CBE 30357
Fall 2017
Homework #5 (revised)
Due 10/13/17

Here are a few questions that arise with fluid flow in living creatures and some related issues. These come directly from statements[1] in the book "Scale" by Geoffrey West.

1. Several times in early chapters he states that as blood vessels branch, say from 1 large artery that splits into 2 smaller arteries, such that the "total area" is preserved.

a. If the "mother" artery has a radius R, what is the radius of the two daughter arteries if the split is equal. (The reason for this type of split is "impedance matching" that reduces losses for pulsatile flows.)
b. However, he later states that blood flow slows down as it moves through the circulatory system. Is this consisted with the equal area branching behavior?
c. As part of the "slow down" observation, he states that we all know this because if we make a small cut in our finger it bleeds slowly but if we cut a major artery, the blood will shoot out fast[2]. (Hopefully you have not experienced the second of these scenarios first hand!) Explain why this observation could be consistent with or unrelated to the area ratios of different stages of the circulatory system.

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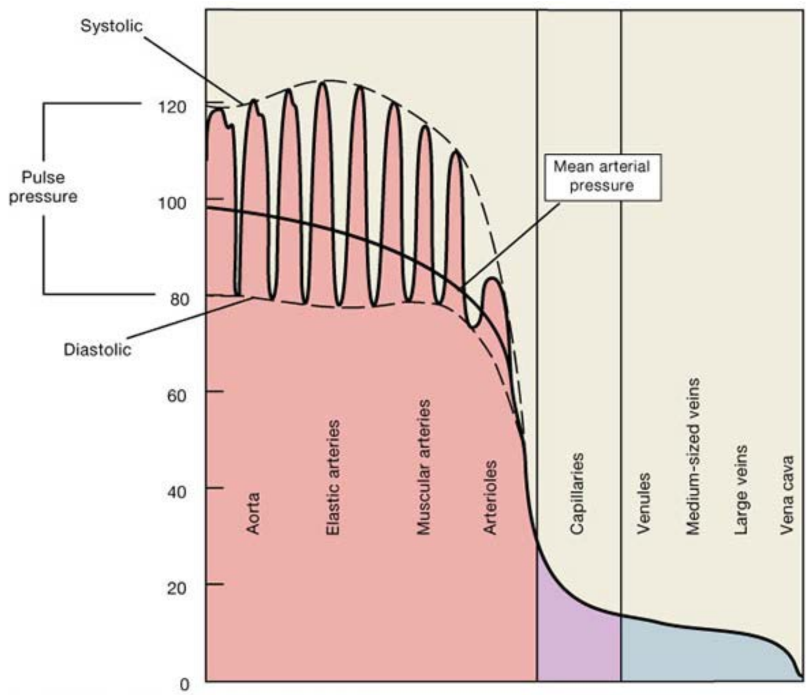
[1] Actually I take a little instructor's license in a couple of cases and write something that either he didn't exactly write or is not what he meant.

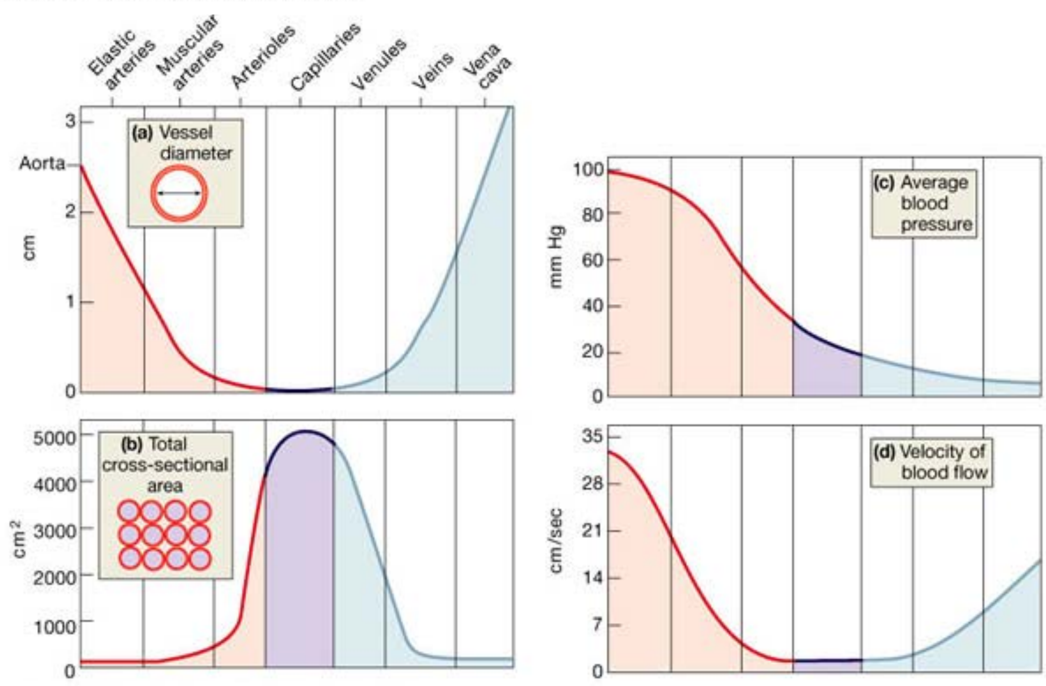
[2] "If you prick your skin, blood oozes out very slowly from the capillaries with scant resulting damage, whereas if you cut a major artery such as your aorta, carotid, or femoral, blood gushes out and you can die in just a matter of minutes.

