CBE 30357 9/21/17

TOPICS

1) LAMINAR & TURBULENT FLOW

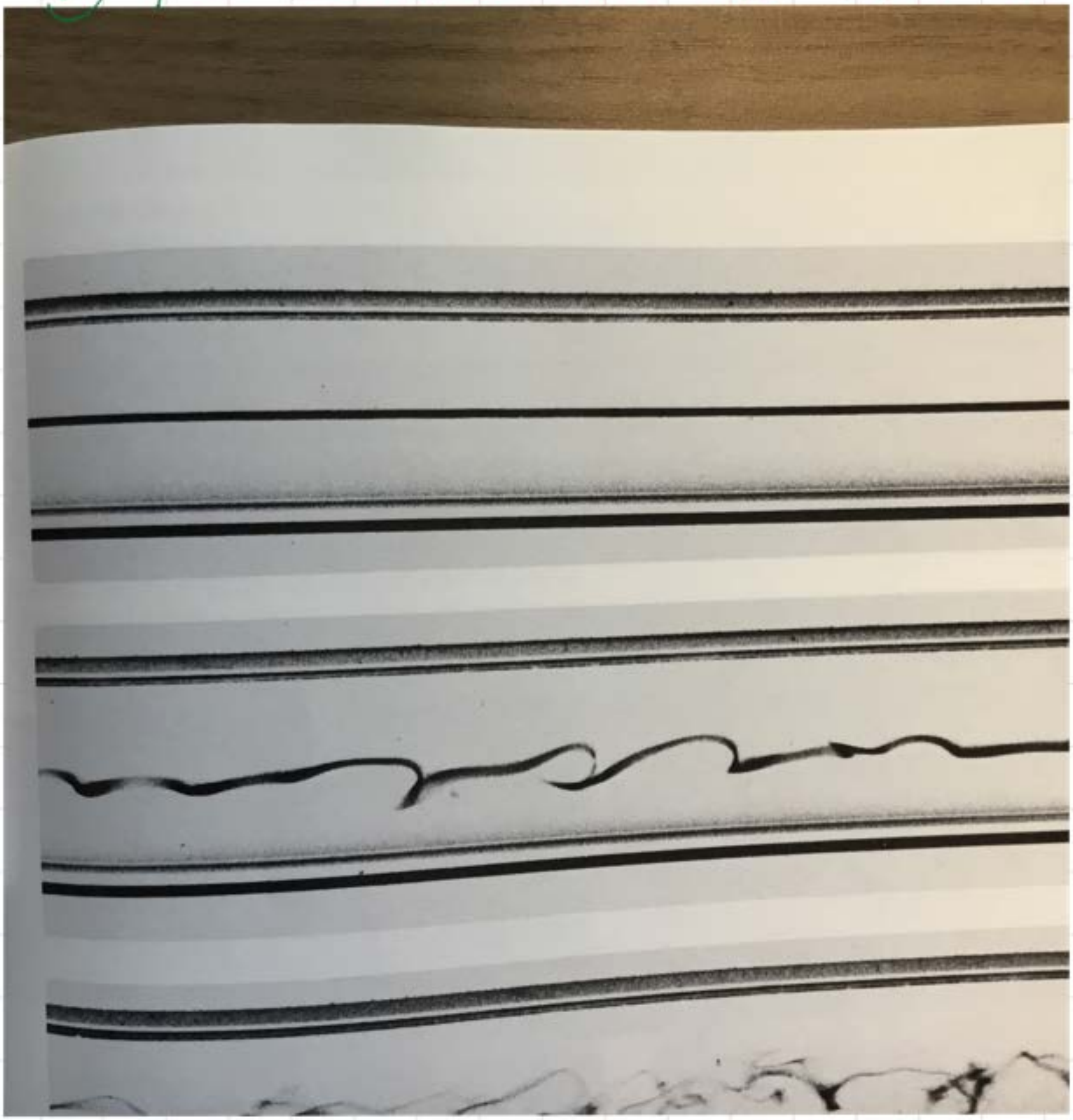
2) QUICK LOOK AT EXAM FROM PRIOR YEAR

3) INTRODUCTION/DESCRIPTION OF NON-NEWTONIAN FLUID BEHAVIOR

4) FLOW OF BINGHAM PLASTIC DOWN AN INCLINED PLATE

5) BLOOD FLOW

DYE STREAM IN PIPE FLOW



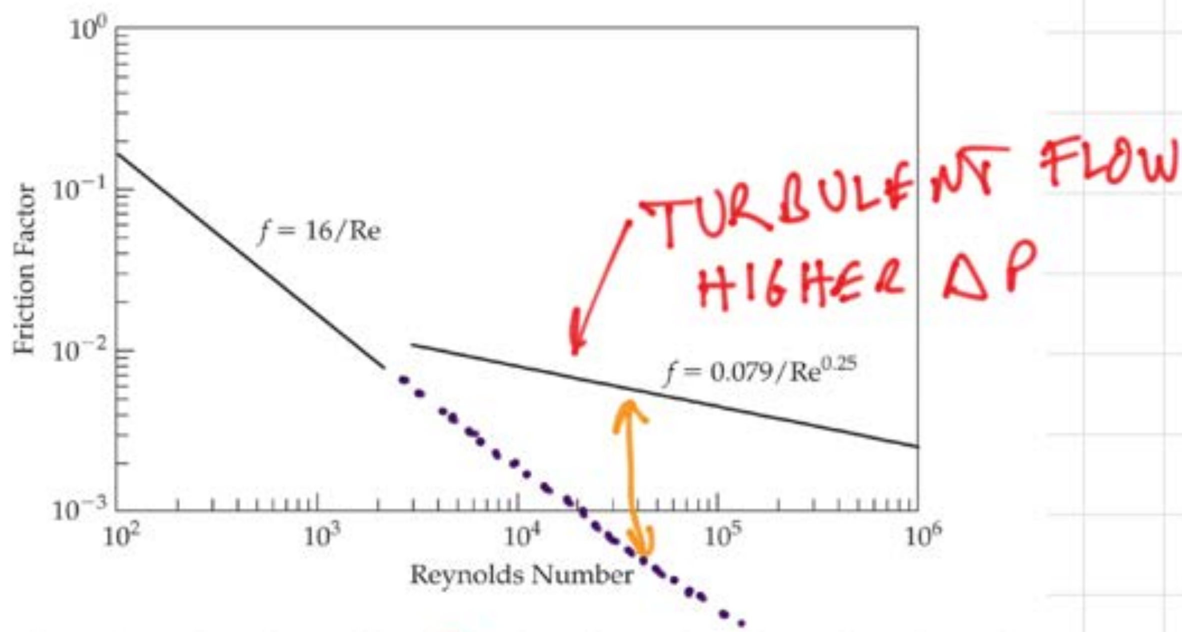
103. Repetition of Reynolds' dye experiment. Osborne Reynolds' celebrated 1883 investigation of stability of flow in a tube was documented by sketches rather than photography. However the original apparatus has survived at the University of Manchester. Using it a century later, N. H. Johannesen and C. Lowe have taken this sequence of photographs. In laminar flow a filament of colored water introduced at a bell-shaped entry extends undisturbed the whole length of the glass tube. Transition is seen in the second of the photographs as the speed is increased; and the last two photographs show fully turbulent flow. Modern traffic in the streets of Manchester made the critical Reynolds number lower than the value 13,000 found by Reynolds.

