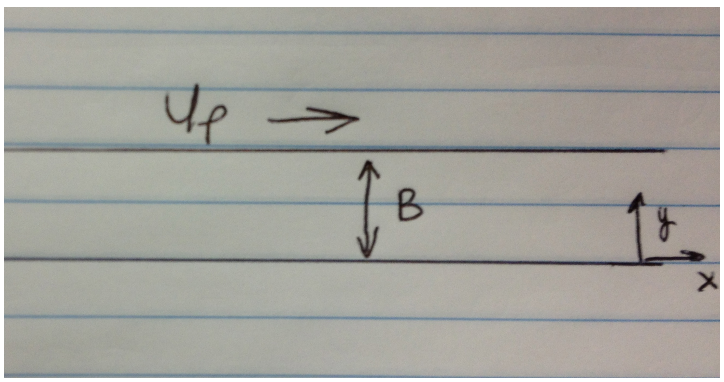
CBE 30357
Fall 2012
Final Exam
12/12/12

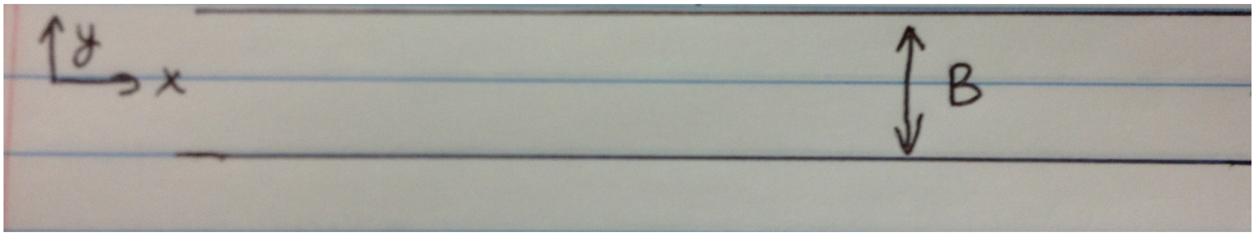
1. Flow between infinite parallel plates.

Consider the flow geometry shown here where a fluid is confined between two infinite parallel plates separated by a distance B.



a. Find the differential momentum equation, which would be valid for any fluid, that governs this situation if the flow is caused only by the motion of the top plate. You can do a shell balance or simplify the complete governing equations.
b. Is inertia important in this flow? If not, explain why?
c. For the case of a moving top plate, find the steady state solution for the velocity field for a Newtonian fluid with viscosity μ and density ρ.
d. Find the average volumetric flow rate.
e. Give an estimate of the time necessary to reach a steady state and defend your answer.

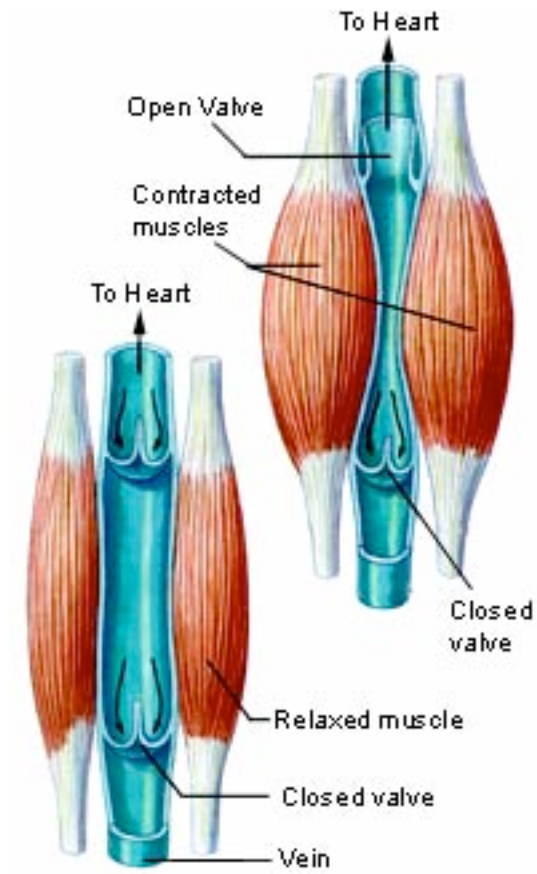
Now consider a pressure driven flow in the same geometry. Both plates are now stationary.



f. Find the differential momentum equation that governs this situation if the flow is caused by a pressure gradient. You can do a shell balance or simplify the complete governing equations.
g. Is inertia important in this flow? If not explain why?
h. Find the steady state solution for the velocity field for a Newtonian fluid with viscosity μ and density ρ.
i. Find the average volumetric flow rate.
j. Give an estimate of the *distance* from the inlet, when the profile is "flat", $v_x =$ constant, that is required to reach a steady state and defend your answer.

