{\mu_f} = 1 + \frac{5}{2}\phi$$

(ALSO DUE TO EINSTEIN)

EITHER DROPS OR BUBBLES WILL SIGNIFICANTLY INCREASE VISCOSITY

DRAG ON A BUBBLE:

$$F_D = 4\pi\mu R U$$

ALMOST AS LARGE AS A SOLID PARTICLE !!

VISCOSITY OF LIQUID IN LIQUID SUSPENSION WILL MOST LIKELY BE HIGHER THAN EITHER COMPONENT

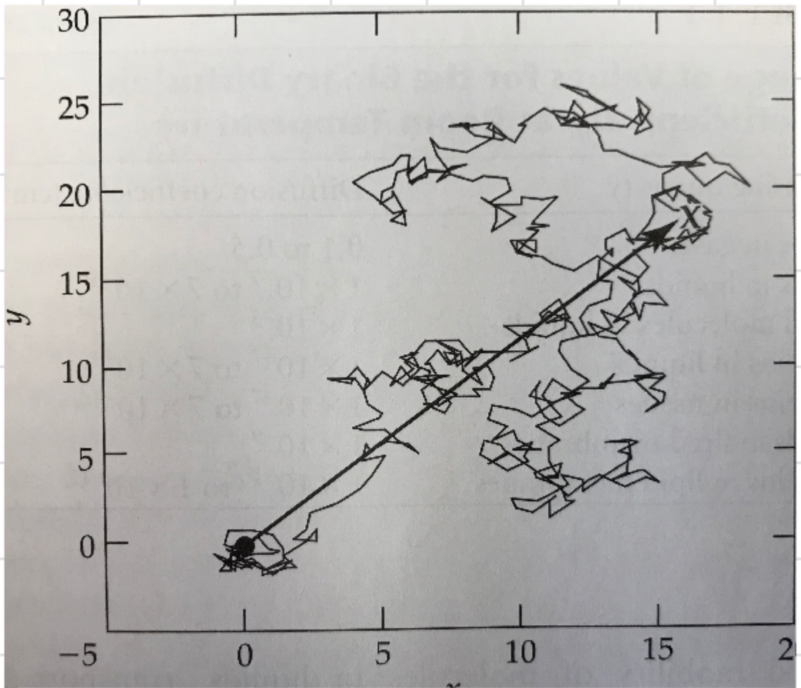


TABLE 1.1

Range of Values for the Binary Diffusion Coefficient, D_{ij}, at Room Temperature

Diffusing quantity	Diffusion coefficients ($cm^2\ s^{-1}$)
Gases in gases	0.1 to 0.5
Gases in liquids	1×10^{-7} to 7×10^{-5}
Small molecules in liquids	1×10^{-5}
Proteins in liquids	1×10^{-7} to 7×10^{-7}
Proteins in tissues	1×10^{-7} to 7×10^{-10}
Lipids in lipid membranes	1×10^{-9}
Proteins in lipid membranes	1×10^{-10} to 1×10^{-12}

