CBE 30357
8/29/17

SPECIFIC QUESTIONS THAT MOTIVATE STUDY OF FLUID FLOW

- WHY DO BLOOD VESSELS "BRANCH" AS THEY DO?
- SAME QUESTION FOR AIR PASSAGES IN YOUR LUNGS
- WHAT PRESSURE CHANGE IS NEEDS TO CAUSE A REQUIRED FLOW
- HOW MUCH "STRESS" DOES THIS PLACE ON A BLOOD VESSEL?

REVIEW OF LAST WEEK...

- OXYGEN SOLUBILITY IN BLOOD...
- TRANSPORT PHENOMENA IS UNIFIED STUDY OF MASS, ENERGY + MOMENTUM TRANSFER

O_2 SOLUBILITY IN BLOOD

PHYSICAL SOLUBILITY

"HENRY'S LAW"

$$C_{O_2} = H_{O_2} P_{O_2}$$

HENRY'S CONSTANT

PARTIAL PRESSURE

LIMITED TO LOW CONCENTRATION ~ PPM, "FLAT EARTH", FIRST TERM IN TAYOR SERIES..

TO SUSTAIN ROBUST LIFE O_2 SOLUBILITY MUCH HIGHER THAN HENRY'S LAW RANGE IS NEEDED

- CHEMICAL COMPLEXATION

A.K.A.: HEMOGLOBIN

THE EQUATION O_2 IN BLOOD IS

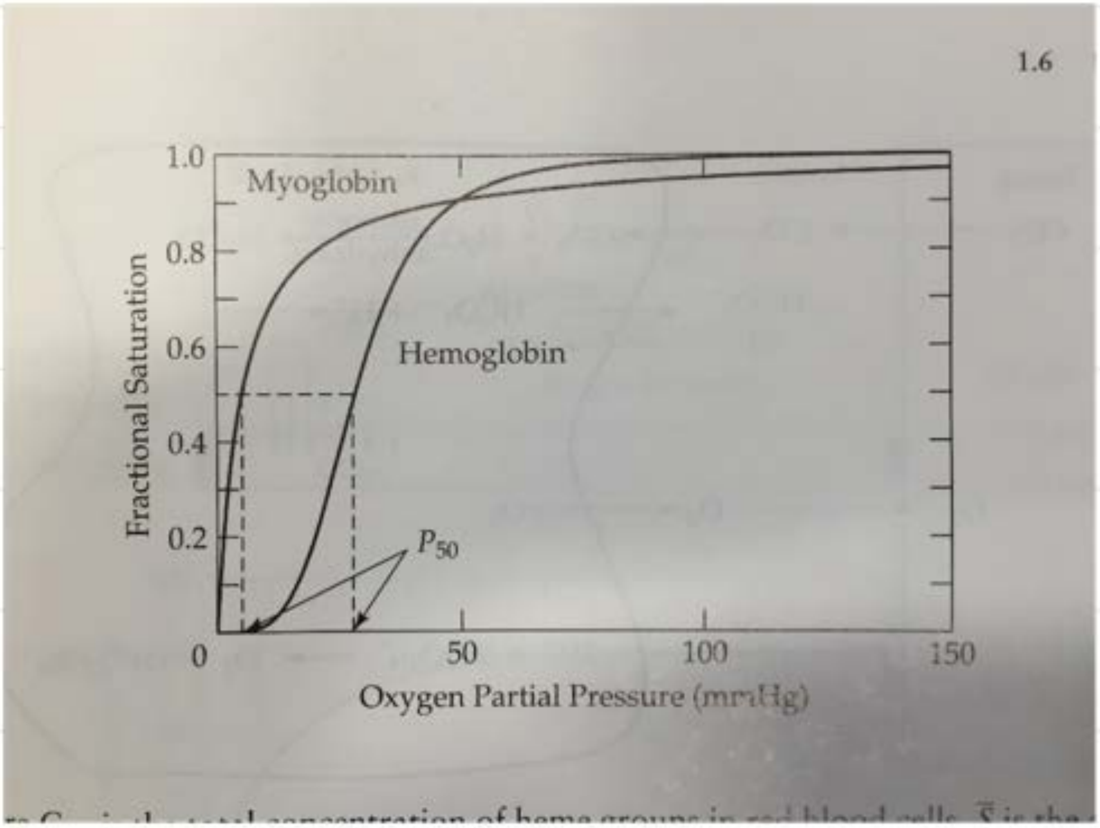
$$C_{O_2} = \underbrace{H_{O_2} P_{O_2}}_{\text{PHYSICAL SOLUBILITY}} (1 - H_{ct}) + \left(\underbrace{4 C_{Hb} \bar{S}}_{\text{CHEMICAL COMPLEXATION}} + \underbrace{H_{Hb} P_{O_2}}_{\text{PHYSICAL SOLUBILITY}} \right) H_{ct}$$

