CBE 30357
11/7/17

TEST 2 REVIEW

- EXACT SOLUTIONS TO NAVIER-STOKES EQUATIONS ARE LIMITED TO SINGLE DIRECTION FLOWS, $\bar{v}\cdot\bar{\nabla}v=0$
- WE NEED TO BE ABLE TO GET USEFUL RESULTS FOR MANY OTHER SITUATIONS...

1) TURBULENT FLOW, INCLUDING FLOWS FOR NON-CIRCULAR CROSS SECTION CHANNELS
   f-Re CORRELATIONS, HYDRAULIC DIAMETER

2) NONDIMENSIONALIZATION OF NAVIER-STOKES EQ.
   - Re CRITICAL PARAMETER
   - $Re \to 0$ LIMIT

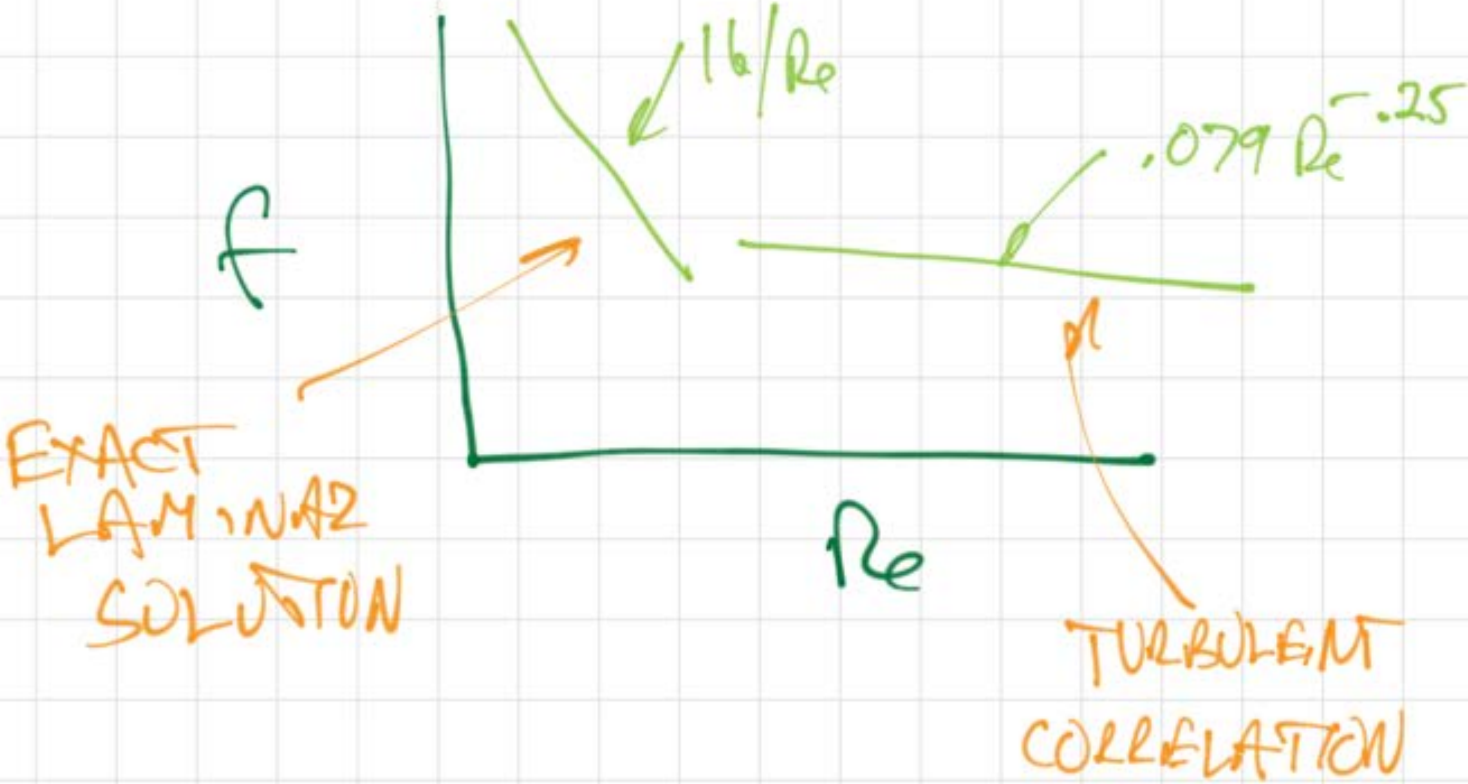
3) INTEGRAL BALANCES

PIPE/CONDUIT FLOW

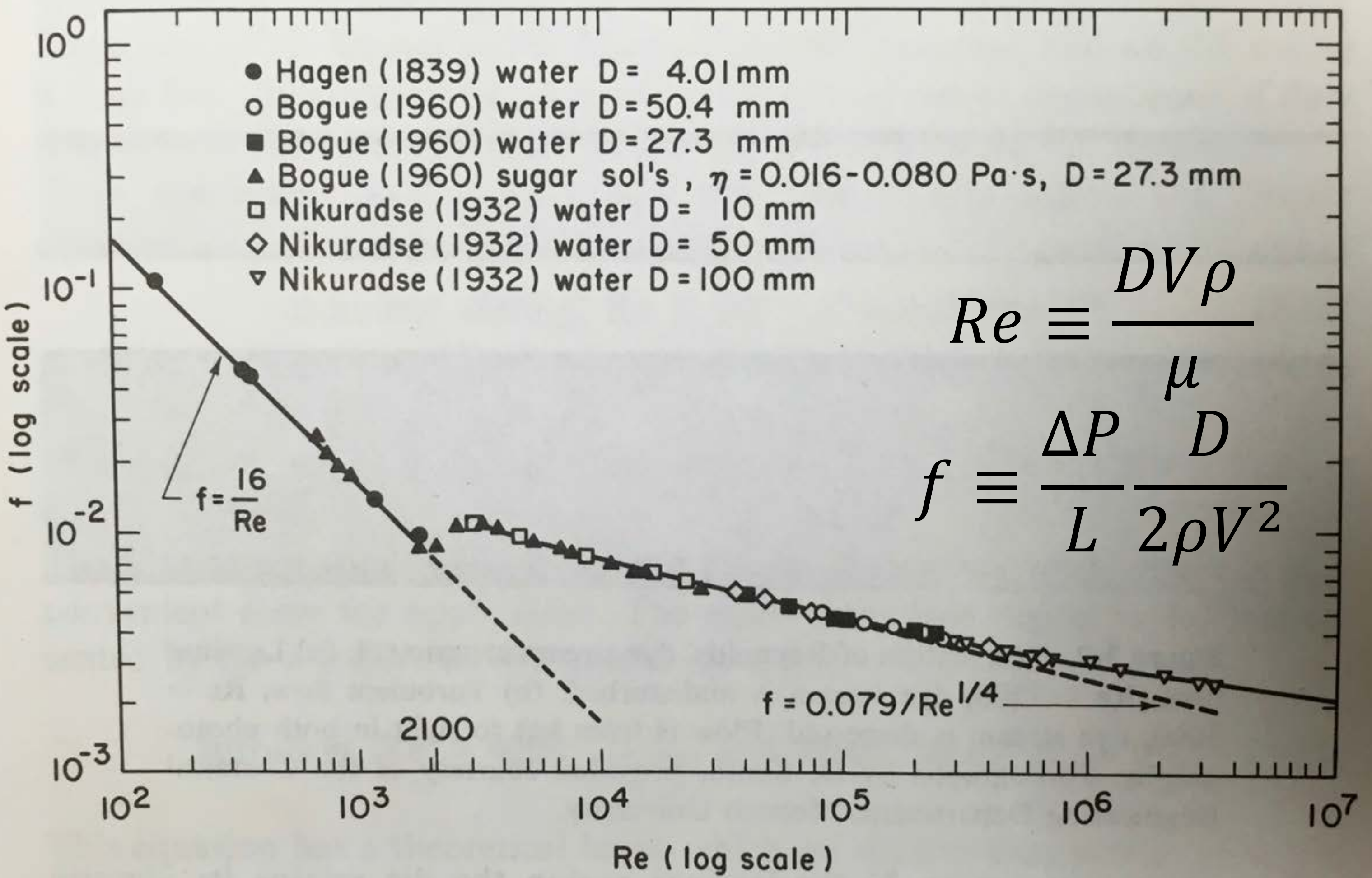
- NEED TO KNOW IF A CHEMICAL ENGINEER

$$f \equiv \frac{\Delta P}{L}\frac{D}{2\rho u^2} = \frac{\tau_w}{\frac{1}{2}\rho u^2}$$

$$Re \equiv \frac{D u \rho}{\mu}$$



Laminar and turbulent pipe flow



$$Re \equiv \frac{DV\rho}{\mu}$$

$$f \equiv \frac{\Delta P}{L}\frac{D}{2\rho V^2}$$

From: M. M. Denn, Process Fluid Mechanics

Figure 3-1. Friction factor as a function of Reynolds number for incompressible Newtonian fluids.