2. **Blood Flow.**

While your heart provides the pumping power to distribute blood throughout the arterial and venous system, by the time blood has passed through the capillaries, the pressure has dropped to only a few percent of its value in the large arteries. The walls of the veins in your extremities are very thin and thus can distend (stretch/expand in radius) and can allow *pooling* of a considerable amount of blood in your legs and feet when you are standing.



To minimize this pooling and to return blood efficiently to the heart, another mechanism is needed. This is accomplished by the *Skeletal Muscle Pump*. A drawing shows what it is and how it works is given here. (Presumably this is why, along with the lubrication issue in your knees, it is more comfortable to walk than to stand for extended periods of time. In particular if you are feeling "light-headed" while standing that flexing your leg muscles might improve the situation.)

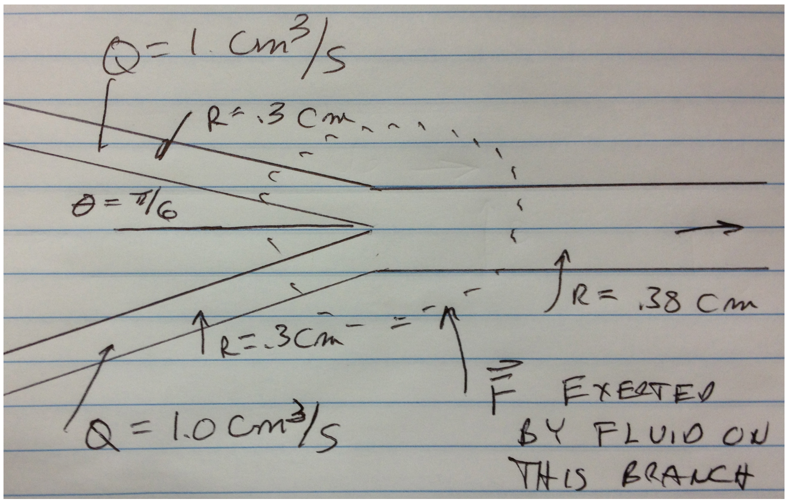
As you may recall, blood has an approximate viscosity, μ, of 3.5 cP (0.035g/(cm-s)), and a density, ρ, of 1.05 g/cm^3. The gravity constant, g, is 980 cm/s^2.

Consider a large vein inside the calf muscle (e.g., the *Posterior Tibial* vein) as a single entity (the scale of the skeletal muscle pump is smaller than this). The vein has a radius of 0.3 cm and the middle of the calf is 100 cm below your heart. For the purposes of this problem, consider that there is 10 cm between the two "check valves".

a. What is the difference in hydrostatic pressure for the blood between your heart and the center of your calf muscle?
b. How much force must be exerted by the muscle over a 10 cm length, to begin to pump the fluid?
c. Does the force change if the vein is distended and the <u>radius</u> is now 0.35 cm?
d. If the flowrate in the vein is 1 cm^3/s, what is the pumping power requirement to return the blood all the way to the heart if there are no viscous losses.
e. If you calculate a Reynolds number and get a friction factor, give an estimate for how important viscous losses will be for the blood return problem. There will be ~3 joining intersections and commensurate increases in diameter -- perhaps according to

Murray's law, but these may not significantly affect the answer. (So just use a 0.3 cm tube all the way to the heart for this calculation!)

f. Consider the return branch flow shown in the figure. What is the magnitude and direction of the force <u>on the branch</u>?