PHYSICAL SOLUBILITY

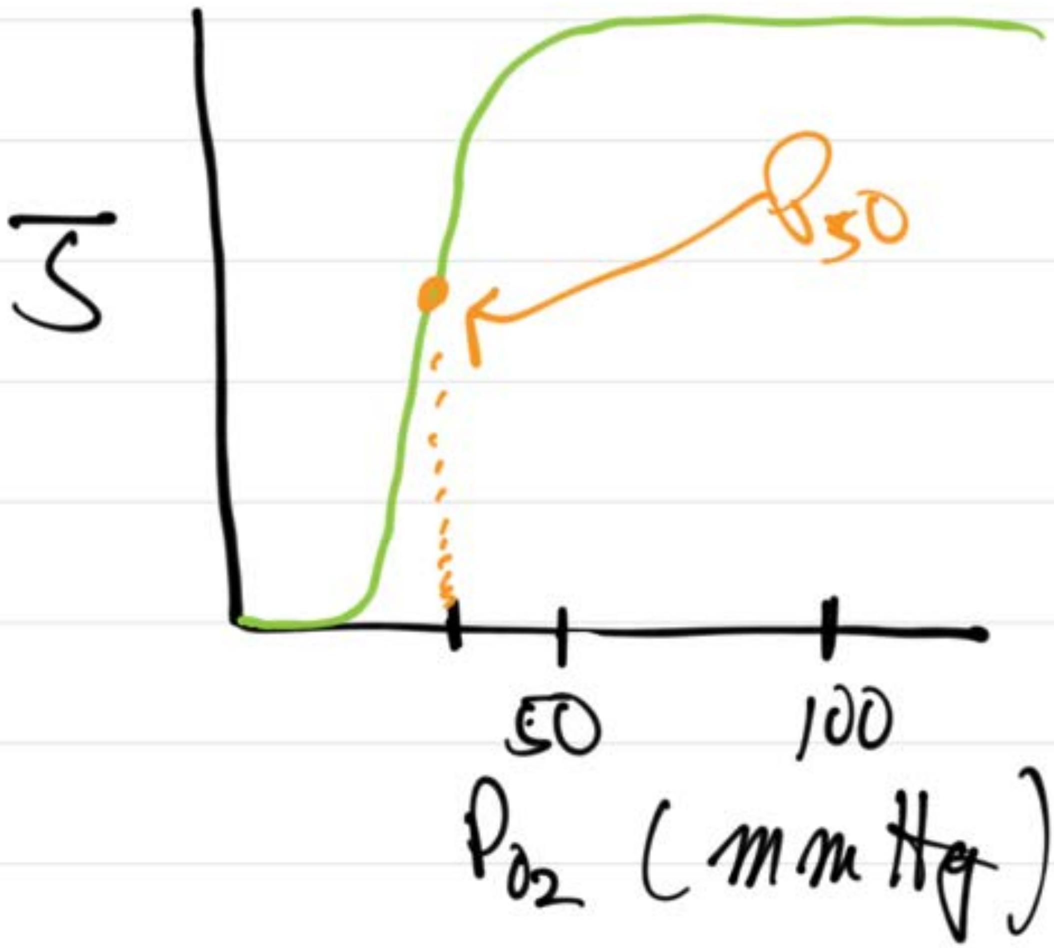
H_{ct}: FRACTION OF BLOOD THAT IS RED BLOOD CELLS

CHEMICAL COMPLEXATION

$\bar{S}$ FRACTIONAL LOADING OF O_2 ONTO HEMOGLOBIN BINDING SITES


1.6
1.0
0.8
0.6
0.4
0.2
0
Fractional Saturation
Myoglobin
Hemoglobin
P_{50}
50
100
150
Oxygen Partial Pressure (mmHg)


Config
120
50
80
ST-II
2.0
0.0
-2.0
0.6
ST-I 0.3
ST-II 0.6
ST-III 0.2
ST-aVR -0.4
ST-aVL 0.0
ST-aVF 0.4
ST-V 0.1
ST-MCL 0.0
PVC
ABP
160
90
96
119/79 (9
PAP
25
10
20
16
0
29/14 (2
SpO2
100
90
97
RR
40
45
8
40
Auto 15 min
15:06
NBP
97/73 (79)
13:54 113/53(68)
14:00 118/69(81)
15:02 93/60(66)
15:05 161/125(133)
15:06 97/73(79)



$$\bar{S} = \frac{\left(P_{O_2}/P_{50}\right)^{2.6}}{\left(1 + P_{O_2}/P_{50}\right)^{2.6}}$$

CHEMICAL COMPLEXATION

.009 MOL/L.

PHYSICAL SOLUBILITY

= .00012 + .00007

RATIO: $\frac{.009}{.00019} = 47$

HW PROBLEM GIVES A CHANCE TO TRY TO SEE WHAT LEVEL OF LIFE IS POSSIBLE WITH ONLY PHYSICAL SOLUBILITY

MASS BALANCE

O_2 USED = BLOOD FLOW IN LUNG $\times \Delta C_{O_2}$ IN LUNG

NET O_2 FLOW INTO LUNGS

1.9 For the following data, determine the oxygen consumption rate $\dot{V}_{O_2}$ under rest and exercise conditions.

	Rest	Exercise
Pulmonary blood flow (L min^{-1})	5.8	25
Arterial P_{O_2} (mmHg)	40	15
Venous P_{O_2} (mmHg)	100	100

DEPLETED: TO LUNGS

RETURN TO HEART

$$\dot{V}_{O_2} = Q\,(C_v - C_a)$$

Note that oxygen in blood is present in red blood cells bound to hemoglobin and freely dissolved in the red cell and the blood plasma. Equation (1.6.3) relates the partial pressure (mmHg) to the concentration in plasma and the total concentration of oxygen in blood is given by Equation (1.6.4), where $4C_{Hb}$ is 0.0203 M; the hematocrit or volume fraction of red cells, Hct, is typically 0.45 for males and 0.40 for females; H_{O_2} is 1.33×10^{-6} M mmHg^{-1}; and H_{Hb} is 1.50×10^{6} M mmHg^{-1}.

