CBE 30357
Fall 2017
HW #4
Due: 9/22/17

1. **Problem 2.20 in the text.**

2.20 *Rheology of Hyaluronic Acid* Hyaluronic acid is a high-molecular polymer of the repeating disaccharide units glucuronic acid and N-acetylglucosamine. Present in all tissues and usually linked with protein in the form of proteoglycans, hyaluronic acid is a component of the synovial fluid that is present between joints and is thought to play a role in joint lubrication and resisting compressive forces.

(a) The data in the table at the bottom of this page were obtained at 25°C in a cone-and-plate viscometer for solutions of 5 mg mL^{-1} hyaluronic acid in phosphate-buffered saline with a pH of 7.2. Determine whether the fluid behaves as a power law fluid, and if so, determine the parameters m and n.

(b) The hyaluronic acid solution was treated with the enzyme hyaluronidase for 5 minutes at 25°C, and the data on the right-hand side of the table were recorded. Determine the effect of the enzyme upon the rheology of hyaluronic acid.

(c) Explain why the enzyme might change the rheology of hyaluronic acid.

2. **Problem 2.21 in the text**

2.21 Consider a Bingham plastic flowing in a cylindrical tube of radius R. Flow is induced by the pressure gradient $\Delta P/L$. The tube is horizontal and flow is steady and fully developed.

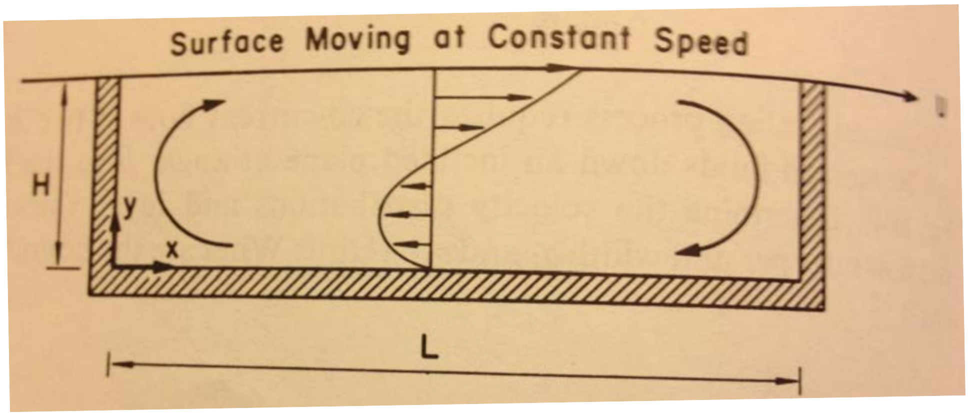
(a) Determine a criterion for the onset of flow.
(b) Determine an expression for the flow rate once flow begins. Compare with a Newtonian fluid of the same viscosity.

3. **"Falling film" (Newtonian) on the outside of a cylinder.**

Consider the steady flow of a Newtonian liquid with viscosity μ and density ρ, caused by gravity down the outside of a cylinder of radius, R.

a. Find an expression for the velocity profile as a function of R, g, μ, ρ and h, the liquid film thickness, (although we will sort its relation more generally in this problem).
b. Find an expression for the shear stress with in the film in terms of the same variables.
c. Find an expression for the volumetric flow, Q, using your velocity profile.
d. Explain if/how for a given value of Q, the liquid film thickness and average liquid velocity are (both?) determined.
e. Taking "King Size Canary" (https://www.youtube.com/watch?v=wjXkc5iC7gU) as motivation, which variable ratio, in what limit, gives the situation where the curvature of tube will become less and less important?
f. Show the in this proper limit that your solution for flow down the tube matches the analogous solution for flow down a flat plate.
g. If the liquid is water with a thickness of 1 mm, what is the average velocity, the velocity at the air-water interface and the total volumetric flow rate if the tube radius is 10cm?
h. If the liquid is water with a thickness of 1 mm, what is the average velocity, the velocity at the air-water interface and the total volumetric flow rate if the tube radius is .3 cm?

4. Moving surface flow in a confined region



a. Determine the differential equation that will govern the velocity profile for this flow by examining the complete differential equations for a Newtonian fluid (Navier-Stokes equations) and selecting the relevant non-zero terms. Some hints are that gravity will not be considered and that <u>you will be looking for a solution in the middle of the box, far away from the vertical walls</u>. As such, the velocity will be only in the x direction and will not be changing in the x direction.

b. Solve the equation for the velocity to find a the velocity profile. Note that the top surface is moving at the constant speed, U and velocity at the bottom of the container is 0. To finish the problem you will need to figure out something "profound" about the pressure gradient as is not "given" and not prescribed from conditions outside of the container.

Drawing from M. M. Denn Process Fluid Mechanics, Wiley 1979.