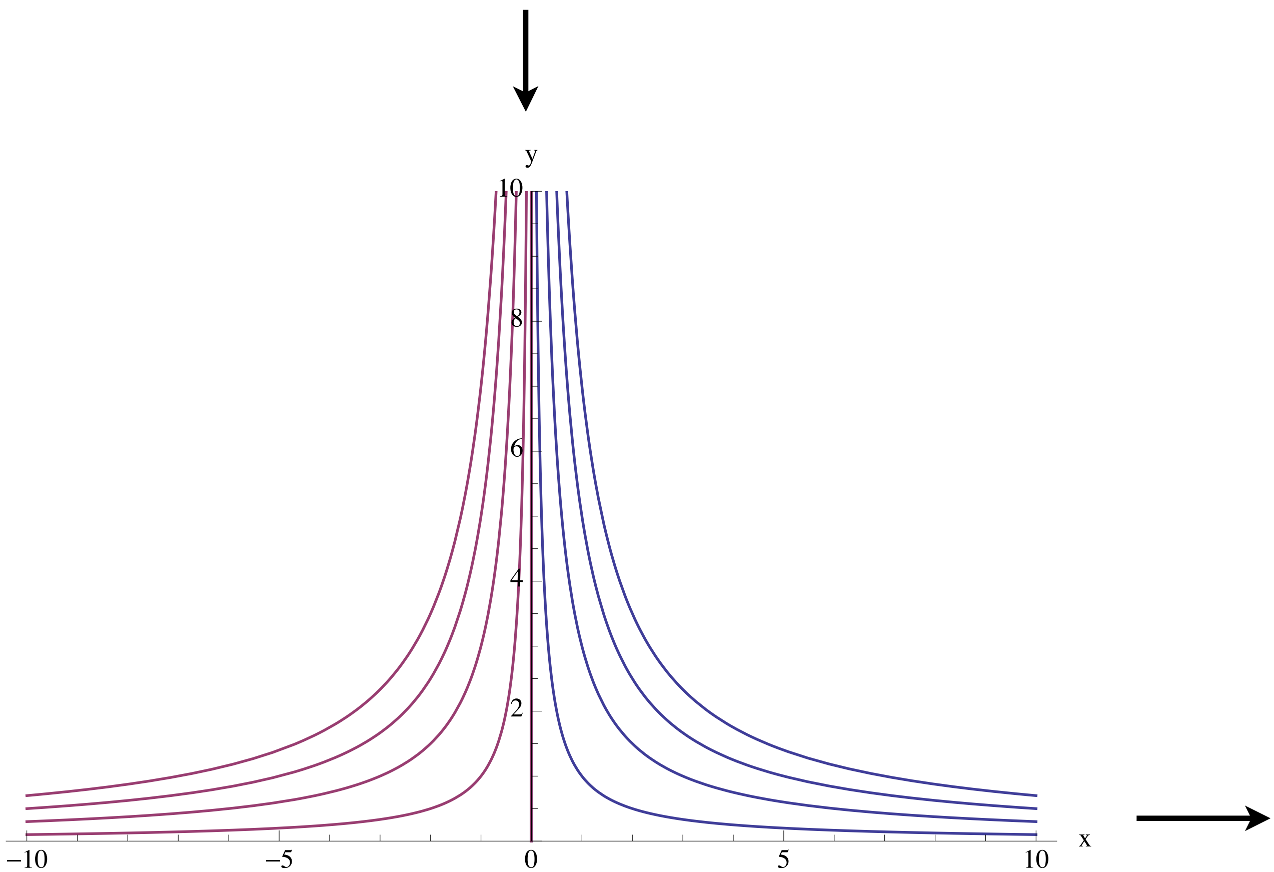


For this problem, we will consider blood flow to be steady laminar flow such that, $Q = \dfrac{\Delta p \pi R^4}{8\mu L}$, where Q is the volumetric flow rate, R is the tube radius, Δp is the pressure drop for length L, and μ is the fluid viscosity.

3. Stagnation point flow

Because of the interesting property of the boundary layer that will result, a stagnation point flow can be useful for growing a uniform biofilm with requires a large supply of oxygen and other nutrients. This geometry could also provide rapid and uniform cooling and oxygenation of blood. (Just be careful about the maximum stress on the cells.)

Consider a high Reynolds number flow impinging on a flat plate as shown here. For this problem the fluid will have 0 viscosity. Hence the no-slip condition is not enforced (Oh!). The lines are streamlines coming in from $y = +\infty$ and impinging on the plate.



The equation for the streamlines, is $\psi(x, y) = kxy$.

<u>For this problem, consider only the region where x and y are > 0!</u>

The x and y velocities can be calculated as

$$v_x = \frac{\partial \psi}{\partial y},\ v_y = -\frac{\partial \psi}{\partial x}$$

a. Show that this formulation satisfies the continuity equation for an incompressible flow.

b. What is happening to the flow field as k is changed? Calculate a few numbers to help explain.

We would like to determine the pressure along a streamline. As the Reynolds number is large, the Bernoulli equation will be valid. While we did not consider a two-dimensional flow in class, the proper formulation for this steady flow case, where there is no gravity, viscous losses or any input work is

$$\Delta\left(\frac{p}{\rho}\right)+\Delta\left(\frac{\left(v_x^2+v_y^2\right)}{2}\right)=0,$$

where Δ is the change in the values between two locations in x and y.

This equation can be used to find the pressure between any two points along a streamline (and actually between any two points in the flow field for this specific, idealized case.)

c. A convenient choice of “numbers” for a calculation is k = 1, $\psi=1$. Calculate the pressure (this will be the pressure change, but consider your first point to have p = 0), along this streamline. Where is the pressure the highest? (Watch the signs, highest means most positive value, not largest absolute value.)