NUMBS

$CO = HR \times SV$

1.11 During exercise, the cardiac output can rise to 25 L min^{-1} from a resting level of 5 L min^{-1}. The heart rate of a well-trained athlete might rise from 60 beats per minute to 105 beats per minute and the mean arterial pressure may rise from 100 to 130 mmHg, whereas the heart rate of a sedentary person might rise from 72 beats per minute to 125 beats per minute and the mean arterial pressure may rise from 100 to 150 mmHg. Determine the volume of blood ejected during each heartbeat (stroke volume) and the peripheral resistance for an athlete and a sedentary person. Assess the power of the left side of the heart for the athlete and the sedentary person.

$$P = Wf = f\int \bar{p}_a dV,$$

where P is power, $\bar{p}_a$ is the mean arterial pressure, V is the ventricular volume, W is work, and f is heart rate in beats per second. Make sure that your units are consistent. Note: 1 mmHg = 133.3 Pa (1 Pa = 1 N m^{-2}).

$$R = \frac{\bar{P}_a}{CO}$$

$\bar{P}_a \sim 100$ mmHg

$(120+80)/2$

$\dot{W} = \bar{P}_a \dot{V}$

$P \sim 0$ ENTERING HEART

TRANSPORT PHENOMENA

UNIFIED TREATMENT OF
MASS, HEAT & MOMENTUM
TRANSPORT

MASS: "C" $\frac{\text{MASS}}{\text{VOLUME}}$

HEAT: $\rho \hat{U} \to \rho C_v T$ $\frac{\text{ENERGY}}{\text{VOLUME}}$

MOMENTUM $m\vec{v} \to \rho \vec{v}$

$\frac{\text{MOMENTUM}}{\text{VOLUME}}$

$$\text{FLUX} \equiv \frac{\text{TRANSPORT OF QUANTITY}}{\text{AREA-TIME}}$$

FLUX

$$J = -D\frac{\partial c}{\partial x}$$ MASS

$$\frac{q}{A} = -k\frac{dT}{dx}$$ HEAT

$$\tau_{yx} = -\nu\frac{\partial \rho v_y}{\partial x}$$

MOMENTUM

THIS GIVES

MASS

TABLE 7.2

Conservation Relations for Dilute Solutions

Rectangular $$\frac{\partial C_i}{\partial t} + v_x \frac{\partial C_i}{\partial x} + v_y \frac{\partial C_i}{\partial y} + v_z \frac{\partial C_i}{\partial z} = D_{ij}\left(\frac{\partial^2 C_i}{\partial x^2} + \frac{\partial^2 C_i}{\partial y^2} + \frac{\partial^2 C_i}{\partial z^2}\right) + R_i$$

HEAT

ion for Open System

$$\rho C_p\left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \dot{Q}_p + \dot{W}$$

MOMENTUM

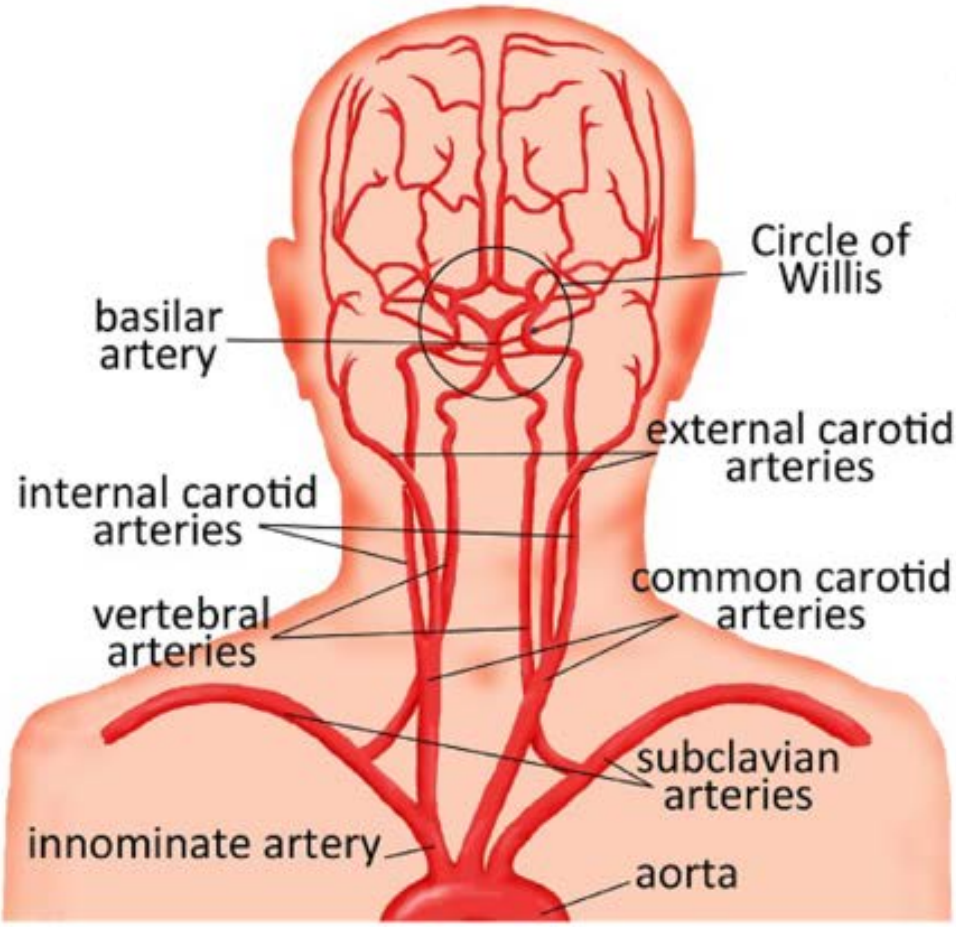
r form of the Navier–Stokes equation is obtained b
1b) into Equation (3.3.16):