Dye Streak Experiment

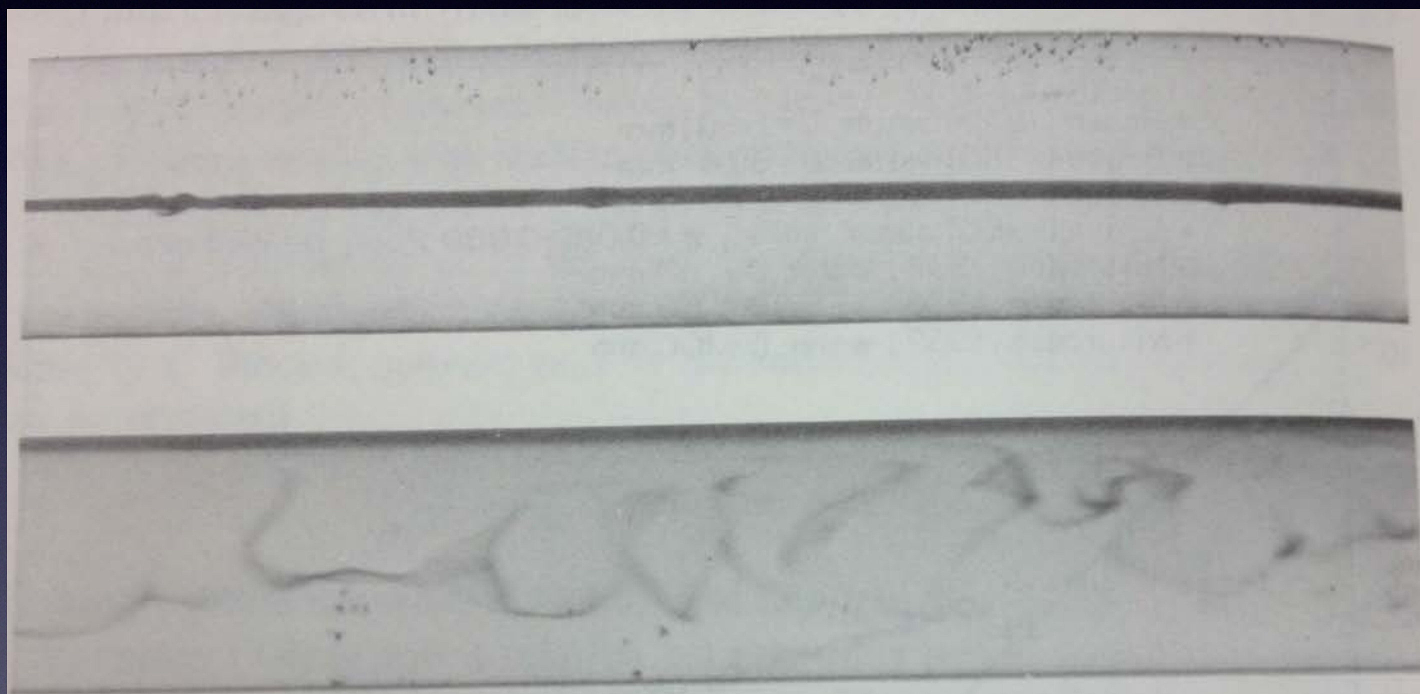


Figure 3-2. Re-creation of Reynolds' dye stream experiment. (a) Laminar flow, Re = 1500, dye stream is undisturbed. (b) Turbulent flow, Re = 3000, dye stream is dispersed. Flow is from left to right in both photographs. Photographs by D. Smith, provided courtesy of the Chemical Engineering Department, Monash University.

"USUAL" PROBLEM IS THAT YOU KNOW YOUR FLUID, HAVE A NEED TO TRANSPORT A GIVEN QUANTITY/TIME

THEN CHOOSE D

$$\hookrightarrow Re \longrightarrow f \Rightarrow \frac{\Delta P}{L}$$

$$\dot{W}_s = \dot{Q}\Delta P$$

VOLUMETRIC FLOW x PRESSURE RISE IN PUMP.

SITUATIONS EXIST WHEN "PIPE" IS NOT CIRCULAR

HYDRAULIC DIAMETER

$$D_H = \frac{4 \text{ CROSS-SECTION AREA}}{\text{"WETTED" PERIMETER}}$$

TOTAL PERIMETER FOR A "FULL" PIPE (ANYCASE WE WOULD DO)

TERMINOLOGY COMES FROM GRAVITY-DRIVEN FLOWS IN DITCHES, SEWERS, ETC....

CIRCULAR PIPE:

$$D_H = \frac{4 \pi D^2/4}{\pi D} = D$$

WE SHOWED THE VALIDITY OF D_H FOR SOME LAMINAR FLOWS (USING EXACT SOLUTIONS OR NUMERICAL SOLUTIONS)

EVENTUALLY BREAKS DOWN

EXPERIMENTS CONFIRM UTILITY FOR TURBULENT FLOWS

HOW TO CHOOSE "D"?

OPTIMIZATION CONSIDERATIONS POSSIBLY:

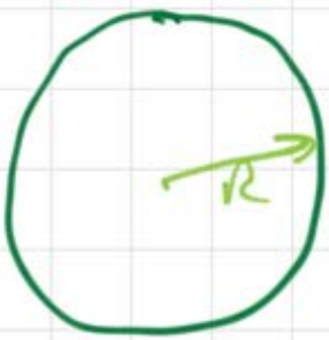
MINIMIZE TOTAL COST = OPERATING COST + CAPITAL COST

FORMAL PROCEDURE TO GET APPROXIMATE SOLUTIONS TO NAVIER-STOKES EQUATIONS

HOW "BIG" ARE THE TERMS IN EQS?

WE CAN START TO LEARN ABOUT TRANSPORT PHENOMENA JUST BY EXAMINING DIMENSIONS OF PHYSICAL QUANTITIES

"COOLING SODA CANS"



$\alpha \equiv$ THERMAL DIFFUSIVITY HAS DIMENSIONS OF $\frac{\text{LENGTH}^2}{\text{TIME}}$

(PURE CONDUCTION)

TIME SCALE $\sim \dfrac{R^2}{\alpha}$ $\quad \dfrac{\ell^2}{\ell^2/\text{TIME}} \sim \text{TIME}$

IF WE SOLVE THE GOVERNING PDE.

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2}$$

THIS WOULD BE THE TIME OF THE TRANSIENT

TERMS IN N-S EQUATIONS...
NONDIMENSIONALIZATION/

DIMENSIONAL REASONING

COMPARE VARIABLES IN OUR PROBLEM TO QUANTITIES WITH SAME DIMENSIONS THAT DEFINE FLOW

$$\frac{\partial v_r}{\partial r} \sim \frac{\Delta v_r}{\Delta r} \hat{=} \frac{\alpha\, U_0}{\beta\, R}$$

$$v_r^* = \frac{v_r}{U_0} \qquad r^* = \frac{r}{R}$$

$$p^* = \frac{p}{\mu U_0 / R}$$

THESE ARE DIMENSIONLESS AND COMPARED TO FLOW SITUATION...