West, Geoffrey. Scale: The Universal Laws of Growth, Innovation, Sustainability, and the Pace of Life in Organisms, Cities, Economies, and Companies (p. 125). Penguin Publishing Group. Kindle Edition. "

2. More blood flow

A some “cartoons” of the blood pressure along the human circulatory system are:





West states that because of the equal areas of the branching in the first several generations of blood vessels, fluid transmission is efficient and that most of the pressure losses for blood flow occur in the later generations of small blood vessels.

a. Is this statement consistent with the top cartoon — or maybe the right top one in the set of 4?

Let's consider the branching flows more generally — you will want to get your spreadsheet warmed up for these, unless you can derive a general formula.

b. Suppose that you have blood (μ = 3.5 cP, ρ= 1.06 g/cc) flowing in a tube of radius = 1.25 cm that is 10 cm long for which the Reynolds number is 1500, what is the pressure drop?
c. Suppose this tube is split into two equal tubes for which the flow area remains the same. Find the Reynolds number and the pressure drop for this generation if the length is the same.
c. Answer the question for 4, 8 and 16 tubes. Can you get a generation relationship for $\Delta p/dz$ of generation "n", where the number of tubes is 2^n to generation 0?
e. If the Reynolds number in generation 0 had been 2500 would anything change? That is, would the pressure drop in the first two splits be higher or lower than the single flow vessel.

3. Murray's Law

a. In the generations after the blood flow is no longer pulsatile, the branching changes such that the wall shear stress remains constant from one generation to the next. Find the relation for the change in radius for an equal split in this case[3]. This result is called Murray's law[4].

Here are some "data" for generations of blood vessels. The sums include all blood vessels of the designated rank (generation)

TABLE II

VESSELS IN TABLE I GROUPED ACCORDING TO RANK

Vessel rank	Σr^2	Σr^3	Σr^4
	mm^2	mm^3	mm^4
0	2.2	3.4	5.1
1	3.8	1.9	0.94
2	4.0	1.2	0.36
3	8.6	0.54	0.043
4	20	0.51	0.013
5	140	1.5	0.021
6	200	1.8	0.019
7	650	2.4	0.0095
6′	380	5.5	0.10
5′	200	7.6	0.30
4′	120	6.3	0.37
3′	39	5.8	1.1
2′	25	19	14
1′	22	26	31
0′	9	27	81

The vessels of Table I have been grouped according to rank and the sums of r^2, r^3, and r^4 have been calculated for each rank.

b. Can you see that perhaps area is preserved for the early generations and that Murray's law is followed for later generation? Give some evidence.

c. Answer the same question for Murray's law as is asked above: "*Can you get a generation relationship for dp/dz of generation "n", where the number of tubes is 2^n to generation 0?*" (It is easier to think of starting with a single tube even if this sort of branching starts in a later generation in a real circulatory system.

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[3] As a point of comparison: What is the split relation for constant area in terms of radius ratio?

[4] This result, by Cecil D. Murray, a biologist published in 1926 used basic principles of optimization — *operating versus capital costs* and was a decade or so ahead of chemical engineers who did the same for flows in process piping. ☹️

4. Etruscan Shrew

West notes that the smallest mammal is the Etruscan shrew that is about 4 cm long and has a heart that weighs 12 milligrams, but still produces an output pressure of something like 120 mm Hg. He says that its aorta has a radius is 0.1 mm. Its heart rate is more that 1000 beats/min. He notes that the oscillatory flow occurs only in the first 1 or two generations.

a. Estimate a Reynolds number for the flow in the shrew aorta.

West says that if a smaller mammal existed it would have a heart beat but no pulse. (How about a baby shrew?) There is no evolutionary advantage to being smaller because the efficiency of blood transport in the oscillatory range is lost— if there are no generations where the oscillatory flow exists.

b. Do insects have something like a circulation system and a heart? Doesn't this contradict the arguments of "no evolutionary advantage for being smaller?

5. Friction factor in rectangular channel

Find a relation between the friction factor and Reynolds number for a rectangular channel that has a height *h* (in y direction) that is very small compared to the width, *w*, (in the z direction). Consider the flow to be in the *x* direction.

6. Comparison of "exact" solution for flow in a rectangular channel with some approximations.

Use equations 3.4.27 and 3.4.28 from the text for this problem.

a. For channels with aspect ratios of 1, 2, 5, 10 and 20, calculate the flow rate for a range of Reynolds numbers (say 10-2000).
b. Use these results to calculate the friction factor versus Reynolds number relation using D_h, the *hydraulic diameter*[5] for the length scale.
c. Compare these results with with nominal relation, 16/Re.
d. Compare these results to the infinitely wide channel result (problem 1) using the channel height as the length scale.
e. How would you decide which approximate result to use?
f. Explain the behavior shown in figure 3.9 and more generally how the wall shear stress is changing with aspect ratio.

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[5] D_h = 4*(cross-section area of conduit)/ (perimeter of conduit)