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho \mathbf{g}.$$

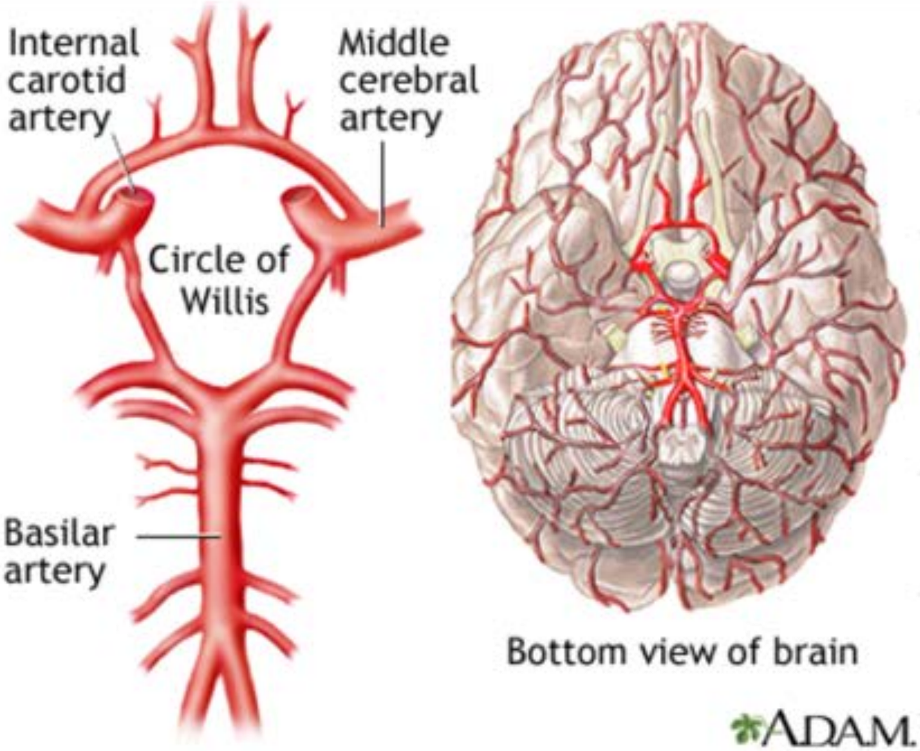
A question that combines a couple of the issues mentioned above is how much power is necessary to pump a required amount of fluid through a certain a vessel network ?

Our brains need a steady supply of oxygen to remain healthy — and to think all of our great thoughts !

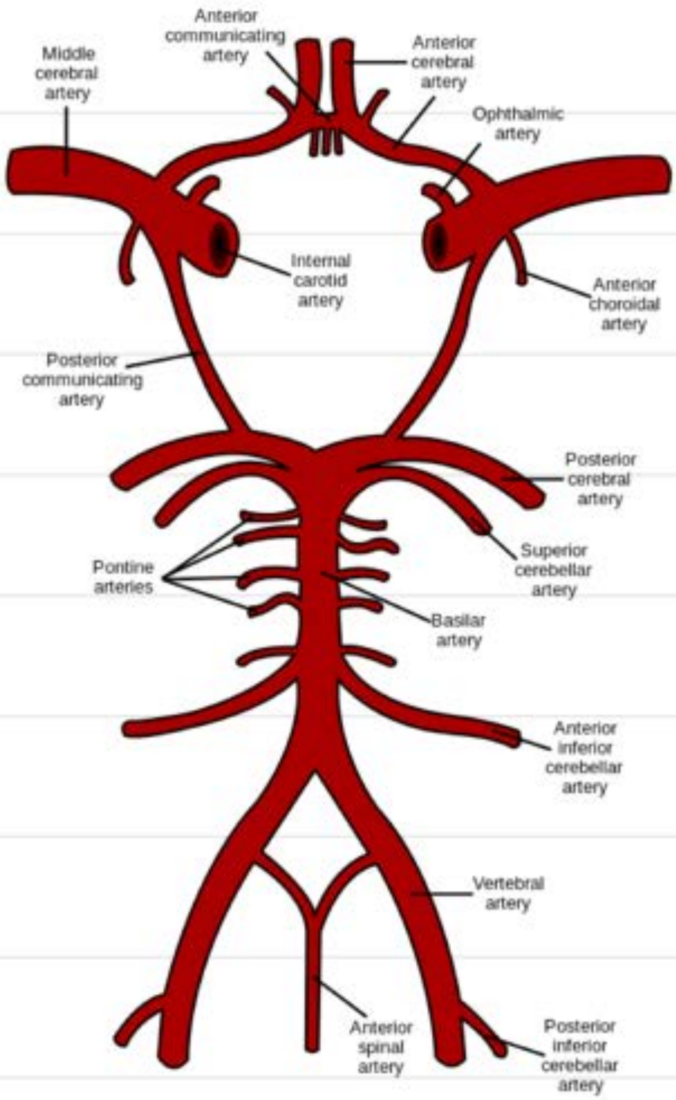
BRAIN BLOOD FLOW



MORE REDUNDANCY THAN IN OTHER PARTS OF BODY


Internal carotid artery
Middle cerebral artery
Circle of Willis
Basilar artery
Bottom view of brain
ADAM.