DIFFUSIVITY

EINSTEIN USED DRAG TO CALCULATE DIFFUSIVITY OF A PARTICLE

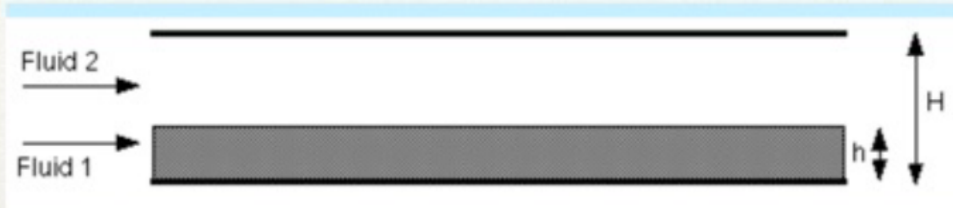
$$D = \frac{kT}{6\pi\mu R}$$

PARTICLE DIFFUSIVITY

BOLTZMANN CONSTANT

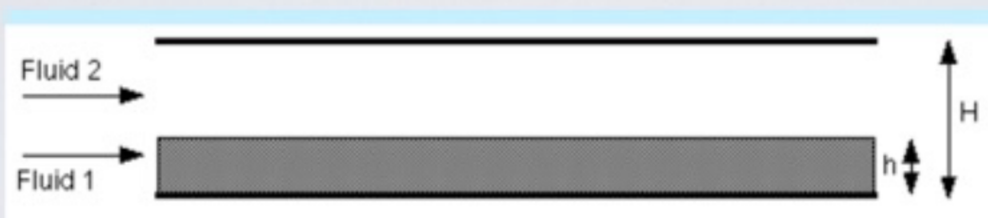
PARTICLE RADIUS

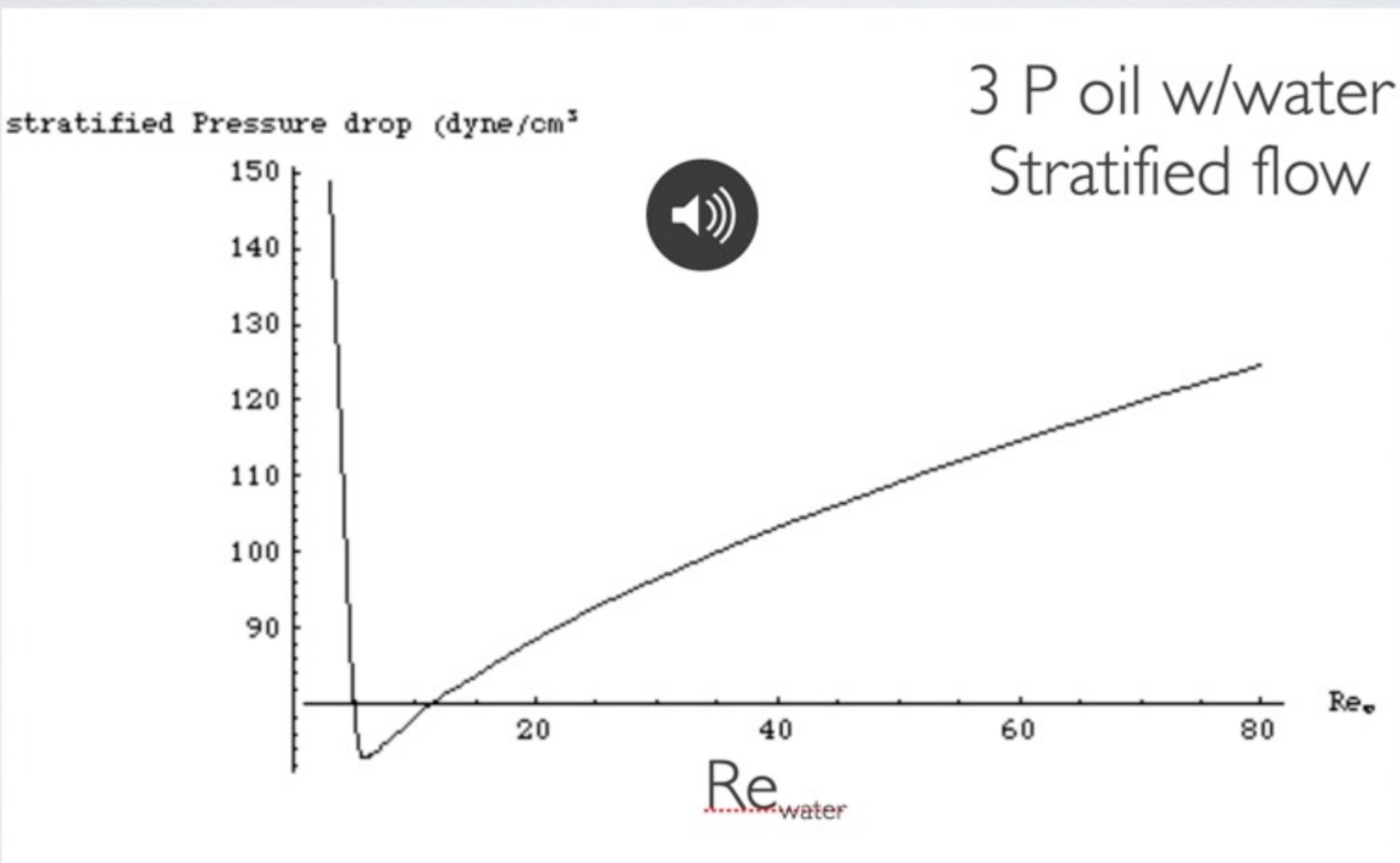
Two layer laminar flow



- It is easy to solve a laminar flow with two different liquids flowing.
- Suppose one of them is much more viscous than the other.
- We normally expect that the pressure drop increases as the flowrate increases…..