61

AS Re ↑ MORE "MIXING"
STRONGER DISTURBANCES

TRANSITION TO TURBULENT FLOW.

Figure 3.11 The Fanning friction factor versus the Reynolds number.



$$f \equiv \frac{\Delta P\, D}{2 L \rho v^2}$$

Turbulent velocity fluctuations "mix" slow moving and fast moving fluid and in doing so transfer the stress from the wall, that is opposing flow, into the middle of the pipe. Result: high pressure drop for a given flow than if the flow were laminar.

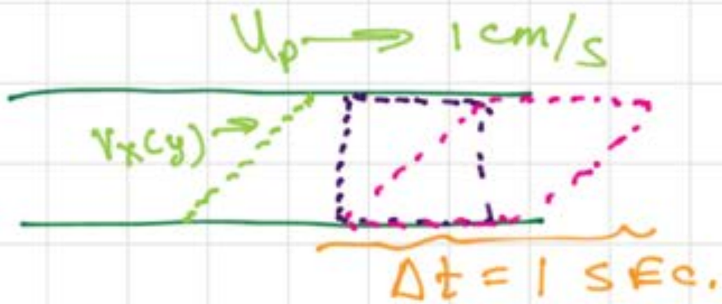
NEWTONIAN FLUID:

- YOU PUSH IT DEFORMS
- YOU PUSH HARDER, IT DEFORMS FASTER 🙂

$$\tau_{yx} = \mu \frac{\partial v_x}{\partial y}$$

$$= \mu \dot{\gamma}_x$$ ← SHEAR RATE

$$\tau_{yx} \sim \dot{\gamma}_x^{\,n} \qquad n = 1$$



$$\dot{\gamma} = \frac{\partial v_x}{\partial y} = \frac{1}{s}$$

ORIGINAL SHAPE IS DISLACED BY ITS SIZE IN 1 S.

NON NEWTONIAN FLUID

- IF YOU DON'T PUSH HARD ENOUGH ... IT DOESN'T FLOW

$$|\tau_{yx}| < \tau_0 \qquad \dot{\gamma}_x = 0$$

$$|\tau_{yx}| > \tau_0 \qquad \tau_{yx} = \pm\tau_0 + \mu\dot{\gamma}_x$$

BINGHAM PLASTIC

- AS YOU PUSH HARDER, IT APPEARS TO FLOW MORE EASILY

$$\eta = m|\dot{\gamma}_x|^{n-1}$$

$\eta \sim$ APPARENT VISCOSITY

$\eta = \mu$ IF $n = 1$

"SHEAR THINNING" IF $n < 1$

NON-NEWTONIAN FLUID

- YOU PUSH HARDER, IT RESISTS MORE

$$\eta = m |\dot{\gamma}_x|^{n-1}$$

"SHEAR THICKENING" IF $n > 1$

- FLUID REMEMBERS ITS PAST FLOW CONDITIONS...

$$\mu = \mu(t)$$

BINGHAM PLASTIC

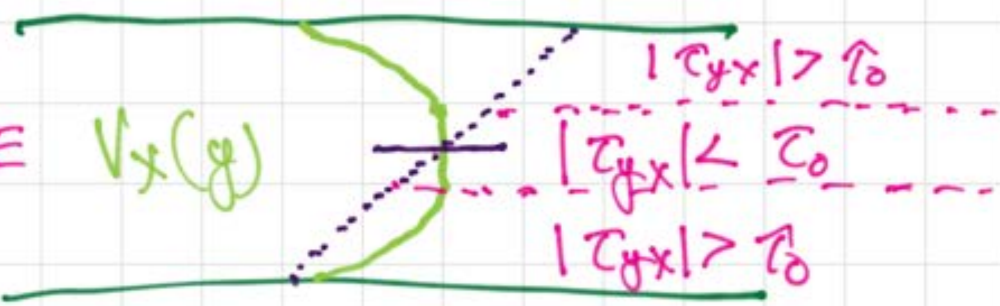
$$|\tau_{yx}| < \tau_0 \qquad \dot{\gamma}_x = 0$$

NO FLOW: FRESH PAINT ON A WALL

$$|\tau_{yx}| > \tau_0$$

$$\tau_{yx} = \pm \tau_0 + \mu_0 \dot{\gamma}_x$$

FLAT PROFILE IN MIDDLE



POWER LAW FLUIDS

APPARENT VISCOSITY CHANGES WITH SHEAR RATE

$$\eta_{APP} = m\,|\dot{\gamma}|^{n-1}$$

$n = 1$ NEWTONIAN

$n < 1$ SHEAR THINNING

$n > 1$ SHEAR THICKENING

$$\tau_{yx} = \left(m \frac{\partial v_x}{\partial y}^{(n-1)} \right) \frac{\partial v_x}{\partial y}$$

