CBE 30357
Fall 2017
Homework #3
Due 9/15/17

1. A problem that can arise when physiological fluids, such as blood, are analyzed is that the various components can separate. The viscosity of plasma at 37C is about 1.3 cP. A nominal value of blood viscosity is 3-4 cP although this is not constant. (You can see the viscosity of a model fluid that contains red blood cells shown in figure 2.32. The viscosity of blood varies with the shear rate.) Consider that blood viscosity will be measured in a parallel plate geometry where one of the plates will be moving. The spacing between the plates is "*b*" and the plate velocity is U_o.

   a. Write or derive the differential equation for the momentum/stress distribution within the device when the top plate is moving. (For consistency, let's pick the moving direction as *x* and the normal direction as *y*. Thus, everything is uniform in the *z* direction.)

   b. If the gap is filled with a homogeneous fluid of viscosity, μ , determine the equation for the velocity profile.

   c. If the blood viscosity is 3 cP, and you want the plate velocity to be 5 cm/s, what is a reasonable value for the gap between the plates? (You don't want to lyse the blood cells.)

   d. Now suppose the the hematocrit separates from the plasma. The plasma viscosity is μ_P and the hematocrit plasma is μ_H . Find the equation for the stress profile.

   e. For part d, derive the equation for the velocity profile.

   f. If the hematocrit fraction is 0.45 and the hematocrit viscosity is 6 times the plasma viscosity, would you measure the wrong value for the blood viscosity? Explain why or why not.

2. Investigation of breathing constriction — Asthma symptoms

For people with such sensitivity, Asthma symptoms — that cause constrictions of airways — can arise either from cold dry air or hot humid air. Since at some point along the path to the lungs, the air will have cooled or warmed to body temperature and reached 100% water saturation. While the major resistance to air intake may not be in the trachea, the sensitivity probably does originate there because by the time air gets further into the lung it has increased in temperature and humid and hence would not elicit the airway constriction.

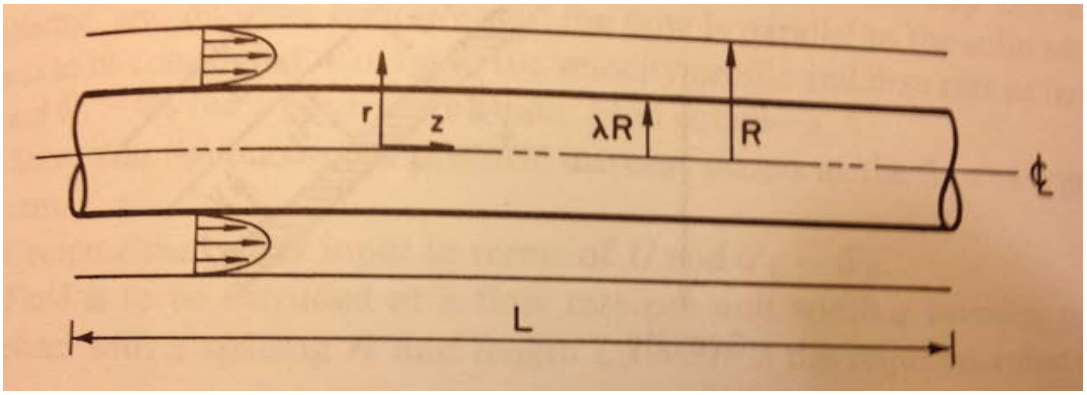
Consider the following questions about air flow in a trachea that has a diameter of 2 cm and is 11 cm long.

Suppose the tidal volume for a breath is 0.75 l and the time of inspiration was 1.5 sec.

a. What is the value of the Reynolds number for this flow?
b. If we assume that the trachea is a straight cylindrical pipe of constant diameter, find an expression for the velocity profile starting with the Navier-Stokes equations.
c. Use this result to find a relation between the flow rate and pressure drop for flow in a cylindrical geometry.
d. Find the definition of the friction factor and use the result from part c to find the equation that relates friction factor to the Reynolds number.
e. What pressure drop is necessary to cause the breathing in flow that is prescribed (i.e., .75 l in 1.5 s)?
f. If a constriction occurs such that the diameter is decreased by 5%, find the new pressure drop and compare to the original as a ratio.
g. For only a 5% change in diameter, why the pressure drop change so much more than this?
h. If the flow were turbulent instead of laminar, what would the pressure drop for part e?

3. Investigation of the flow "outside" of a circular tube

Consider the flow of a Newtonian fluid in the geometry shown below[1] that models flow past a catheter inserted in a blood vessel.



a. Determine the differential equation that will govern the velocity profile for this flow by examining the complete differential equations for a Newtonian fluid (Navier-Stokes equations) and selecting the relevant non-zero terms. Some hints are that gravity will not be considered, no velocity should be induced in the r direction and there is complete symmetry in the theta direction.
b. Find an equation for the velocity field.
c. Find an equation for the volumetric flow.

Consider flow of blood ($\rho = 1.05\text{g/cm}^3, \mu = 0.035\text{g/(cm-s)}$ in a 2 cm long section of a (circular) coronary artery for which for d = 0.28 cm, and the flow rate is 1cc/s.

d. What is the pressure drop in this section of artery if there is no catheter in it?
e. Now consider the same artery with a 1 mm catheter inserted (as in the figure above) if the flow rate remains the same, what is the pressure drop?
f. If the pressure drop is the same as the clear artery, what is the flow rate?

[1] Figure from M. M. Denn Process Fluid Mechanics, Prentice-Hall 1980.

4. Comparison of flow flow inside and outside of an artery

Recall the video for cerebral spinal fluid in the brain where the claim is made that the CSF flows along the outside of arteries. (It may be some years before this is confirmed.) This flow is motivation for us to compare the flow inside and outside of cylindrical blood vessels.

a. For equal pressure change and equal length and for the same fluid, what value of λ will give an equal flow rate inside and outside the vessel for the geometry of problem 3?

b. Does the numerical value of R matter in your answer?