Increasing flow of water **decreases** pressure drop





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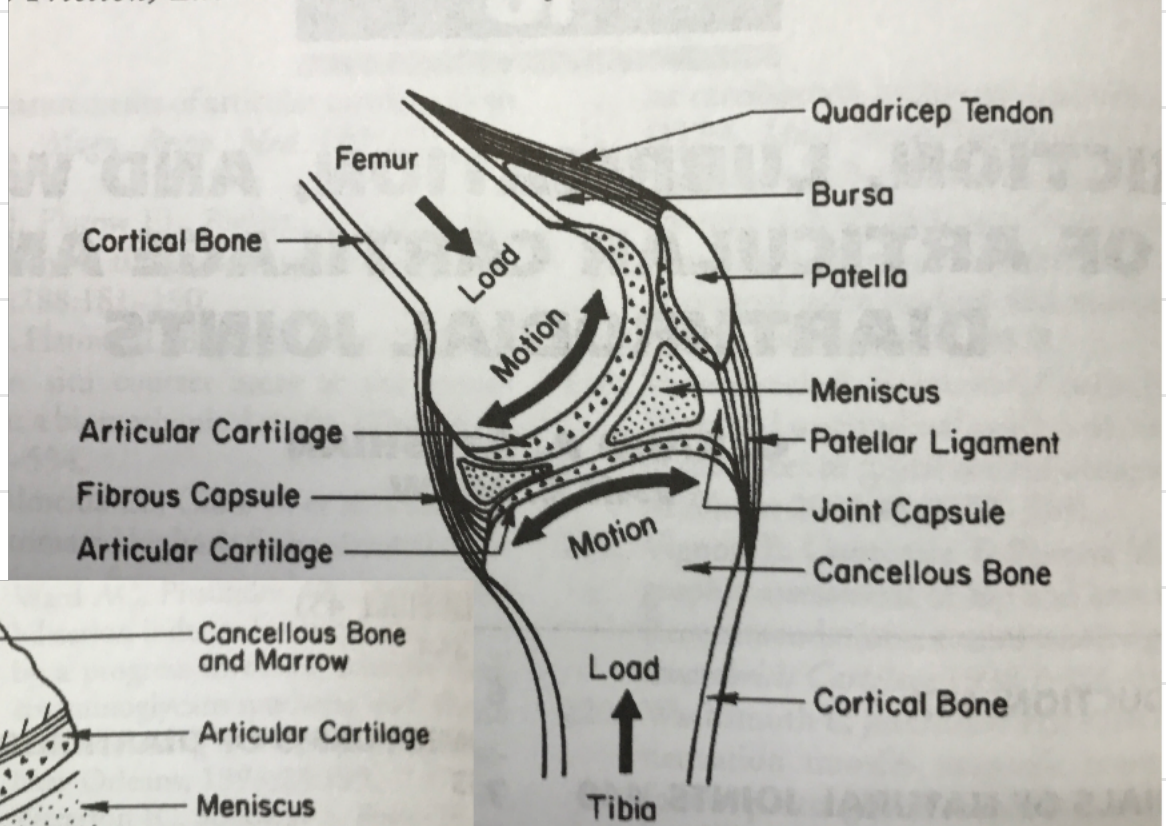
HYDRODYNAMIC LUBRICATION THEORY APPLIED TO:

DIARTHROIDAL JOINTS

10

FRICTION, LUBRICATION, AND WEAR OF ARTICULAR CARTILAGE AND DIARTHRODIAL JOINTS

GERARD A. ATESHIAN
VAN C. MOW



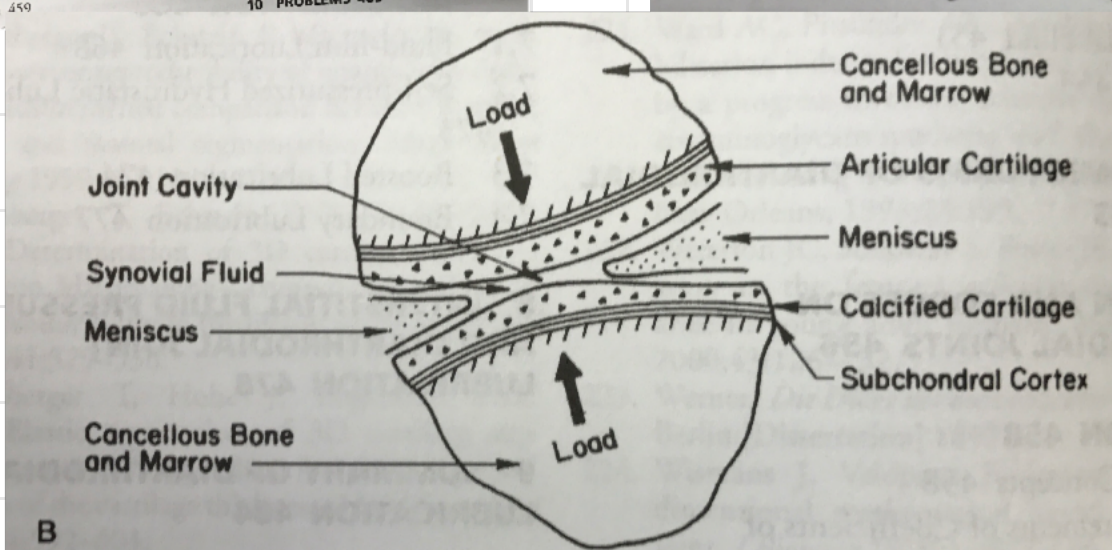
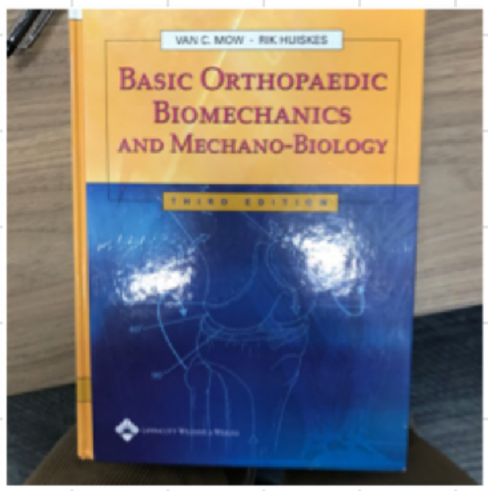


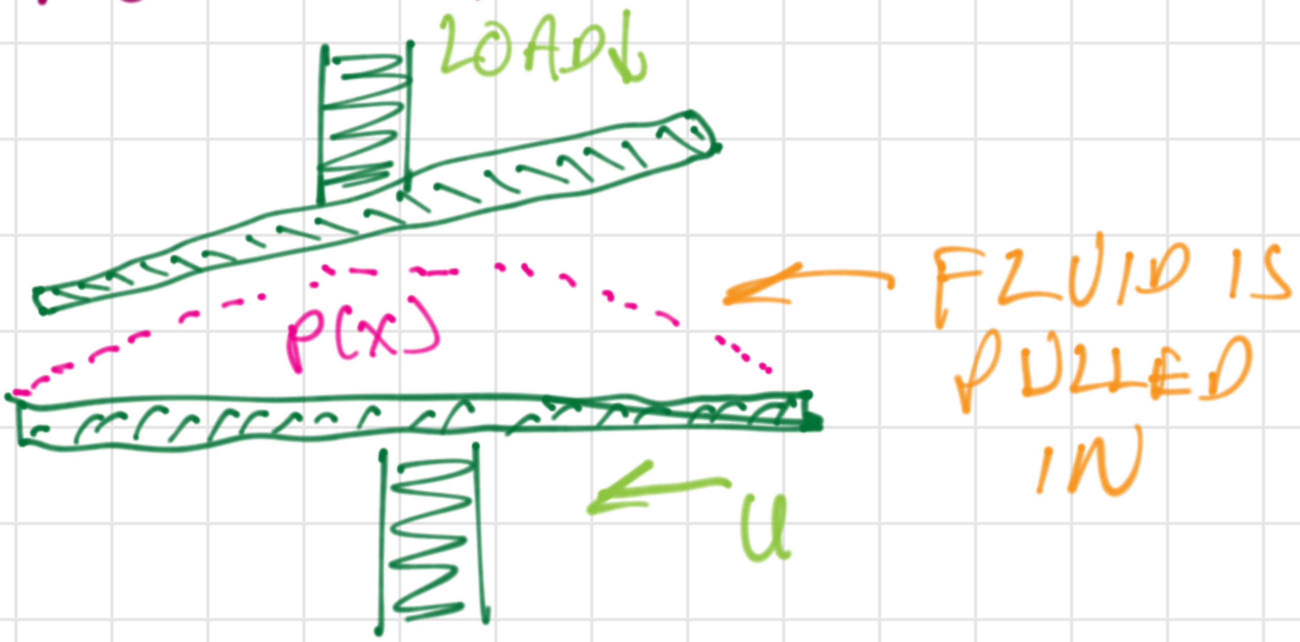
FIGURE 10-1. (A) Schematic representation of the human knee joint showing important anatomical features for mechanical function [171]. (B) Enlargement of the load-bearing region in the knee, depicting a thin layer of synovial fluid (<50 μm) and two layers of articular cartilage (each <7 mm) [9,171]. Each layer of articular cartilage contains approximately 80% fluid.