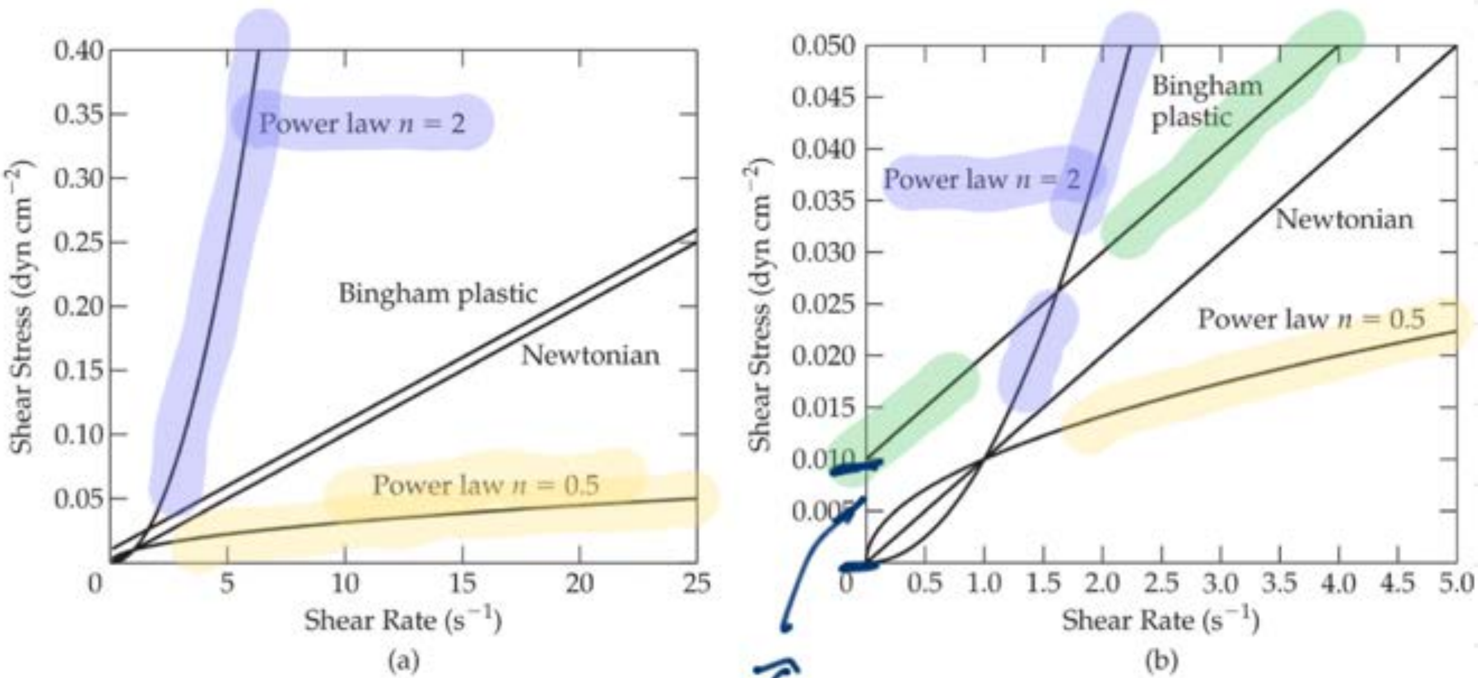
$n > 1$

$n < 1$

" μ "

rest file format.

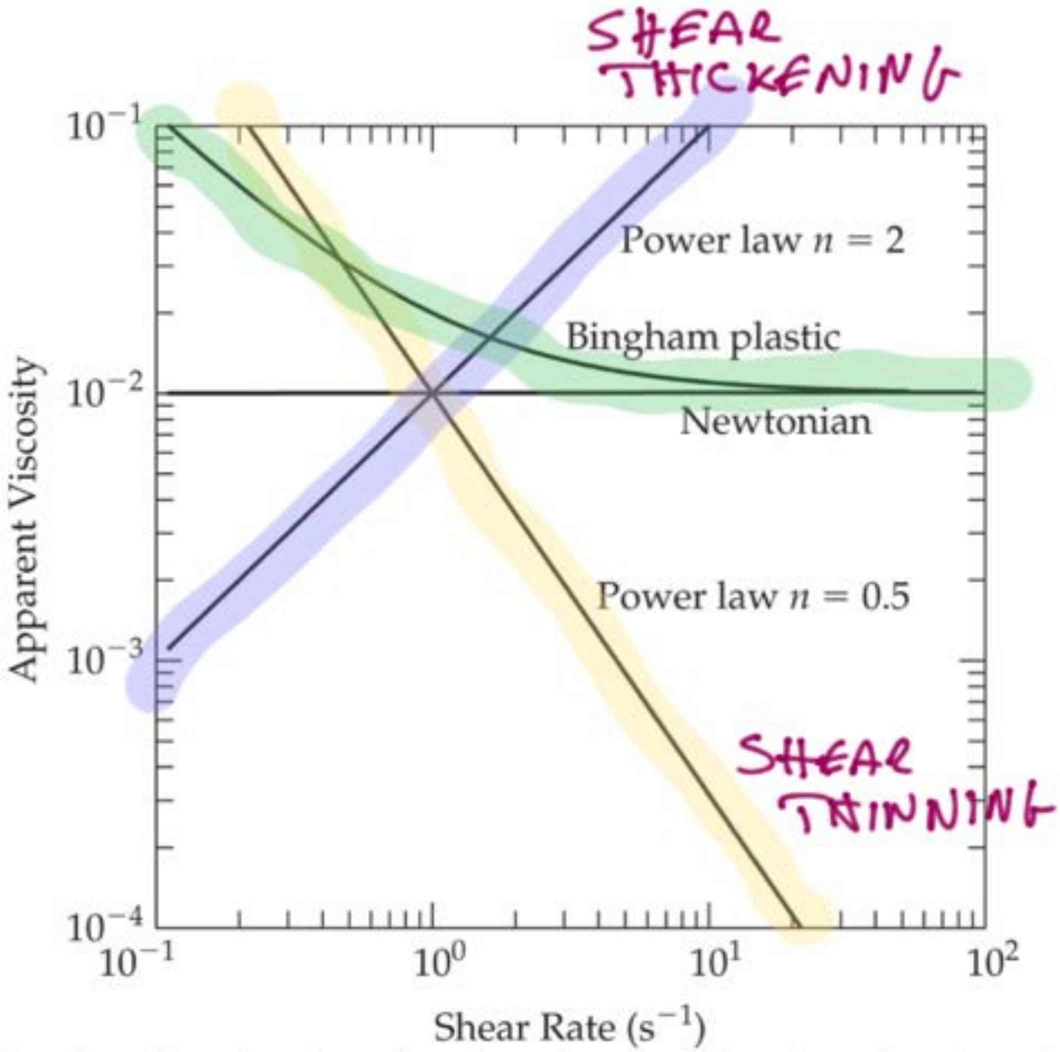
Figure 2.17 Relationship between the shear stress and shear rate for Newtonian and non-Newtonian fluids. Fluid properties are Newtonian fluid, μ = 0.01 g cm^{-1} s^{-1}; Bingham plastic, τ_0 = 0.01 dyn cm^{-2}, μ_0 = 0.01 g cm^{-1} s^{-1}; shear-thickening fluid (dilatant), m = 0.01 g cm^{-1}, n = 2.0; and shear thinning fluid (pseudoplastic), m = 0.01 g cm^{-1} s$^{-1.5}$, n = 0.5.