NONDIMENSIONALIZE N.S. EQ'S

$$Re\left(\overset{O(1)}{\frac{\partial \vec{V}^*}{\partial t^*}} + \overset{O(1)}{\vec{V}^* \cdot \vec{\nabla}^* \vec{V}^*}\right) = -\overset{O(1)}{\vec{\nabla}^* P} + \overset{O(1)}{\nabla^{*2} \vec{V}^*}$$

LIMIT $Re \Rightarrow 0$

$$\begin{cases} \vec{\nabla}^* P = \nabla^{*2} \vec{V}^* \\ \vec{\nabla}^* \cdot \vec{V}^* = 0 \end{cases}$$

LINEAR EQ'S

CHARACTERISTICS OF LOW Re FLOWS

1) EFFECTIVELY NO INERTIA

CAN'T JUST SCALE DOWN MACROSCOPIC DEVICES

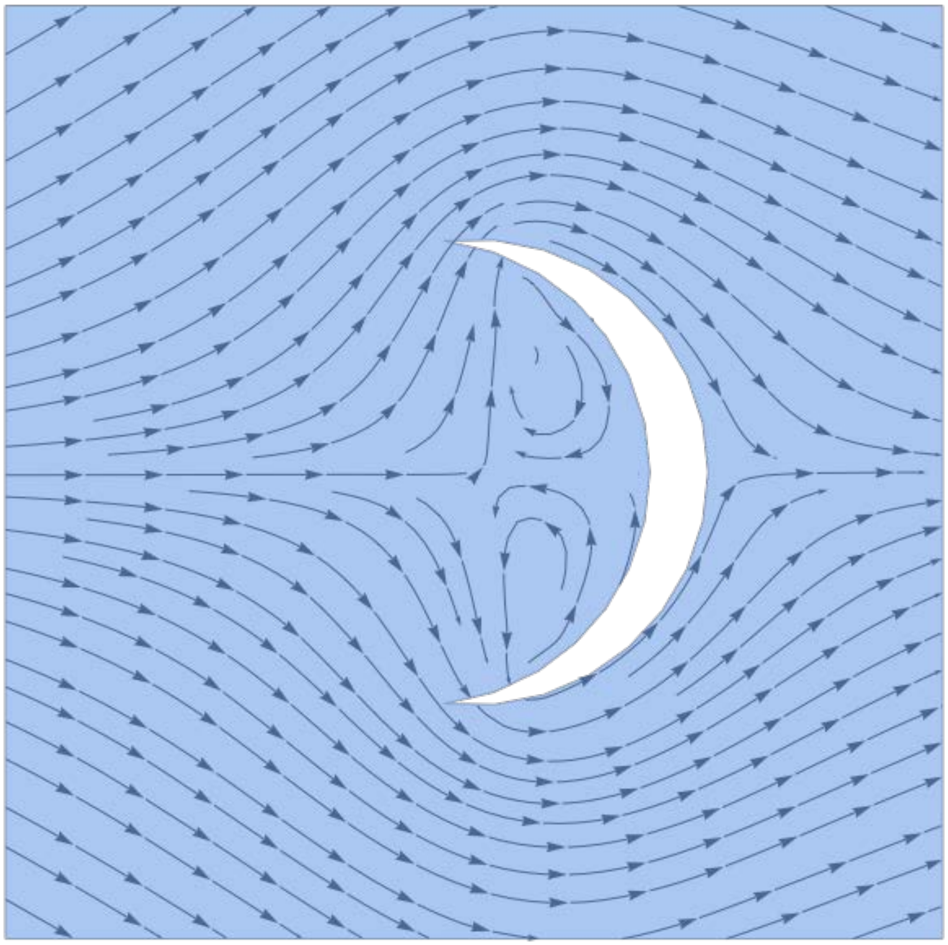
"PROPELLER" WON'T WORK?!

2) NO CONVECTIVE MIXING

TURBULENCE FROM STIRRING IS HOW WE MIX MOST SUBSTANCES

Left to right

```
rmf = RegionMember[Ω];
Show[BoundaryDiscretizeRegion[Ω],
 StreamPlot[{xVel[x, y], yVel[x, y]}, {x, -1, 1}, {y, -1, 1},
  RegionFunction → Function[{x, y}, rmf[{x, y}]], AspectRatio → Automatic], ImageSize → 600]
```