CIRCLE OF WILLIS



IS THIS THE CAUSE OF SYMPTOMS

Under normal conditions, blood flow in the communicating arteries is negligible. However, if a subject has an atypical Circle of Willis, e.g., missing one of the main arteries or communicating arteries or under pathological conditions such as complete or partial occlusion of one of the cerebral or carotid vessels, the flow can be redirected to perfuse deprived areas [22, 23]. The borderzones are then perfused through the network of communicating arterioles. The ring-like structure of the Circle of Willis is often incomplete or not fully developed. It has been found that in more than 50% of healthy brains [2, 42, 43] and in more than 80% of dysfunctional brains [51], the Circle of Willis contains at least one artery that is absent or underdeveloped. The most common topological variations include missing communicating vessels, fused vessels, string-like vessels, and presence of extra vessels [3]. These topological variations may

In modern times, if there is a problem with a chemical plant, or if we wish to do an modification or expansion, we first use our mathematical models of the process to determine what is best, and to make sure it will work.

Why not the same before surgery?

Sometimes there are multiple options for how to fix something.

There may be a question if the successful surgery will really fix the problem.

So in addition to the medical tests and imaging, perhaps you would like to consult an engineer to do some calculations! (Not likely today.)

Schematic of circle of Willis For calculations

8

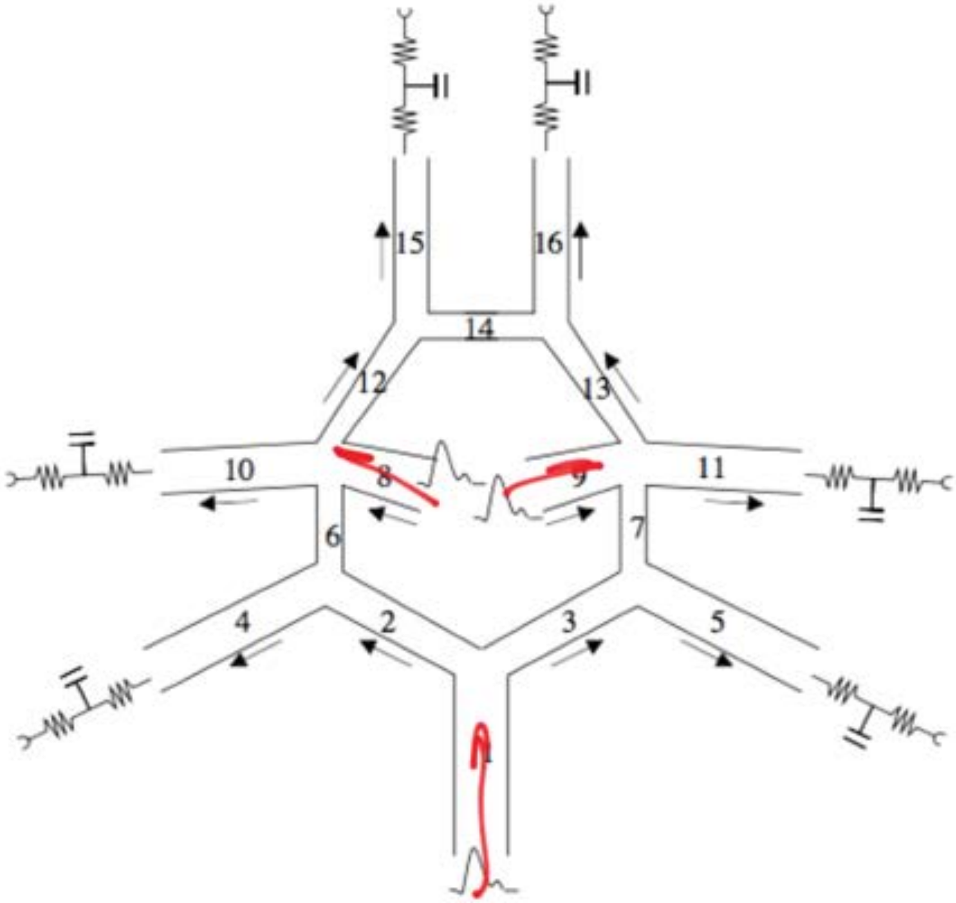


FIG. 3.1. *Topology of the Circle of Willis and boundary conditions and numbering convention, see also Table 3.1.*

A common question from fluid flow is how much power does it take to maintain a specific flow rate of a liquid through a conduit of radius R?

