CBE 30357
Fall 2017
Homework 2
Due 9/8/17

1. Fluid flow in the Atmosphere

You have no doubt heard of the consequences of Hurricane *Harvey* on Texas. If you have looked at the satellite photos or listened to weather forecasters or had occasion to study the issue, you may know that large storms rotate counter-clockwise in the Northern Hemisphere and clockwise in the Southern Hemisphere. The question that arises is, why?

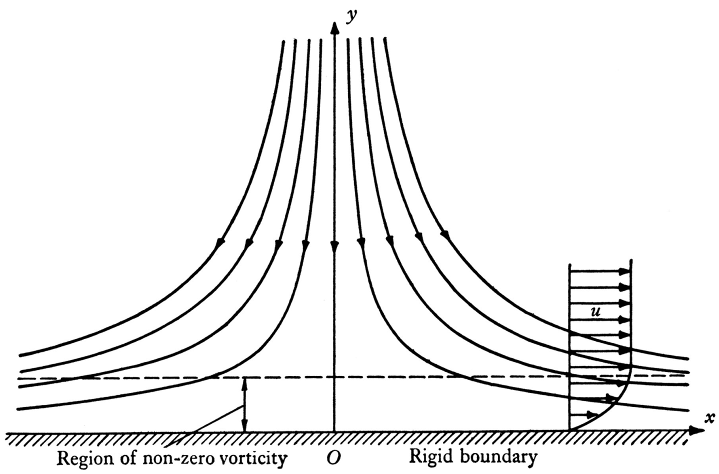
We will be finishing the derivation of the different equations for momentum this week. The development in the text is in the beginning of chapter 3. So you will want to read this up to about p 134. (To complete the derivation we will be referring to chapter 2, up to section 2.4.2. So please read this as well.)

The answer to this question is contained in the differential equations of motion — which govern fluid (in this case air) over any length scale. If you compare the left side of the equations in table 3.4 for Cartesian to either cylindrical and spherical coordinates, you will note that there are terms in the cylindrical and spherical equations that have no analog in the Cartesian equations. Specifically, you will see terms that contain, v_θ, or v_ϕ, that represent rotating flows, but that are different in mathematical form than any of the left side terms in the Cartesian equation.

A. Find these terms and write them down for both cylindrical and spherical coordinates and note from which of the vector component equations they come.
B. Use some information source to give "names" to each of the terms.
C. If any of these terms are non-zero, is the fluid *accelerating* — even if there is no change in time for the flow?
D. If the fluid is accelerating, must this be caused by a force? — (for simplicity consider cylindrical coordinates and a liquid flow with only a nonzero v_θ component.)
E. Which coordinate system is most convenient to use when describing atmospheric flows on the scale of miles?
F. Which of the terms could be associated with the fluid flow in the earth's atmosphere that is associated with the rotation of the earth?
G. Explain how the rotation of the earth causes an apparent deflection of trajectories of moving objects to the right in the Northern hemisphere?
H. Given this, explain the origin of the <u>counterclockwise</u> rotation of <u>low</u> pressure regions in the Northern Hemisphere.

2. **Stagnation point flow:**

Suppose that a liquid is flowing directly into a solid wall such that velocity in the y direction is -U far from the wall (located at y=0) which is a "y-face" and hence forces the flow in the x direction.



(The z direction is uniform and there is no flow in the z direction.) The "lines" are called "streamlines". These are determined such that no flow is occurring across a streamline.

a. Write down the mass balance equation for this flow field.
b. At large y and anywhere where there is no x direction flow, explain why the "y" velocity cannot change with y.
c. In the region, where there is no x velocity, what is the direction of the streamlines?
d. Show that if a scalar function, ψ, a "stream function" is defined, then it is possible to write:

$$v_x = -\frac{\partial \psi}{\partial y}, v_y = \frac{\partial \psi}{\partial x}$$

e. List any restrictions to the validity of this formulation.
f. If the functional form for $\psi = kxy$, what value of k matches this flow field?

The stream function is useful for visualization and is sometimes convenient for analysis or numerical computation.

3. "Couette" Flow

Consider the flow between two flat, parallel plates are separated by a distance, *b* in the *y* direction. The top plate moves in the *x* direction with a speed *U*. The fluid is Newtonian.

a. Find the simplified momentum equation for this flow?
b. Find an expression for the shear-stress between the plates.
c. Which form of the Navier-Stokes equations describes this flow?
d. What are the boundary conditions?
e. Find a relation for the velocity field if the viscosity is μ.