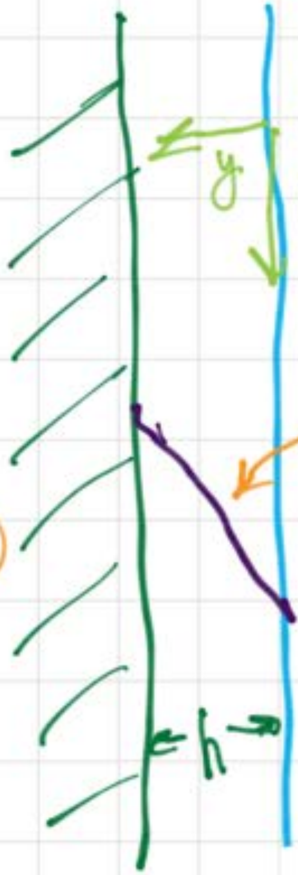
τ_0

$$\tau_{yx} = \tau_0 + \mu \frac{\partial v_x}{\partial y}$$

Figure 2.18 Apparent viscosity versus shear rate for fluids shown in Figure 2.17.



$$0 = \frac{\partial \tau_{yx}}{\partial y} + \rho g_x$$



NO FLOW

$$\tau_{yx} = -\rho g_x y$$

$$\tau_{yx}(h) = -\rho g_x h$$

IF $|\tau_{yx}(h)| < \tau_0$

$$|\tau_{yx}(h)| > \tau_0$$

$$\tau_{yx}(y) = \tau_0$$

τ_{yx}

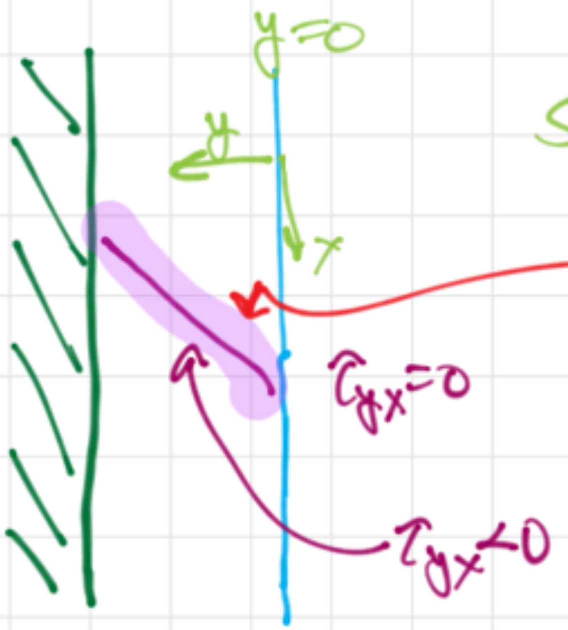
PARABOLIC

FLAT

$$\tau_{yx} = -\rho g_x y$$

FOR A BINGHAM PLASTIC:

$$\tau_{yx} = -\tau_0 + \mu \frac{\partial v_x}{\partial y}$$



SO y IS POSITIVE INWARD

CORRECTED DIRECTION 9/27
UP, NOT "DOWN"

SUBS CONSTITUTIVE RELATION INTO INTEGRATED MOMENTUM EQ.

$$-\tau_0 + \mu \frac{\partial v_x}{\partial y} = -\rho g_x y$$

$$\mu \frac{\partial v_x}{\partial y} = -\rho g_x y + \tau_0$$

$$\int dv_x = \int \left(\frac{-\rho g_x y}{\mu} + \tau_0 \right) dy$$

$$v_x = \frac{-\rho g_x}{\mu} \frac{y^2}{2} + \frac{\tau_0}{\mu} y + C_1$$

$$v_x(h) = 0$$

$$\therefore \quad C_1 = \frac{\rho g_x}{\mu} \frac{h^2}{2} - \frac{\tau_0}{\mu} h$$

$$\therefore \quad v_x = \frac{\rho g_x}{2\mu} \left(h^2 - y^2 \right) + \frac{\tau_0}{\mu} (y - h)$$

VALID IN $|\tau_{yx}| > |\tau_0|$

FOR $\tau_{yx} \leq \tau_0$

$$v_x = v_{MAX}$$

HOW DO WE FIND $y_0 \Rightarrow (\tau_{yx} = \tau_0)$?