$$\text{POWER} \; [=] \; \frac{\text{WORK}}{\text{TIME}}$$

$$\text{WORK} \Rightarrow \int \bar{F} \cdot d\bar{l}$$

For "force" we can use pressure, $P \; [=] \; \frac{\text{FORCE}}{\text{AREA}}$

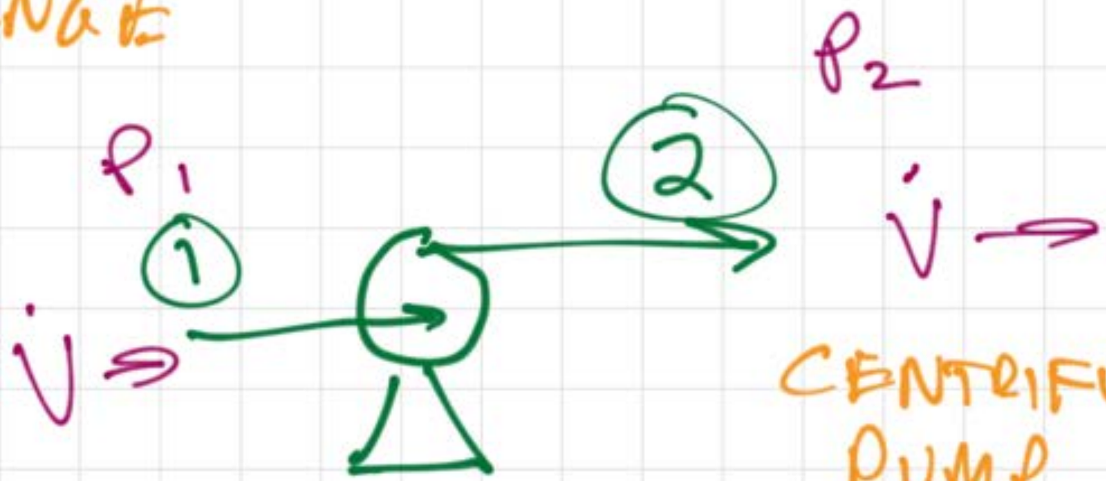
$$\text{POWER} = \frac{\text{FORCE}}{\text{AREA}} \times \frac{\text{AREA} \times \text{DISTANCE}}{\text{TIME}}$$

PUMPING POWER:

$$\Delta P \quad \dot{V}$$

PRESSURE CHANGE

VOLUMETRIC FLOW

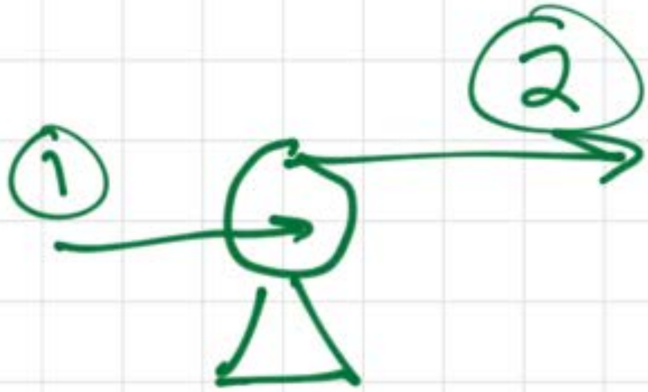


CENTRIFUGAL PUMP

$$\Delta P \equiv P_2 - P_1$$

DOES FIRST LAW OF THERMO HELP?

$$\boxed{\frac{d(mU)_{\mathrm{cv}}}{dt} + \Delta\left[\left(H + \frac{1}{2}u^2 + zg\right)\dot{m}\right]_{\mathrm{fs}} = \dot{Q} + \dot{W}}$$



$$\boxed{\frac{d(mU)_{cv}}{dt} + \Delta\left[\left(H + \frac{1}{2}u^2 + zg\right)\dot{m}\right]_{fs} = \dot{Q} + \dot{W}}$$

$$0 = \dot{m}\left(\hat{H}_1 - \hat{H}_2\right) + \dot{W}_s$$

$$d\hat{H} \equiv T\,d\hat{S} + \hat{V}\,dP$$

PUMP $\Delta S \sim 0$

$$d\hat{H} = \hat{V}\,dP$$

$$\Delta H = \int \hat{V}\,dP \qquad \hat{V} \approx \text{CONST}$$

$$\therefore \; \dot{m}(\hat{H}_2 - \hat{H}_1) = \underbrace{(\dot{m}\hat{V})}_{= \dot{V}}\Delta P$$

$$\boxed{\therefore \; \dot{W}_S = \dot{V}\,\Delta P}$$

MATCHING DIMENSIONAL ANALYSIS ..

This gives some general insight, but what about a specific blood vessel? What is ΔP?

FLOW IN TUBE

What is relation between ΔP and velocity?

? USE MASS BALANCE ?

