CBE 30357
Fall 2017
HW 9
Due 12/8/17

1. Set up of boundary-layer equations

Revisit the nondimensionalization and scaling arguments that lead to the “boundary-layer” equations for flow around a body of arbitrary shape. Do as little or much algebra as you need to answer the following questions. (see pp 188-191 in text for help on this.)

A. Which equation allows you to determine the magnitude of the velocity normal to the surface? Explain how it is used.
B. Why can the the second-derivative term be neglected in the x equation?
C. Explain why you need to keep both inertia terms in the x equation.
D. Why does the y-direction equation reduce to just the pressure gradient?
E. What further simplification of the x-direction equation occurs if the plate is “flat” (0 angle of incidence.)?
F. In general, explain how the Bernoulli equation provides specific information to enable solution of the x-direction momentum equation.

2. Startup of a parallel plate flow.

As usual you wish to solve your PDE by changing it to an ODE. For the *Heat equation* on a finite domain (and some other PDE’s that *arise* in mathematical physics) this can be achieved by using a “separation of variables” approach. The Mathematica Notebook shows this and helps you get the answers for the problem below.

a. Consider plates with a spacing b. The flow is in the the x direction with the direction normal to the bottom plate being y. Initially the fluid is not moving but at time = 0, the bottom plate is set to U0. What equation and boundary conditions govern this flow? Give justification for your answer.
b. Find the “exact” analytical solution. (Using Mathematica and perhaps the “plate_startup_2017” notebook is recommended!)
c. What scaling relation exists between the penetration depth of the disturbance and time?
d. Can <u>you</u> get the “coefficient” (i.e., the number in front of the algebraic relation) for the scaling relation from the exact solution?
e. What is the steady state solution? Explain how this comes from the complete time dependent solution?
f. “When” is t= infinity?
g. At t <<1, does the solution still work? What problem arises and how can you fix it?

3. Startup flow on an infinite domain

Now suppose that the "box" of fluid has infinite extent in the y direction. In this case the geometric length scale "b" is absent. Commensurate with this defining eigenfunctions with defined "frequencies" or wavenumber — the eigenvalues — is problematic. "The violin string is not pinned at the ends!"

Thus a different mathematical approach is needed. This problem is similar to the boundary-layer discussion in class where there was no "top" to the flow. The thickness of the boundary layer grew with distance along the plate. So δ was a function of x. In this problem, the depth of penetration of the disturbance will be growing in time by a rate prescribed by the kinematic velocity.

a. What length scale exists for this problem?
b. What special relation between the spatial dimension and time do you expect to exist on an infinite domain?
c. Show how to use a similarity variable to convert the pde to an ode (and combine two boundary conditions).
d. Find the analytical solution to this problem. (Solve yourself, or "find" it.)
e. Find a numerically accurate answer to "what is the relation between penetration distance and time".

4. Problems from boundary-layer calculations (see the Mathematica Notebook)

1. How does the boundary layer thickness change with Reynolds number ??
2. How does the wall shear stress change with Reynolds number ??
3. What is the primary physical characteristic of a boundary layers ??
4. Run the code to find is the value of η for which the velocity is 95% of the free stream.
5. Run the code to find the value of the stress at the wall for a flat plate. Can you get more significant figures? Check elsewhere to see if my value of f'''(0) == .332058 matches other calculations.
6. Run the code to show how the boundary-layer thickness and shear stress change when the imposed pressure gradient changes (i.e. the wedge problem) Find the angle at which separation occurs. Look up the solutions for flow around a cylinder. How does the angle for separation for the wedge compare to the angle of separation for flow around a cylinder (from "Boundary-Layer Theory, by H. Schlichting, p161, this occurs at θ = 108.8 as measured from the side that the flow impinges on the cylinder) Note that these angles are measured differently so make a sensible comparison.
7. The Reynolds numbers for cars, planes and boats is very large. What is the boundary layer thickness in real variables (say cm) 25 cm from the leading edge of

a. A WWII Spitfire airplane
b. A 787 landing
c. An "8's" racing shell
d. Your first new car cruising to your new job at 55 mph!

8. For different values of the wedge angle, find wall shear stress and compare the exact result to just using $\tau = \frac{U}{\delta}$
9. Explain the relationship between the variable for the suddenly started plate: $y/\sqrt{\nu t}$, and $\eta = \frac{y}{\sqrt{\frac{x\nu}{U}}}$ for steady flow past a flat plate.

Note that ν is "nu" even if the font is not clear.

5. Lubrication "theory"

a. It was mentioned in class that for either a "slider" (slightly miss aligned surfaces that are moving relative to each other) or the "squeeze film", (parallel surfaces with the spacing decreasing, it is essential for the spacing to be sufficiently small. Explain why quantitatively and qualitatively using the requisite equations.

b. Consider a "squeeze film" flow. The "Reynolds equation" 4.7.7 in the text is a good starting point. (note that above it in eq. 4.7.5, the two integrals should be "dy", not "dx".)

For the case of constant "load", find a relation for the speed of the plate convergence as a function of the gap spacing. Plot this for values that might be present in your knee (2 Poise for the fluid viscosity, 20 cm^2 of area, a gap from about .1 mm and lower.)

c. Find an expression that gives the spacing as a function of time for a constant load. Plot this as well for the "realistic" values given here.

d. Look up a "value" for the roughness of cartilage. While it certainly is not "sand glued to steel", note the limits of the calculation you are doing.