Right to Left!

```
rmf = RegionMember[Ω];
Show[BoundaryDiscretizeRegion[Ω],
 StreamPlot[{xVel[x, y], yVel[x, y]}, {x, -1, 1}, {y, -1, 1},
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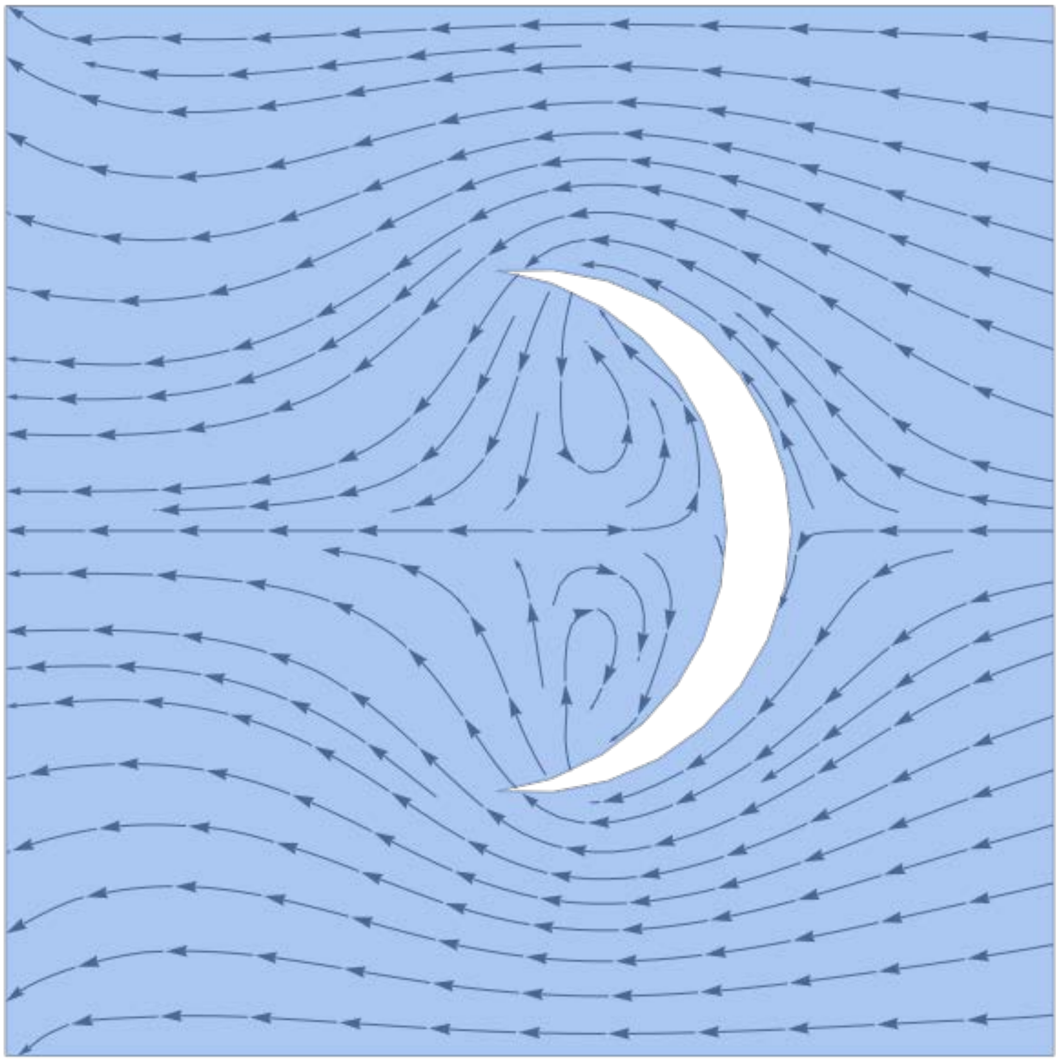
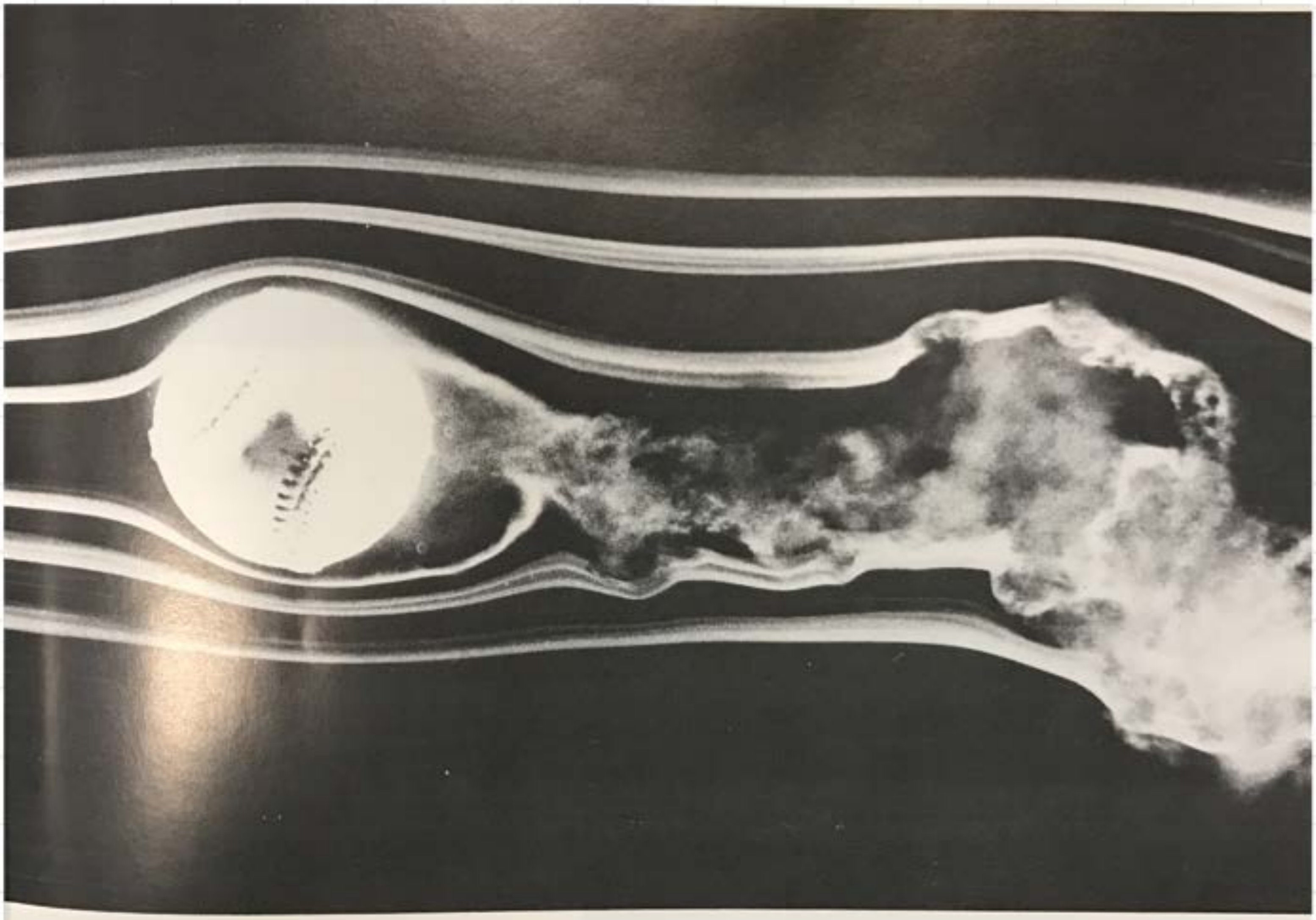


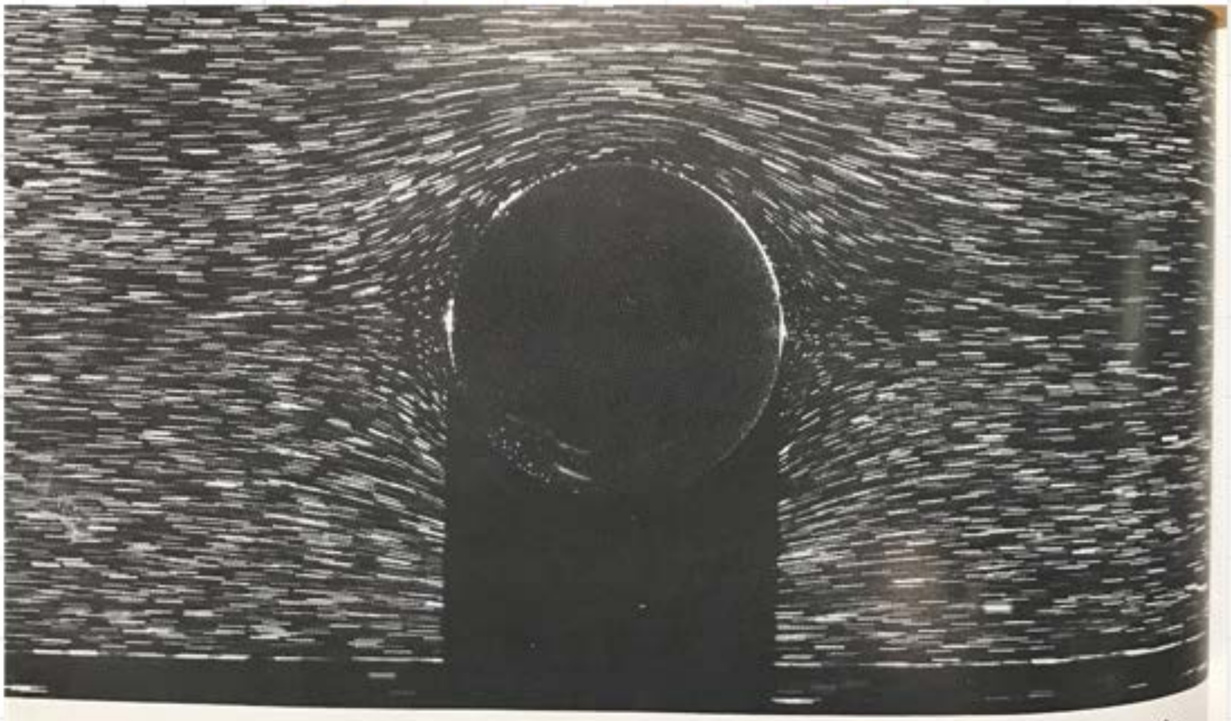
WHAT WOULD FLOW PAST A SPHERE AT LOW Re LOOK LIKE?

NOT THIS:



66. Spinning baseball. The late F. N. M. Brown devoted many years to developing and using smoke visualization in wind tunnels at the University of Notre Dame. Here the flow speed is about 77 ft/sec and the ball is rotated at 630 rpm. This unpublished photograph is similar to several in Brown 1971. *Photograph courtesy of T. J. Mueller*

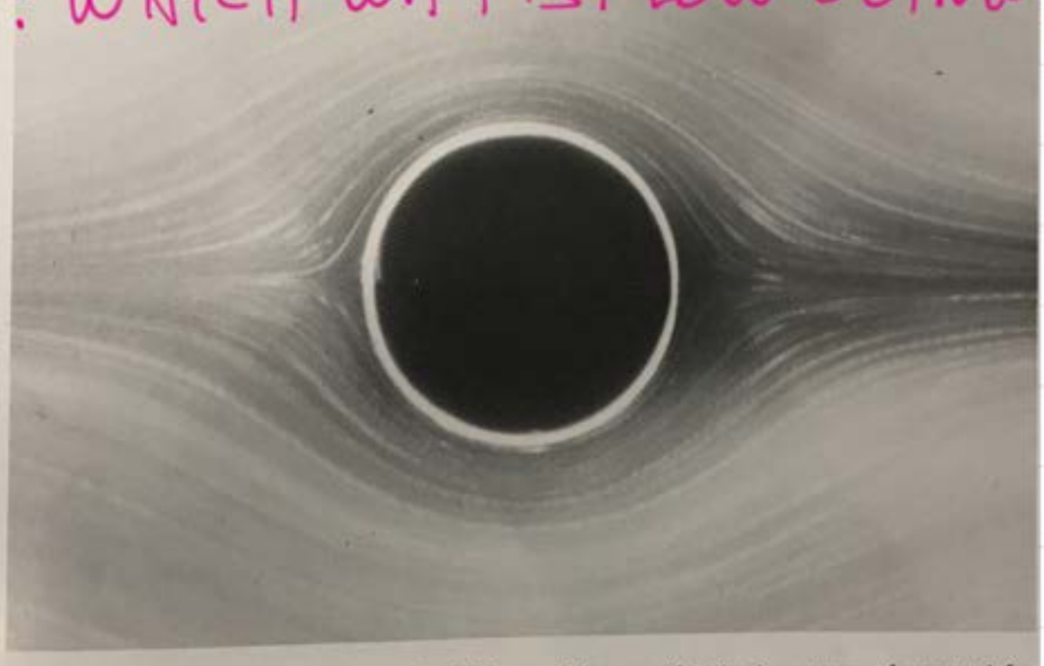
LOW Re FLOWS....



NO WAKE

8. Sphere moving through a tube at $R=0.10$, relative motion. A free sphere is falling steadily down the axis of a tube of twice its diameter filled with glycerine. The camera is moved with the speed of the sphere to show the flow relative to it. The photograph has been rotated to show flow from left to right. Tiny magnesium cuttings are illuminated by a thin sheet of light, which casts a shadow of the sphere. *Coutanceau 1968*

SYMMETRIC: WHICH WAY IS FLOW GOING?