2 Chapter 2: Conservation of Mass

Table 2.2-1 The Mass Conservation Equation

	Mass Basis	Molar Basis
Rate-of-change form of the mass balance		
General equation	$\frac{dM}{dt} = \sum_{k=1}^{K} \dot{M}_k$	$\frac{dN}{dt} = \sum_{k=1}^{K} \dot{N}_k$
Special case:		
Closed system	$\frac{dM}{dt} = 0$ $M = \text{constant}$	$\frac{dN}{dt} = 0$ $N = \text{constant}$
*Difference form of the mass balance**		
General equation	$M_2 - M_1 = \sum_{k=1}^{K} \Delta M_k$	$N_2 - N_1 = \sum_{k=1}^{K} \Delta N_k$
Special cases:		
Closed system	$M_2 = M_1$	$N_2 = N_1$
Steady flow	$M_2 - M_1 = \sum_{k=1}^{K} \dot{M}_k \Delta t$	$N_2 - N_1 = \sum_{k=1}^{K} \dot{N}_k \Delta t$

*Here we have used the abbreviated notation $M_i = M(t_i)$ and $N_i = N(t_i)$.

water leaves the tank by evaporation. How much water is in the tank at the end of the time period?

$$\cancel{\frac{dM}{dt}}^{\,0} = \dot{M}_{IN} - \dot{M}_{OUT}$$

$$\dot{M}_{IN} = \dot{M}_{OUT}$$

$$\rho_1 \langle v \rangle_1 A_1 = \rho_2 \langle v \rangle_2 A_2$$

BUT: $\rho_1 = \rho_2$

$A_1 = A_2$

$\therefore \langle v \rangle_1 = \langle v \rangle_2$

OK. HOW ABOUT ENERGY BALANCE!!

$v_1 = v_2$
$V_1 = V_2$

$$\frac{d}{dt}\left[U + m\left(\frac{u^2}{2} + gz\right)\right] = \dot{Q} + \dot{W} + \sum_{i=1}^{k} \dot{m}_i \left(\underline{H}_i + \frac{u_i^2}{2} + gz_i\right)$$

COULD MAKE $\dot{Q} = 0$

$\underline{H}_1 = \underline{H}_2$

$\underline{H} \equiv \underline{U} + P\underline{V}$

$P_1 \neq P_2$, T MUST CHANGE?

WE ARE NOT MAKING MUCH PROGRESS. !!

WE WILL NEED ANOTHER PHYSICAL LAW:

CONSERVATION OF MOMENTUM !!

RESOLVED ON A DIFFERENTIAL SCALE...

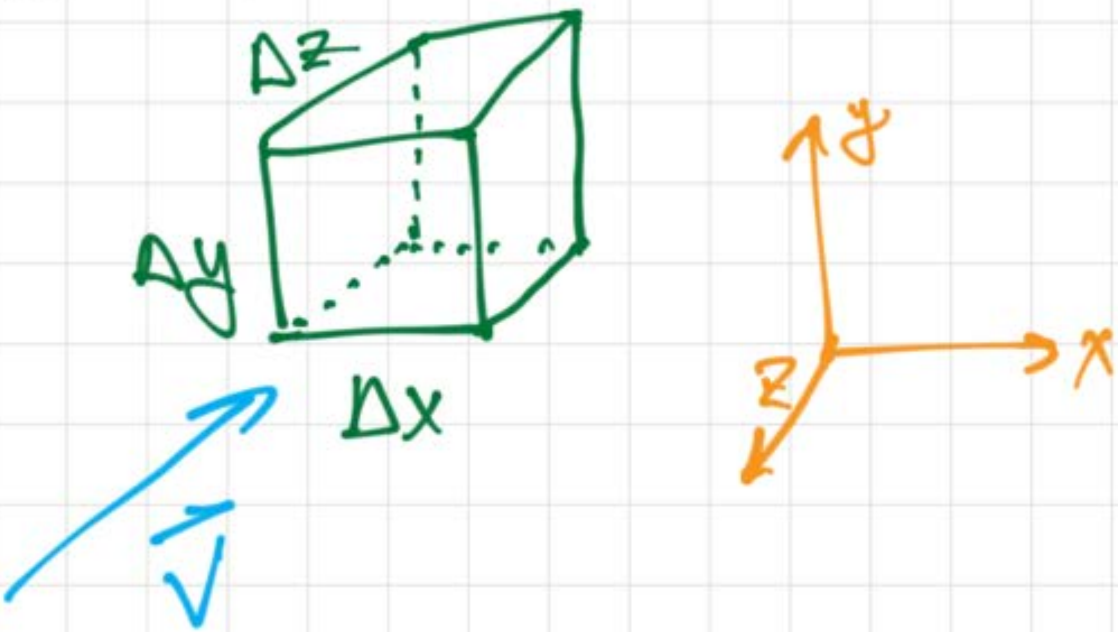
To proceed we need a formalism that can describe mass, momentum and energy transport and can be adapted to all problems of interest.

This formalism is the set of partial differential equations that I showed before.

We will start by explaining where these come from.

