CBE30357
Fall 2017
Homework #6
Due 10/27/17

1. Pure radial flow

Consider the situation where a long, thin hypodermic needle is used to inject (red dyed… look it up if you are curious) kerosene into a large container of pure glycerine. The volumetric flow rate is held constant at a value of *Q*.

Solve the problem first without any numbers.

a. Find relations for the flow fields of the kerosine and glycerin. The container is big enough to consider that *r*—> infinity.
b. Find expressions for the pressure field inside and outside the growing drop. Note that in my copy of the text on page 134, the "v_r" equation has a typo in the second term. It should be $v_r \partial v_r/\partial r$. You can consider that far away from the drop, the pressure is p_0 and there there is no effect of gravity.
c. Find expressions for the viscous stresses at the interface of the drop.
d. How could the flow rate be adjusted to provide a constant τ_{rr} at the drop interface for all values of *R*?
e. Use the requisite fluid properties to find the pressure field everywhere and τ_{rr} at the interface if the volumetric flow is 1 cm^2/s and the drop radius is 0.1 cm.
f. Show (with numbers) that a typical inertia term really is smaller than a typical viscous term if the Reynolds number is small.

2. Rotating sphere at very low Reynolds number

Consider a sphere of radius *R* that is rotating (around the ϕ axis) with an angular velocity of 2π ω in a large quantity of fluid that is otherwise quiescent. The coordinate r, is the distance from the center of the sphere and the angle θ, measures the angle starting at the "north pole". (So the equator has a θ value of π/2. Note also the θ varies only from 0 to π while ϕ varies from 0 to 2 π.

a. Find all the non-zero terms of the Navier-Stokes equations if the velocity is only in the ϕ direction. (Ignore gravity.)
b. Find an expression for the velocity field for this case. You might find that the problem will go well if you assume that the velocity will have the form $v_\phi(r)\sin\theta$. (Substitute and try!) The resulting ODE is an "Euler" differential equation that will have solutions that are powers of *r*, $v_\phi \sim r^\alpha$. (Again substitute and try.) Once you do this, match the boundary conditions at *r* = *R* and *r* —> infinity.
c. Now suppose that the rotation causes fluid to be forced outward along the equation (and hence back in at the poles. This flow field will have *r,* θ and ϕ components, but still have perfect ϕ symmetry. Find all the non-zero terms of the Navier-Stokes equations if the velocity is only in the ϕ direction. (Ignore gravity.)

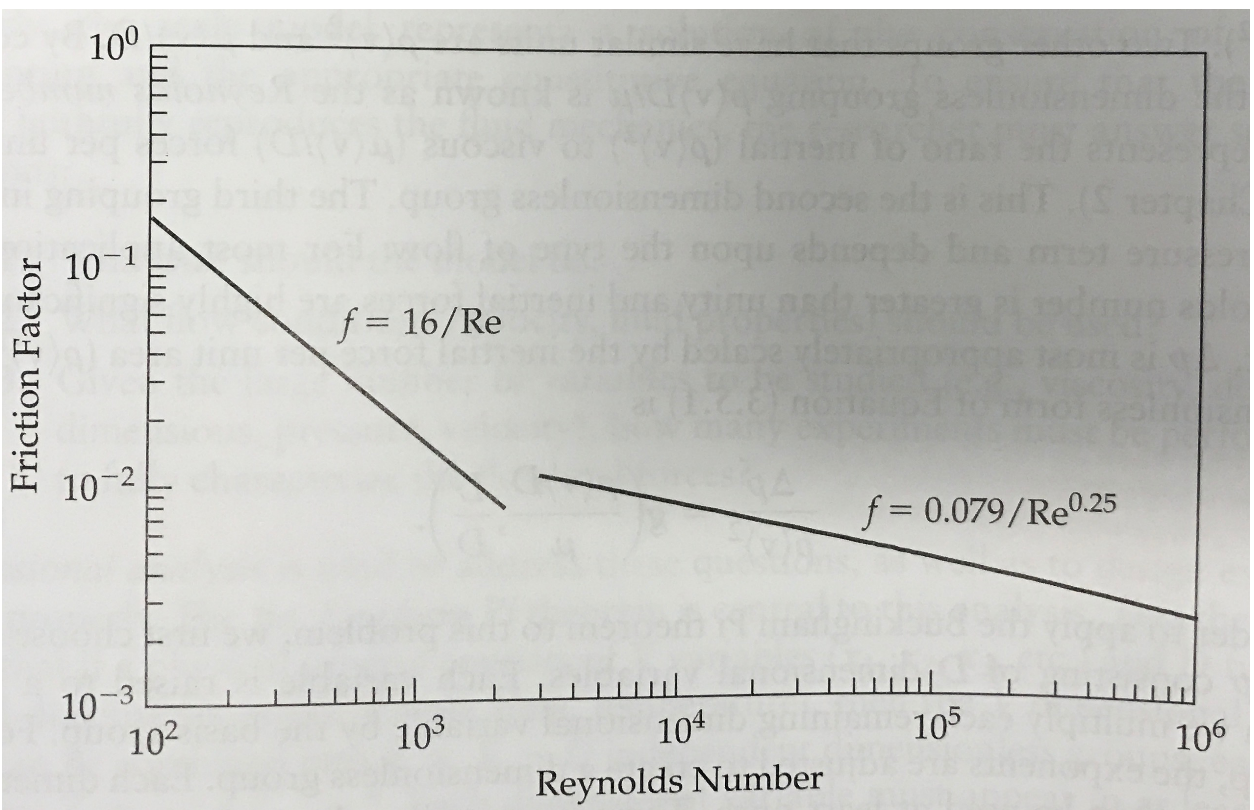
d. Considering either the v_r or v_θ equation, pick a typical inertia term and a typical viscous term (not a term with v_Φ) and determine what would need to be true so that inertia could be neglected compared to viscous effects.