FIND $\frac{dv_x}{dy} = 0$

$$\frac{dv_x}{dy} = -\frac{\rho g_x}{\mu} y + \frac{\tau_0}{\mu} = 0$$

$$y_0 \equiv y\Big|_{\tau=\tau_0} = \frac{\tau_0}{\rho g_x}$$

SO FOR $y < y_0$

$$V_x = \frac{\rho g_x}{2\mu}\left(h^2 - y^2\right) + \frac{\tau_0}{\mu}(y - h)$$

$$V_{x\,MAX} = \frac{\rho g_x}{2\mu}\left(h^2 - \left(\frac{\tau_0}{\rho g_x}\right)^2\right) + \frac{\tau_0}{\mu}\left(\frac{\tau_0}{\rho g_x} - h\right)$$

$$V_{MAX} = \frac{\left(\tau_0 - \rho g_x h\right)^2}{2 g_x \mu \rho}$$

DOWNWARD

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In[62]:= Show[%, %%, PlotRange -> All]
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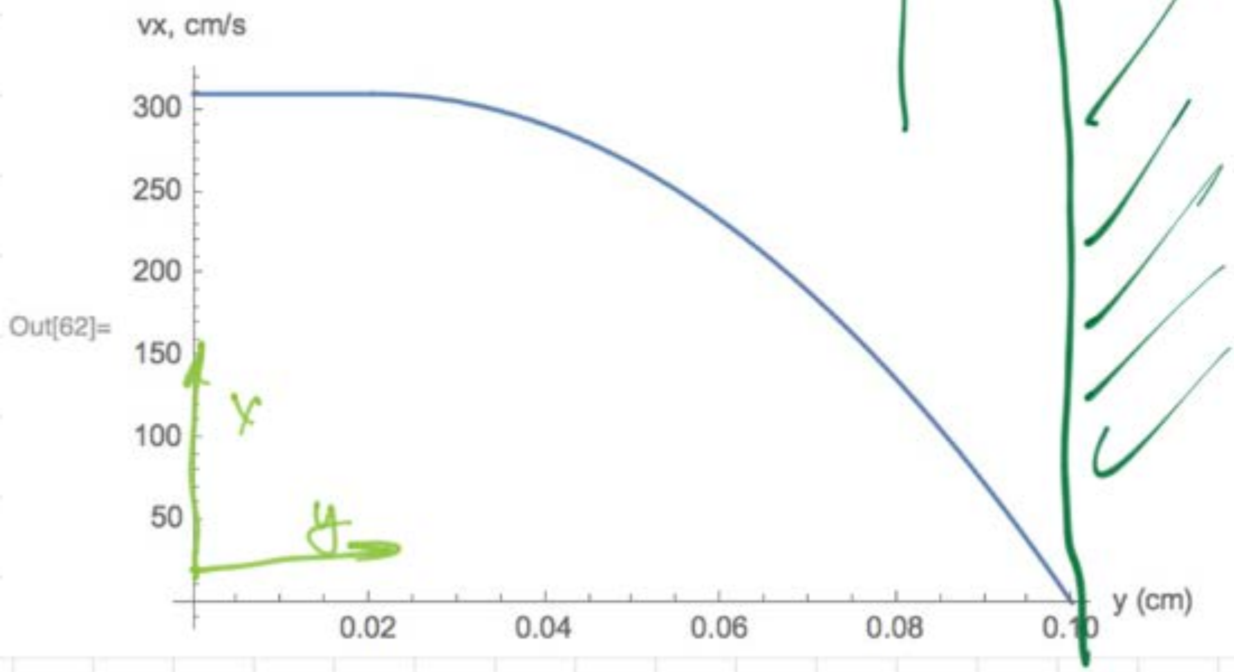
Out[62]=




```
Plot[veex /. {gx → 980, ρ → 1, h → .1, τ0 → 20, μ → .01}, {y, 20 / 980, .1},
 AxesLabel → {"y (cm)", "vx, cm/s"}]
```

TO GET FLOWRATE INTEGRATE 2 PIECES

$$Q = W \int_0^{\tau_0/\rho g_x} V_{MAX}\, dy +$$

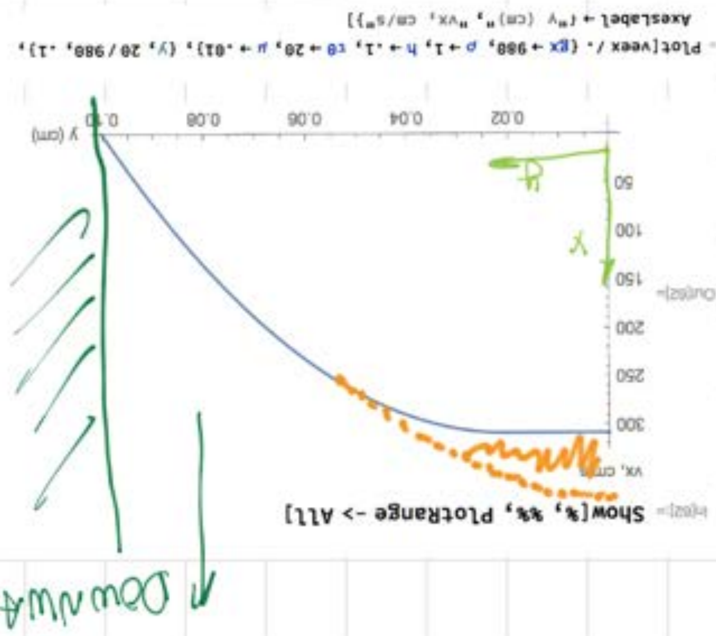
$$W \int_{\tau_0/\rho g_x}^{h} \left(\frac{\rho g_x}{2\mu}\left(h^2 - y^2\right) + \frac{\tau_0}{\mu}(y-h) \right) dy$$

$$Q = W\frac{(\tau_0 - \rho g_x h)^2 (\tau_0 + 2\rho g_x h)}{6 g_x^2 \mu \rho^2}$$