NONE OF THE CANDIDATE MECHANISMS WHERE LARGE MOLECULES TOUCH "SURFACES" GET THIS LOW:

SO CONSIDER

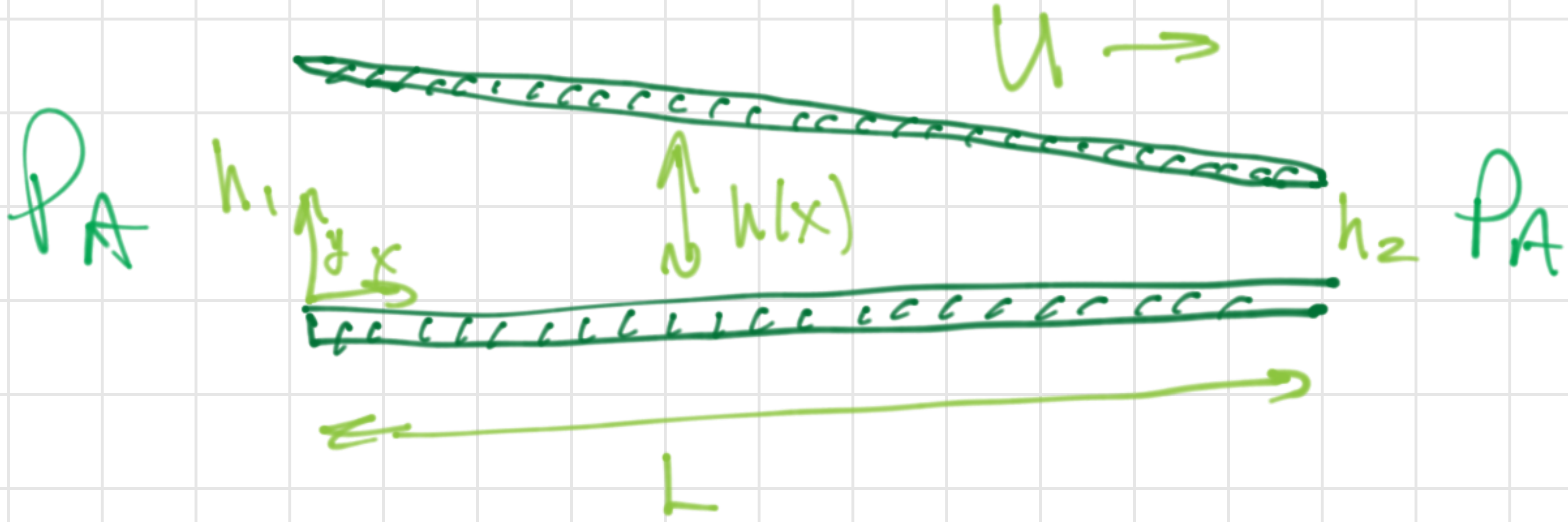
(ELASTO) HYDRODYNAMIC LUBRICATION



PRESSURE PROFILE IS RESULT OF LIQUID BEING FORCED THROUGH SMALL GAP !!

LUBRICATION ANALYSIS
(SECTION 4.7 IN TEXT)

CONSIDER THE "SLIDER" AS A SIMPLE EXAMPLE



$h \ll L$

"SLIGHTLY" NON-PARALLEL FLOW

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$\frac{U}{L}\frac{\partial v_x^*}{\partial x^*} + \frac{V}{h}\frac{\partial v_y^*}{\partial y^*} \Rightarrow V \sim \frac{h}{L} U$$

SAME RESULT AS BOUNDARY-LAYER