3. Questions from the movie: Low Reynolds number flows

Even though the title of the movie (https://www.youtube.com/watch?v=51-6QCJTAjU) is "Low Reynolds Number Flows", there is demonstration of a **turbulent jet**.

a. Explain why for pipe flow, if the fluid flow is turbulent, the momentum transfer is more efficient (you have to pick the appropriate direction to make sense of the question) and hence why the pressure drop will be higher, at the same Reynolds number, than if the fluid flow were laminar.

b. By using the two standard relations for friction factor and Reynolds number, find the ratio of the pressure drop for turbulent flow to laminar flow in terms of the Reynolds number,



Lubrication (spinning “tetotum”)

c. We will do some analysis on this flow in a few weeks, but sketch the flow region that occurs between the paper and the solid surface and explain why in one direction this allows the weight of the “teetotum” to be supported by the air. Estimate the Reynolds number for this flow.

Reversibility

d. Explain why if the fluid has inertia, the flow field is not reversible. Then if the flow field is not reversible, why the dye blob experiment would not work.

Propulsion at Low Reynolds number.

A standard marine impeller relies on being able to expel a jet of fluid behind the boat, which causes a force in the opposite direction as well as the generation of lower and higher pressure regions along the blades of the propeller to add to the thrust.

e. Explain why neither of these mechanisms will work at Re <<1 and hence why the movie *Fantastic Voyage* has some scientific plot holes!

4. Use the Mathematica Notebook, “Creeping Sphere” to help answer the following questions.

a. Write down the pressure and viscous terms that are non-zero for flow around a sphere for (separately) the radial and tangential velocity
b. Write down separately) the non-zero inertia terms for the radial and tangential velocity.
c. When the equations have been non-dimensionalized, what parameter appears? What is its physical significance?
d. What is the specific argument that is made that allows neglect of the inertia terms?
e. What is the primary strategy for solving a PDE analytically?
f. What functional forms are “assumed” (chosen) for the radial and tangential velocities? Where do these forms come from?
g. How is pressure eliminated during the solution for the velocities?
h. What is an “Euler” differential equation? What is the solution form?
i. How is the pressure field obtained?
j. Does the velocity increase above the free stream value? Why/Why not?
k. Explain how this confirms the argument for neglecting the inertia terms (see part d).
l. Does the pressure increase above the free stream value?

5. Stokes-Einstein diffusivity

Estimate the diffusivity of:

oxygen, r_w = .21 nm
glucose, R_H = .47 nm
insulin R_H = 2.7 nm

in blood plasma for a healthy human, μ = 1,6 cP at 37C.

Note that R_H is the "hydrodynamic" radius of these molecules and r_w is the "van der Waals" radius.

How do these values compare to table 1.5 in the text? Is there any general realization that can be made about this particular way of relating molecule size to diffusivity?

TABLE 1.5

Relative Importance of Diffusion and Convection

Molecule	MW (g mol^{-1})	D_{ij} (cm^2 s^{-1})	Diffusion time, L^2/D_{ij} (s)	Pe = Lv/D_{ij}
Oxygen	32	2×10^{-5}	5	0.05
Glucose	180	2×10^{-6}	50	0.50
Insulin	6,000	1×10^{-6}	100	1.0
Antibody	150,000	6×10^{-7}	167	1.67

Particle	Diameter	D_{ij} (cm^2 s^{-1})	Diffusion time (s)	Pe
Virus	0.1 μm	5×10^{-8}	2,000	20
Bacterium	1 μm	5×10^{-9}	20,000	200
Cell	10 μm	5×10^{-10}	200,000	2,000

Note: For L = 100 μm, and if v = 1 μm s^{-1}, the time for convection is always equal to L/v = 100 s for all molecules and particles.