IF WE DO A SERIES EXPANSION

$$Q = \frac{\rho g_x h^3}{3\mu} - \frac{\tau_0 h^2}{2\mu} + O(\tau_0^3)$$

EXTENT OF REDUCTION IN FLOW



BLOOD FLOW

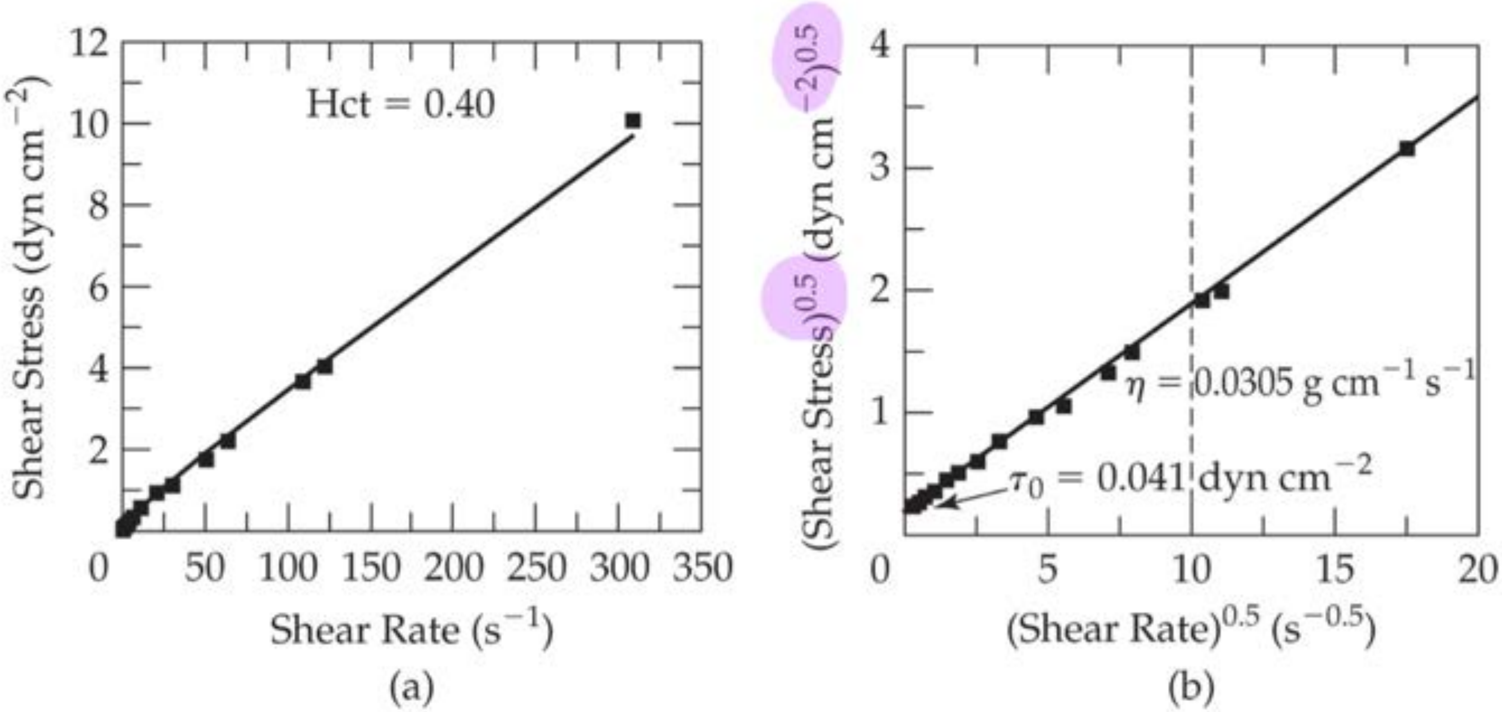
In the largest arteries for humans, the Reynolds number is large enough that we would expect turbulent flow and for the first few generations of the arterial system the flow is pulsatile.

So there are some limits to exact application of the analysis that we can do.

However, we can get some useful insights so we shall proceed. (You can read more at another time.)

ile format.

Figure 2.30 (a) Shear stress versus shear rate for blood at a hematocrit of 0.40. (b) Data in (a) replotted according to Equation (2.8.6). (Adapted from Ref. [25].)