. **Uniform flow past a circular cylinder at $R=0.16$.** hat the flow is from left to right can scarcely be deduced om the streamline pattern, because in the limit of zero eynolds number the flow past a solid body is reversible, nd hence symmetric about a symmetric shape. It resembles superficially the pattern of potential flow but the disturbances to the uniform stream c more slowly. The flow of water is shown b dust. *Photograph by Sadatoshi Taneda*

3) FLOWS ARE REVERSIBLE
DIE BLOB EXPERIMENT

WE EXAMINED RESULTS OF
SOLUTIONS FOR
a) ROTATING SPHERE
b) FLOW PAST A SPHERE

SOLUTION FOR A ROTATING SPHERE ...

$$V_\phi \Big|_{r=R} = \Omega R \sin\theta$$

WE CAN SHOW:

$$V_\phi(r) = \frac{R^3 \Omega \sin\theta}{r^2}$$

SOLUTION FOR FLOW PAST A SPHERE

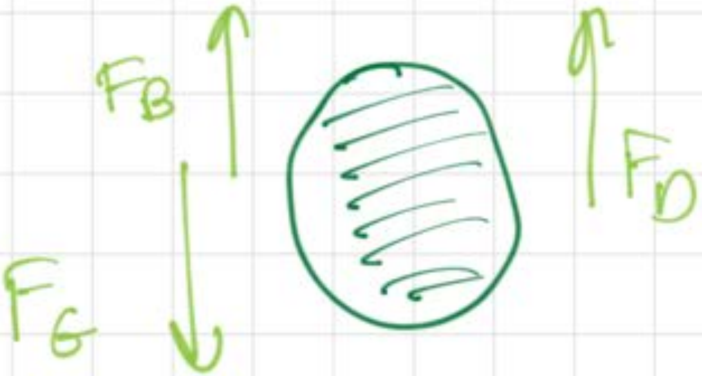
$$\begin{cases} V_r = u_0 \cos\theta \left(1 - \dfrac{3R}{2r} + \dfrac{R^3}{2r^3}\right) \\ V_\theta = u_0 \sin\theta \left(-1 + \dfrac{3R}{4r} + \dfrac{R^3}{4r^3}\right) \end{cases}$$

VELOCITY IS ALWAYS u_0 OR LOWER, NO FLUID ACCELERATION.

$1/r$ IS SLOW DECAY SO DISTURBANCE IS FELT FAR FROM SPHERE

FORCE BALANCE ON A SPHERE

IF U = TERMINAL VELOCITY



$F_D = 6\pi\mu U R$
(IF $Re \to 0$)
OTHERWISE, USE CORRELATION

$$F_G = mg = \frac{4}{3}\pi R^3 \rho_s g$$

$$F_B = \frac{4}{3}\pi R^3 \rho_f g$$

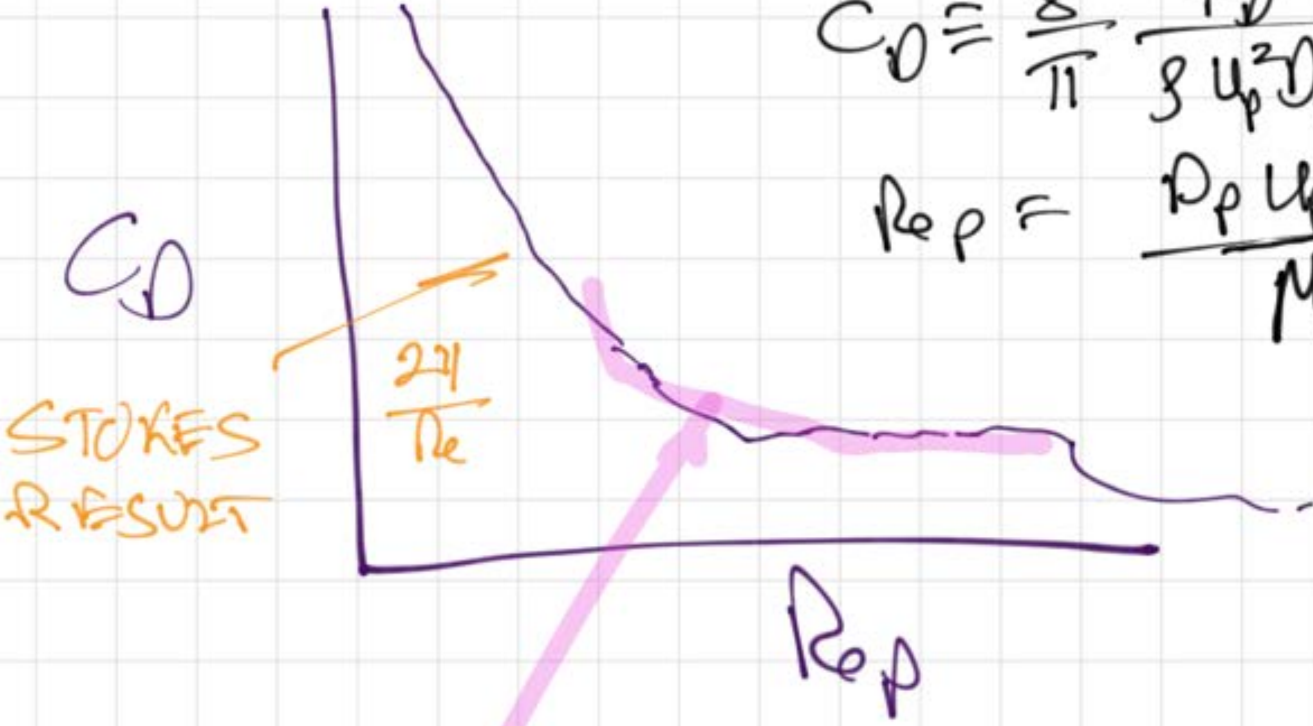
$$0 = F_G - F_B - F_D$$

$$0 = \frac{4}{3}\pi R^3 \rho_p g - \frac{4}{3}\pi R^3 \rho_f g - 6\pi\mu U R$$

$$U = \frac{2}{9}\frac{R^2(\rho_p - \rho_f)g}{\mu}$$

$$C_D \equiv \frac{8}{\pi}\frac{F_D}{\rho U_p^2 D_p^2}$$

$$Re_p = \frac{D_p U_p \rho_f}{\mu_f}$$