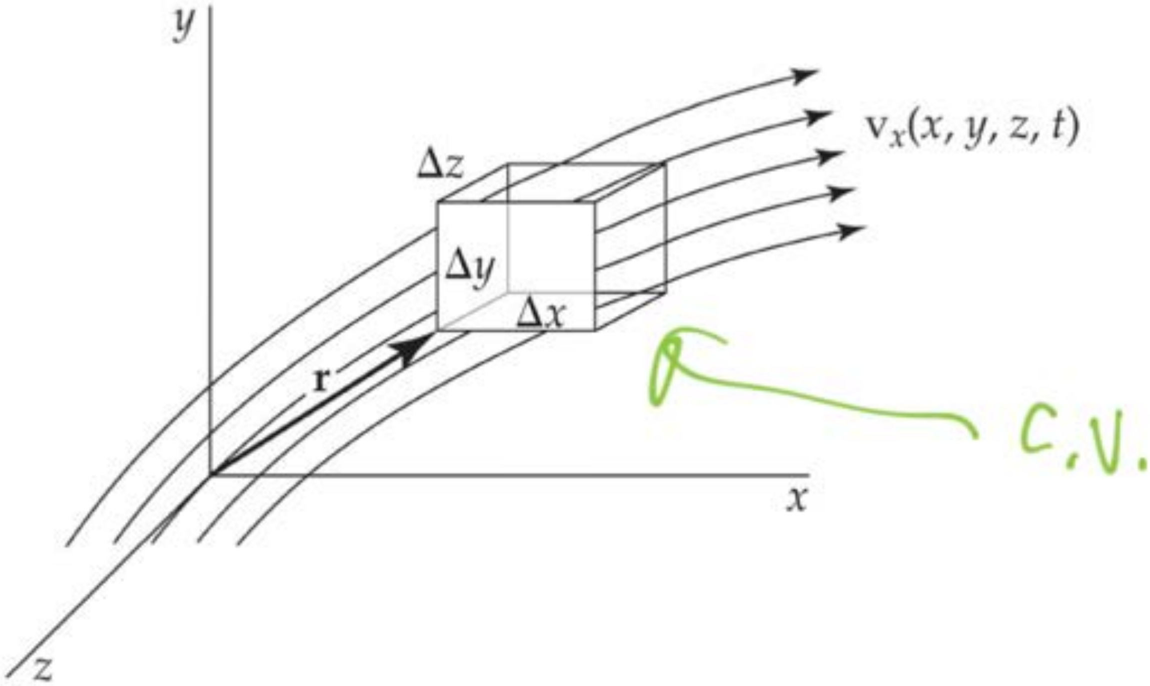
Then, we will use them!

WE WILL START WITH
A "CONTROL VOLUME"
THAT IS OF DIFFERENTIAL
SIZE



$$\vec{V} = V_x \hat{e}_x + V_y \hat{e}_y + V_z \hat{e}_z$$

Figure 2.3 Motion of fluid through a volume $\Delta x\ \Delta y\ \Delta z$.



PEARSON
Transport Phenomena in Biological Systems, Second Edition
George A. Truskey, Fan Yuan, and David F. Katz

CONSERVATION PRINCIPLE:

RATE OF CHANGE OF MASS IN C.V.
= FLOW RATE OF MASS INTO C.V
− FLOW RATE OF MASS OUT
OF C.V.

WHAT IS RATE OF CHANGE OF MASS IN C.V. ?

$$\frac{d}{dt} M = \frac{d}{dt}(\rho \Delta x \Delta y \Delta z)$$

ONLY $\rho = \rho(t)$ $\therefore$

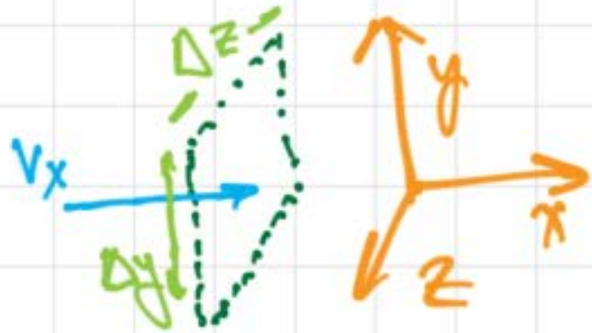
$$\Delta x \Delta y \Delta z \frac{\partial \rho}{\partial t}$$

HOW ABOUT AN "INFLOW"?

Need a way to keep track of any flow situation

Thus we consider each face separately and allow 1 in_flow and 1 out_flow for each coordinate direction.

v_x will carry fluid in or out across "x" faces



MASS FLOW ACROSS X-FACE

$$\frac{\text{CONSERVED QUANTITY}}{\text{VOLUME}} \times \text{VOLUMETRIC FLOW RATE}$$

$$\rho \quad \times \quad v_x \Delta y \Delta z$$

SO FOR THE JUST THE X- DIRECTION:

$$\Delta x \Delta y \Delta z \frac{\partial \rho}{\partial t} = \left(\rho V_x \big|_x - \rho V_x \big|_{x+\Delta x} \right) \Delta y \Delta z$$

$$\frac{\partial \rho}{\partial t} = - \frac{\left(\rho V_x \big|_{x+\Delta x} - \rho V_x \big|_x \right)}{\Delta x}$$

LIMIT $\Delta x \Rightarrow 0$

$$\frac{\partial \rho}{\partial t} = - \frac{\partial}{\partial x} \rho V_x$$

$$= -V_x \frac{\partial \rho}{\partial x} - \rho \frac{\partial V_x}{\partial x}$$

IF ρ = CONST

$$\frac{\partial v_x}{\partial x} = 0$$

V_x IS NOT A FUNCTION OF X

FOR 3-D

$$\frac{\partial \rho}{\partial t} = -\frac{\partial \rho V_x}{\partial x} - \frac{\partial \rho V_y}{\partial y} - \frac{\partial \rho V_z}{\partial z}$$

$$= -\rho\left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}\right)$$

$$- V_x \frac{\partial \rho}{\partial x} - V_y \frac{\partial \rho}{\partial y} - V_z \frac{\partial \rho}{\partial z}$$

IF $\rho = \text{CONST}$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

$$\vec{\nabla} \cdot \vec{v} = 0$$

Called: "Continuity equation"