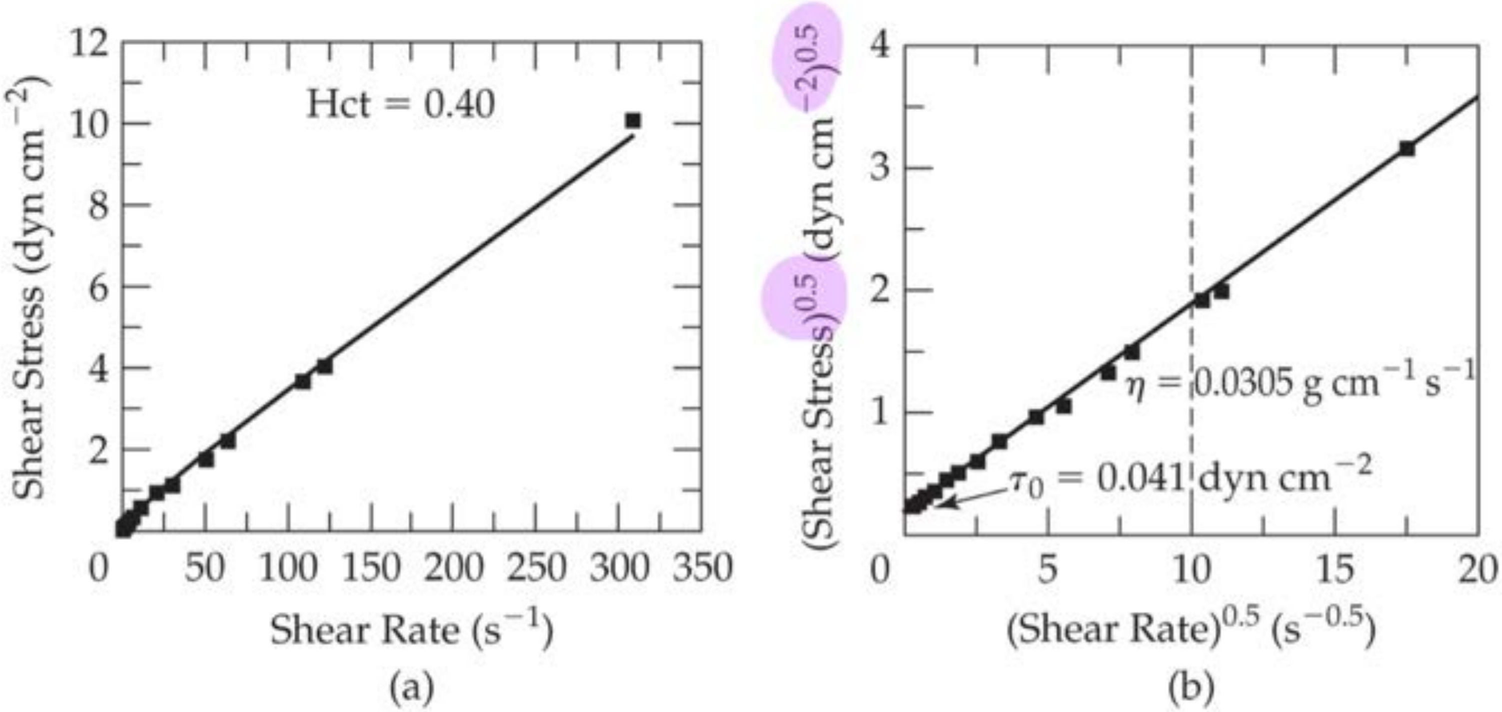


$$\sqrt{\tau} = \sqrt{\tau_y} + \sqrt{\eta\dot{\gamma}}$$

YEILD STRESS

SHEAR THINNING

Figure 2.30 (a) Shear stress versus shear rate for blood at a hematocrit of 0.40. (b) Data in (a) replotted according to Equation (2.8.6). (Adapted from Ref. [25].)



$$\sqrt{\tau} = \sqrt{\tau_y} + \sqrt{\eta\dot{\gamma}}$$

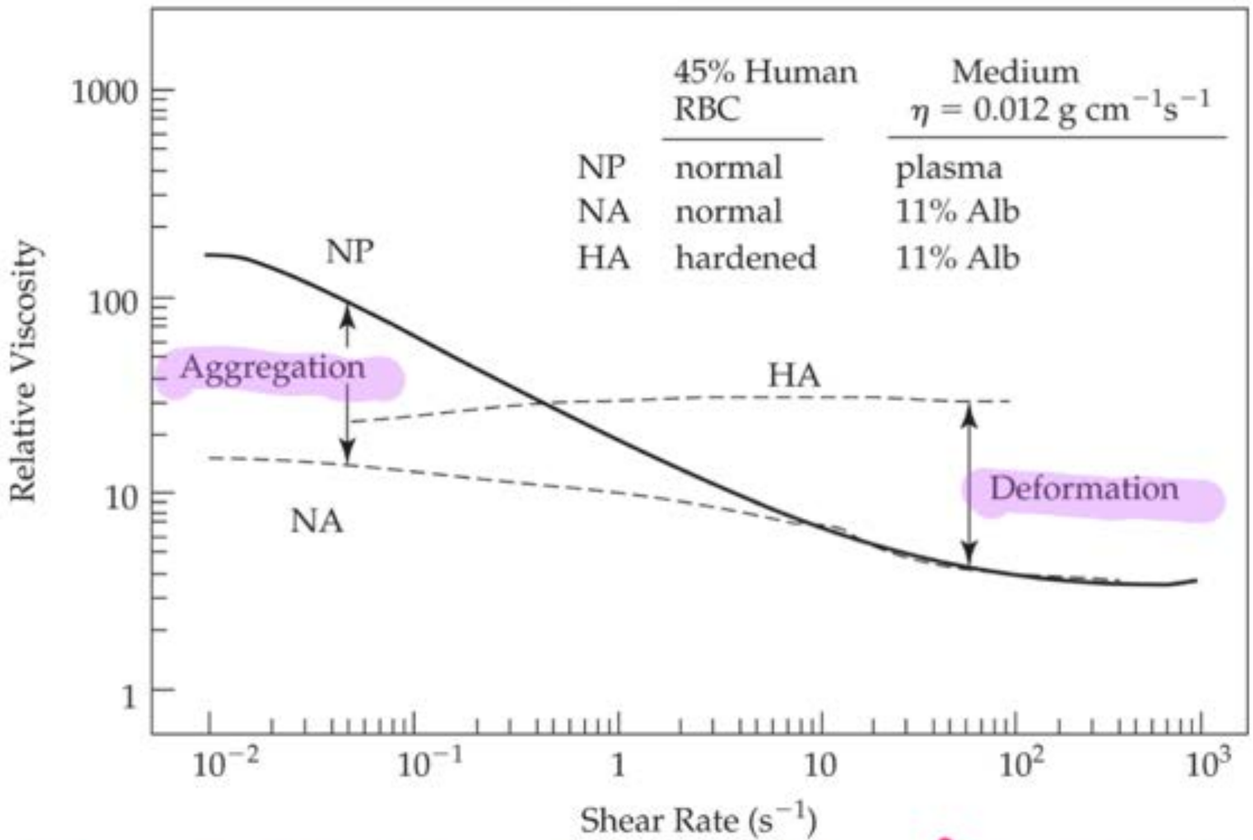
SHEAR THINNING

YEILD STRESS

$$\sqrt{\tau} = \sqrt{\tau_0} + \sqrt{\eta\dot{\gamma}}$$

file format.

Figure 2.32 The apparent viscosity of RBC suspensions (divided by the plasma viscosity) as a function of shear rate. NP = normal RBC in plasma; NA = normal RBC in isotonic saline containing 11% albumin in order to make the liquid viscosity equal the plasma viscosity; HA = glutaraldehyde- fixed RBC in the same saline solution. (Reprinted with permission from Ref. [29], © 1970 American Association for the Advancement of Science.)