Rep COULD BE OUT HERE

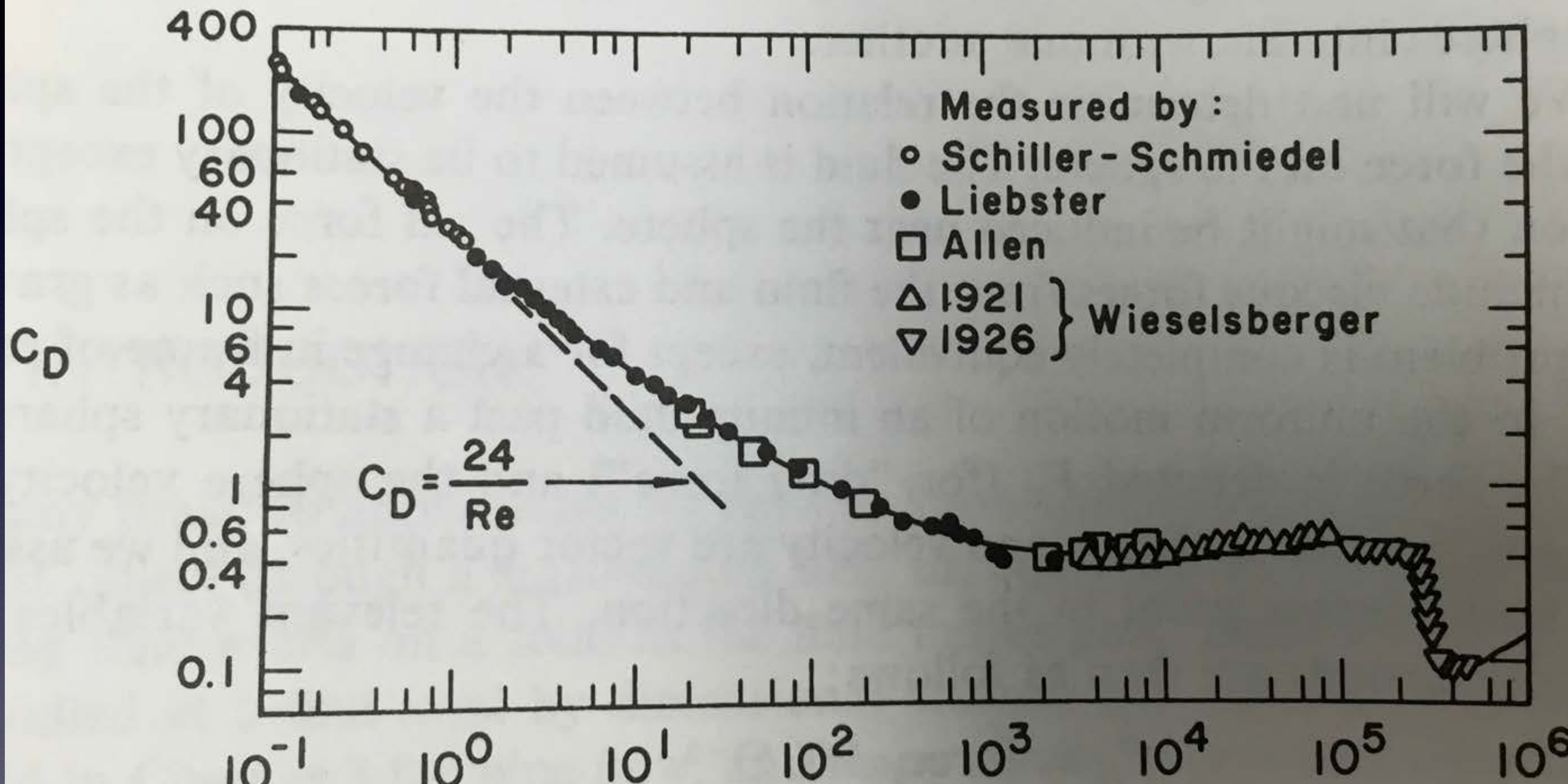
DISCUSSION OF "INHALERS" FOR DRUG DELIVERY ENSUED

YOU HAVE DEFENSES ...

Drag coefficient

Reynolds number: $\mathrm{Re} = \dfrac{D_p V_p \rho}{\eta}$

drag coefficient: $C_D = \dfrac{8}{\pi} \dfrac{F_D}{\rho V_p^2 D_p^2}$



From: M. M. Denn, Process Fluid Mechanics

Figure 4-1. Drag coefficient as a function of Reynolds number for flow past a sphere. (Reproduced from H. Schlichting, *Boundary Layer Theory*, 6th ed., McGraw-Hill Book Company, New York, 1968, by

STOKES-EINSTEIN RESULT

PREDICTION OF DIFFUSIVITY FOR MOLECULES + SMALL PARTICLES

BOLTZMAN CONSTANT

$$D = \frac{kT}{6\pi\mu R}$$

PARTICLE RADIUS

EINSTEIN RESULT FOR VISCOSITY OF SUSPENSION

SUSPENSION

$$\frac{\mu_s}{\mu_f} = 1 + \phi\left(\frac{\mu_f + \frac{5}{2}\mu_p}{\mu_f + \mu_p}\right)$$

FLUID

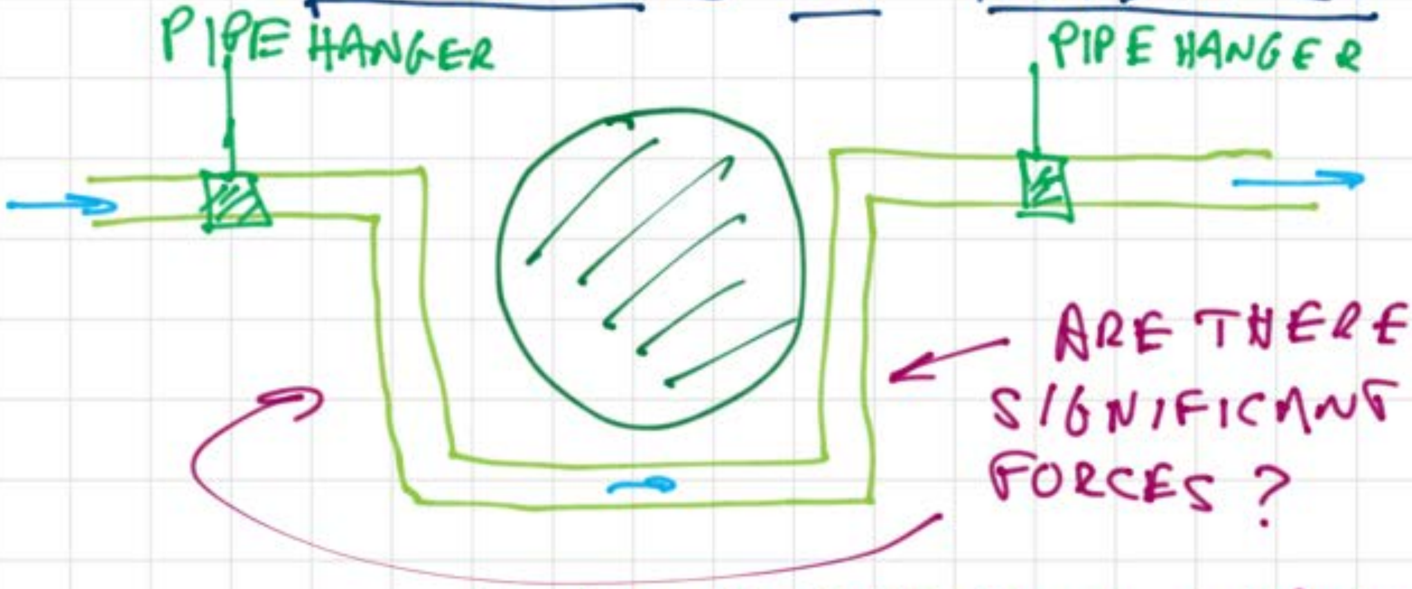
VOLUME FRACTION OF PARTICLES

VISCOSITY OF PARTICLES

$\mu_p \rightarrow \infty$ FOR SOLID

$$\frac{\mu_s}{\mu_f} = 1 + \frac{5}{2}\phi$$

PROBLEMS OF INTEREST



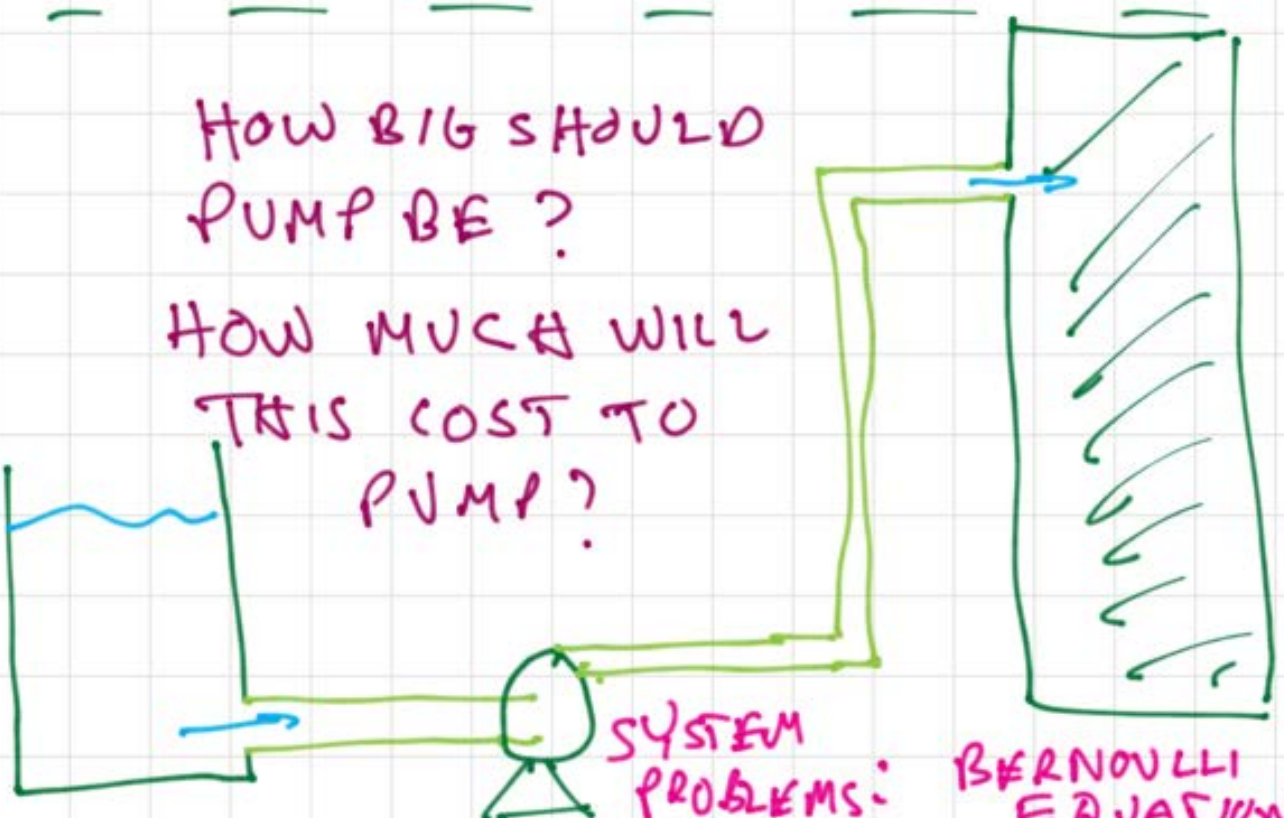
ARE THERE SIGNIFICANT FORCES?

DO YOU NEED ADDITIONAL SUPPORTS?

IF QUESION INVOLVES "FORCE" YOU 🍕 WILL NEED MOMENTUM EQUATION!

HOW BIG SHOULD PUMP BE?

HOW MUCH WILL THIS COST TO PUMP?



SYSTEM PROBLEMS: BERNOULLI EQUATION

"AVERAGED" (LESS DETAILED) ANALYSIS OF FLOWS IN BENDS, BRANCHES, ETC.

INTEGRATE MASS + MOMENTUM EQUATIONS OVER VOLUMES OF INTEREST, USE DIVERGENCE THEOREM AS NEEDED:

MASS BALANCE

$$\frac{dm}{dt} = -\int \rho(\bar{v}\cdot\bar{n})\,dS$$

$$\frac{dm}{dt} = \rho \sum_{\substack{i \\ IN}} \langle v\rangle_i A_i - \rho \sum_{\substack{j \\ OUT}} \langle v\rangle_j A_j$$

MOMENTUM BALANCE

DIFFERENTIAL MOMENTUM EQ

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nabla\cdot\boldsymbol{\tau} + \rho\mathbf{g}.$$

INTEGRATED OVER CONTROL VOLUME:

$$\int_v \rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right)dV = \int_V (-\nabla p + \nabla\cdot\boldsymbol{\tau} + \rho\mathbf{g})dV.$$

WE DO SOME MATH...

$$\frac{\partial \int_V \rho\mathbf{v}dV}{\partial t} + \int_S \mathbf{v}\rho(\mathbf{n}\cdot\mathbf{v})dS = -\int_S p\mathbf{n}dS + \int_S \mathbf{n}\cdot\boldsymbol{\tau}dS + m\mathbf{g}.$$

SLIGHTLY MORE USEFUL TO BREAK OUT "$\bar{F}$"

$$\frac{d}{dt}\left(\int \rho\bar{v}\,dV\right) + \int_S \bar{v}\rho(\bar{n}\cdot\bar{v})dS = -\underbrace{\int p\bar{n}\,dS}_{\text{OPEN SURFACE}} + \underbrace{\int (\bar{n}\cdot\bar{\tau})dS}_{\text{OPEN SURFACE}} + m\bar{g} + \bar{F}$$

↑ONLY RARELY WOULD THIS BE NEEDED

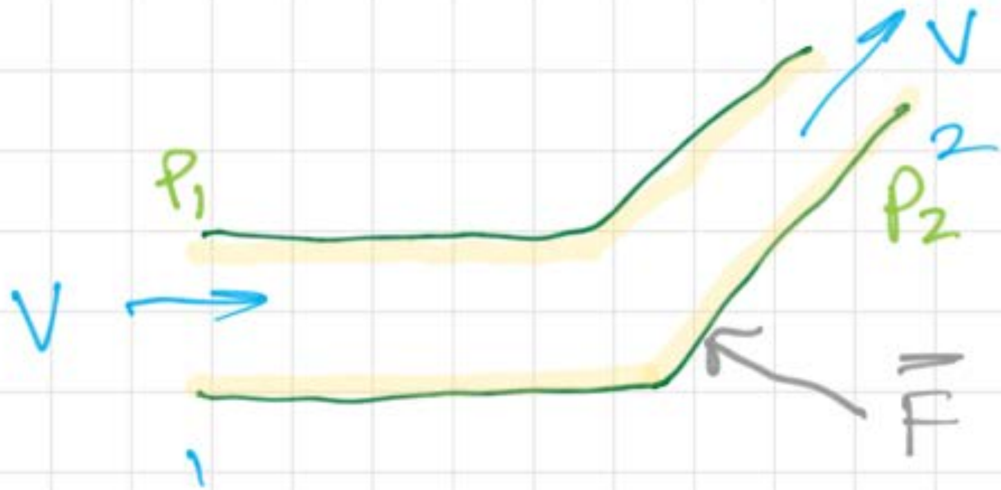
$$\bar{F} \equiv -\underbrace{\int p\bar{n}\,dS}_{\text{SOLID SURFACE}} + \underbrace{\int \bar{n}\cdot\bar{\tau}\,dS}_{\text{SOLID SURFACE}}$$

MOMENTUM:

Momentum equations for single flow inlet in + x direction.

$$-\rho\langle V_x V_x\rangle A_1 + \rho\langle V_2 V_2\rangle A_2\cos(\theta) = P_1 A_1 - P_2 A_2\cos(\theta) + F_x$$