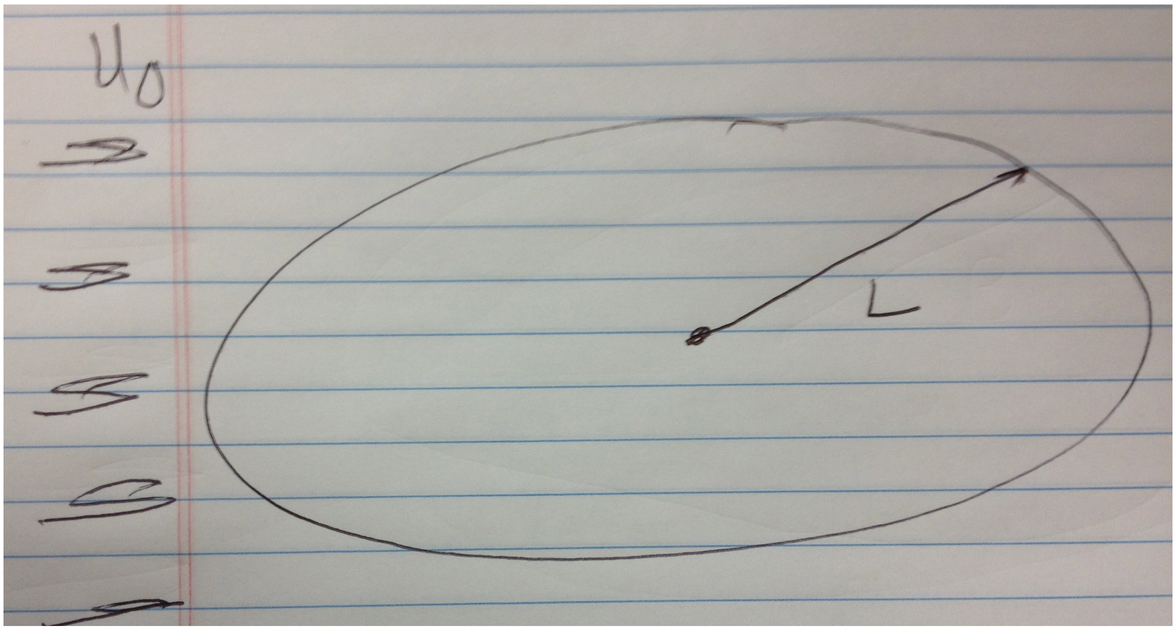
d. Explain why your answer makes sense based on other flows for which we have used the Bernoulli equation.

e. Where is the pressure highest if you consider all >0 and $y>0$? Explain why this is physically consistent with what we have said about the Bernoulli equation in the past.

Since this is a final exam…. we will come back to this flow in a minute….

4. Boundary - layer flows.

Consider the standard case of a high Reynolds number flow past a large solid body. We expect that a boundary-layer will be present starting at some point at the front (left side) of the body.



a. What are the important physical characteristics of a boundary layer?
b. Could a boundary layer exist for Reynolds << 1?
c. What key property do we calculate using boundary-layer analysis that we can't get from the high Reynolds number analysis of the entire flow field?
d. How does this last answer relate to the case of heat or mass transfer across a boundary layer?

Now, use nondimensionalization and scaling analysis to

e. Determine the important terms in the x-direction Navier - Stokes equation. (Choose x to be the main flow direction.)
f. Determine the order of magnitude of the y direction velocity in the boundary layer.
g. Explain why we can say that the "pressure from the flow outside the boundary layer is imposed on the boundary layer".
h. If the boundary layer is on a "flat plate", explain why there is no pressure gradient term.
i. In general, how do you solve a partial differential equation?
j. Provide the arguments that motivate our efforts to find a "similarity" variable that could be used to help solve this system of partial differential equations.

For a flat plate, we used a numerical technique to solve the equation,

$$f'''(\eta)+\frac{1}{2}f(\eta)f''(\eta)=0$$

For flow past a wedge, we solved

$$f'''(\eta)+\frac{m+1}{2}f(\eta)f''(\eta)+m\left(1-f'(\eta)^2\right)=0$$

The difference between these equations is that for a wedge, there is a pressure gradient in the x direction.

k. If the pressure is decreasing in the flow direction, as opposed to not changing, the boundary - layer thickness will decrease. For this case, what happens to the wall shear stress. Why?

l. For the stagnation flow above, the boundary layer thickness is constant for all x. Explain how this is consistent with the observed pressure gradient behavior that you have calculated in the problem above.