$$+\rho\langle V_2 V_2\rangle A_2\sin(\theta) = -P_2 A_2\sin(\theta) + F_y$$



$$\int (\bar{n} \cdot \bar{\tau}) ds = F_x \hat{i} + F_y \hat{j}$$

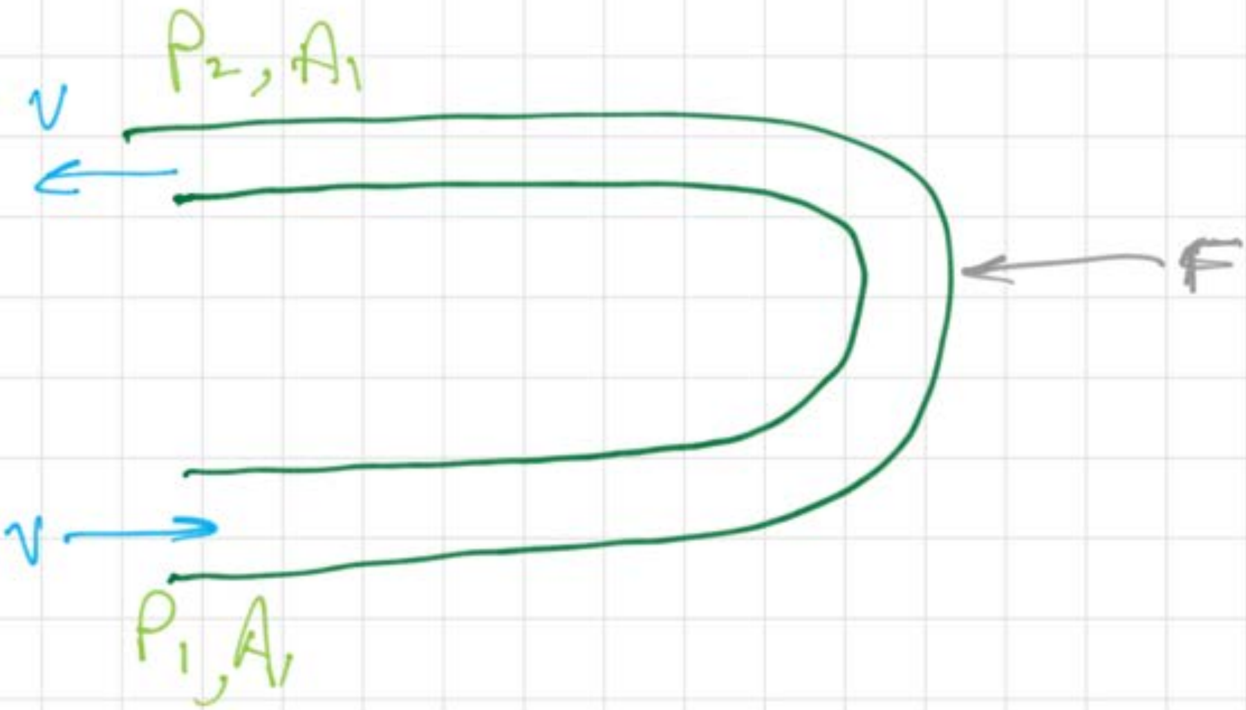
COMBINATION OF PRESSURE AND SHEAR STRESS
USUALLY CAN'T EVALUATE INTEGRALS EXACTLY

SO: COMPLETE BALANCE

$$x - \quad -\rho V_1^2 A_1 + \rho V_2^2 \cos\theta A_2 = P_1 A_1 - P_2 A_2 \cos\theta + F_x$$

$$y - \quad 0 + \rho V_2^2 \sin\theta A_2 = -P_2 A_2 \sin\theta + F_y$$

A USUAL QUESTION IS "FIND $\bar{F}$"



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$$-\rho V_1 V_1 A_1 + \rho V_1 V_1 A_1 \underbrace{\cos\pi}_{(-1)} = P_1 A_1 - P_2 A_1 \overbrace{\cos\pi}^{(-1)} + F_x$$

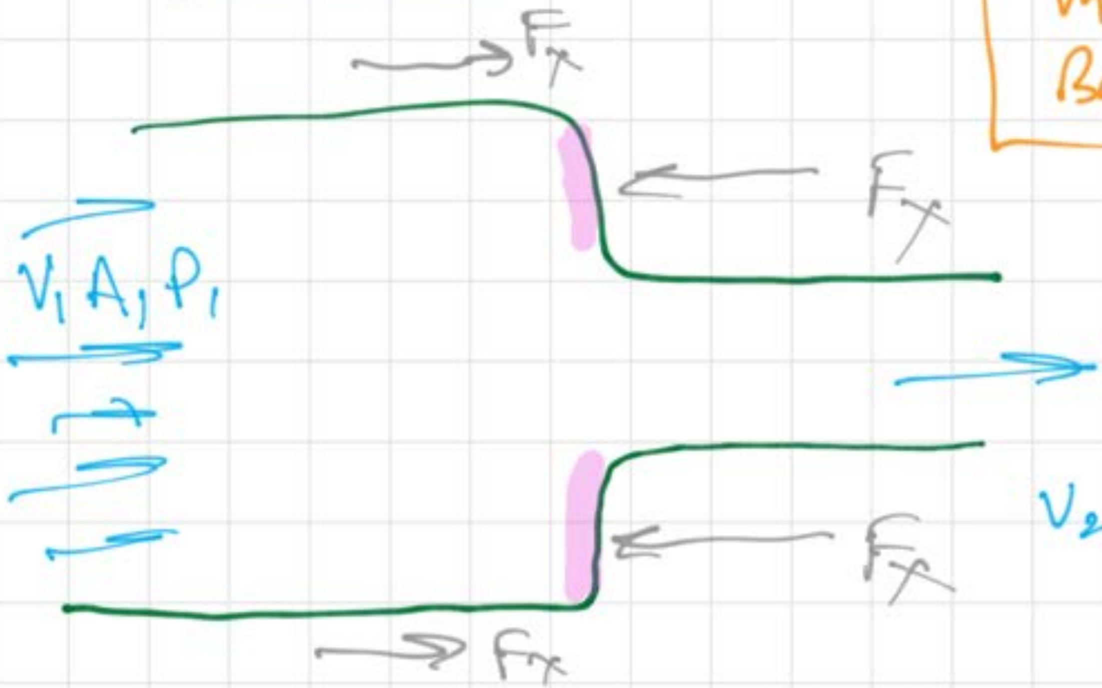
$$F_x = -2\rho V_1^2 A_1 - A_1(P_1 + P_2)$$

BALANCES 2X MOMENTUM

BALANCES ALL PRESSURE FORCE

AREA CHANGE

WILL NEED MASS BALACE!



$$0 = \rho_1 A_1 V_1 - \rho_2 A_2 V_2 \qquad \rho_1 = \rho_2$$

$$V_2 = V_1 \frac{A_1}{A_2}$$

SPEEDS UP.

$$-\rho V_1 V_1 A_1 + \rho V_2 V_2 A_2 = P_1 A_1 - P_2 A_2 + F_x$$

$$F_x = -\rho V_1^2 A_1 + \rho V_2^2 A_2 - P_1 A_1 + P_2 A_2$$

$$= -\rho V_1^2 A_1 \left(1 - \frac{A_1}{A_2}\right) - P_1 A_1 + P_2 A_2$$

$$F_x = -\rho V_1^2 A_1 \left(1 - \frac{A_1}{A_2}\right) - P_1 A_1 + P_2 A_2$$

< 0 NET BALANCE ON PRESSURE

FOR HIGH Re FLOWS,

NORMAL FORCES ARE MUCH LARGER THAN SHEAR FORCES

SO, JUST NEED TO CONSIDER PRESSURE