BLOOD VISCOSITY DECREASES BECAUSE RBC'S DEFORM AS $\dot{\gamma}$ INCREASES

BLOOD ALSO EXHIBITS A YIELD STRESS

$\tau_y \cong .05\ \mathrm{dyne/cm^2}$

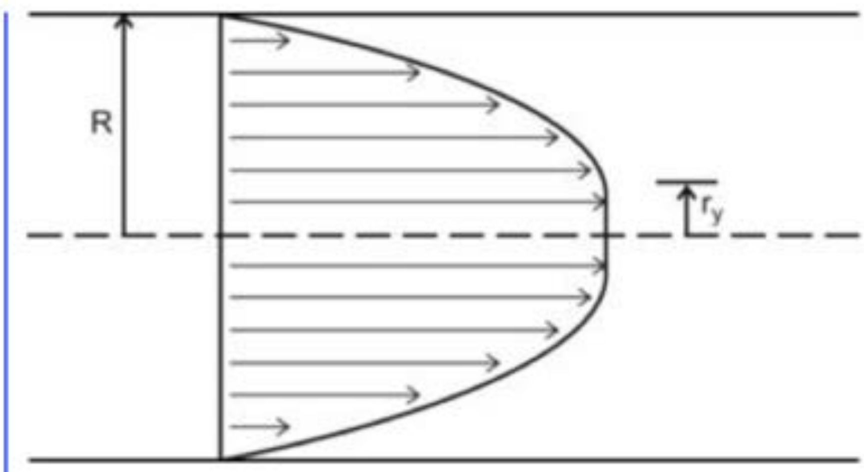


Figure 5.7 The velocity profile of blood flowing

$$\sqrt{\tau} = \sqrt{\tau_y} + \sqrt{\eta\dot{\gamma}}$$

BLOOD CONTAINS

LIQUID: PLASMA

PARTICLES: RED BLOOD CELLS... ETC

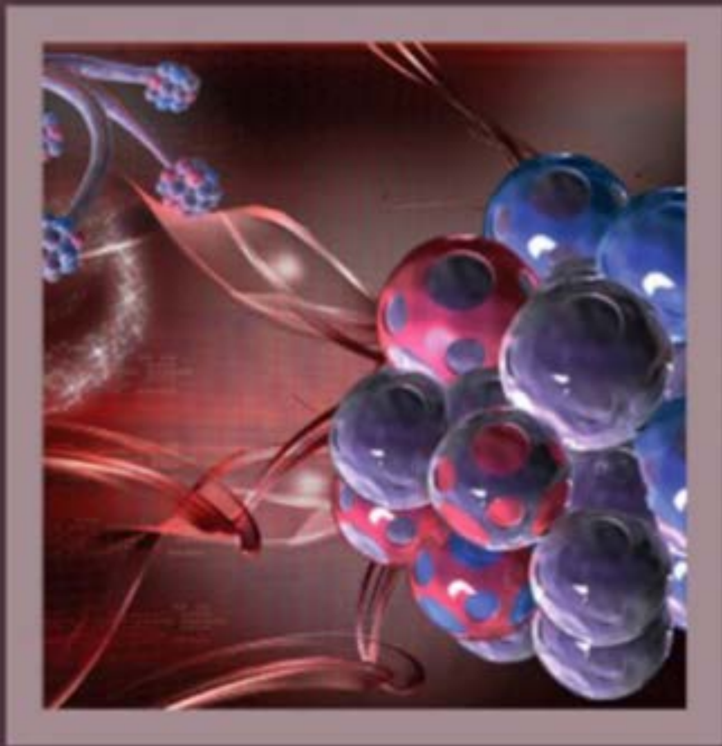
SO WE COULD MODEL VISCOSITY WITH EINSTEIN'S RELATION:

$$\eta = \mu_0 (1 + 2.5\phi)$$

VOLUME FRACTION OF PARTICLES

VALID FOR SUSPENSION OF SOLID SPHERES $\phi \leq .05$

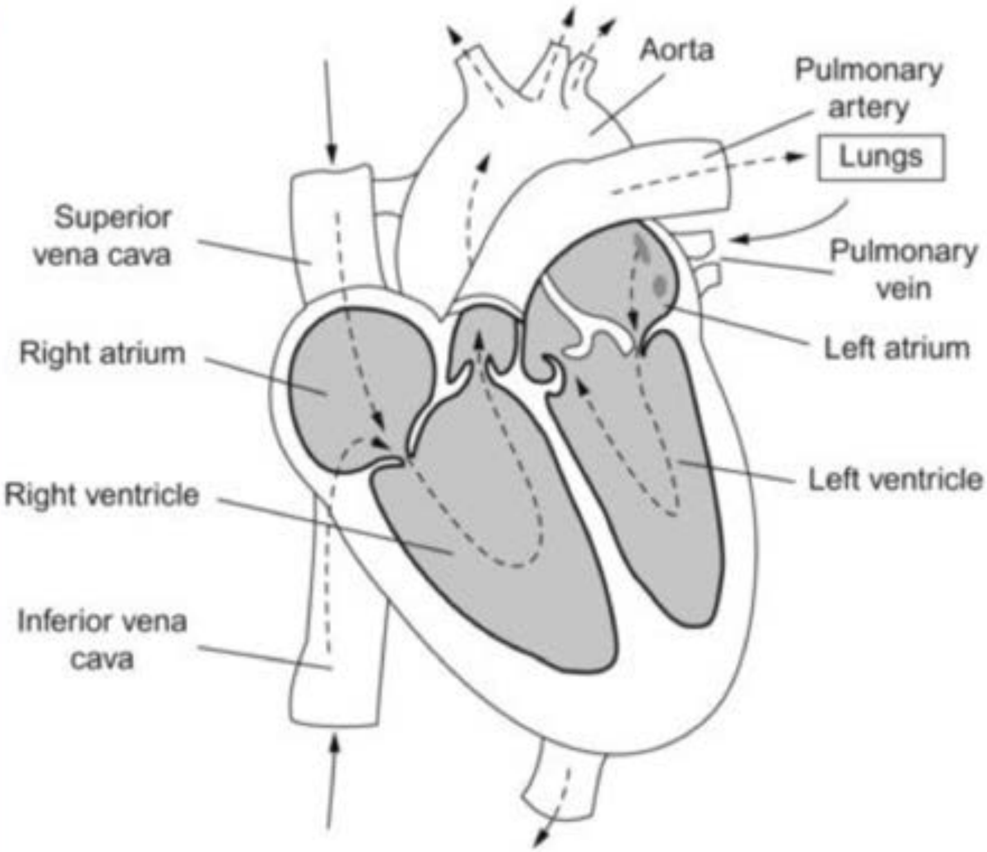
ACADEMIC PRESS SERIES IN BIOMEDICAL ENGINEERING

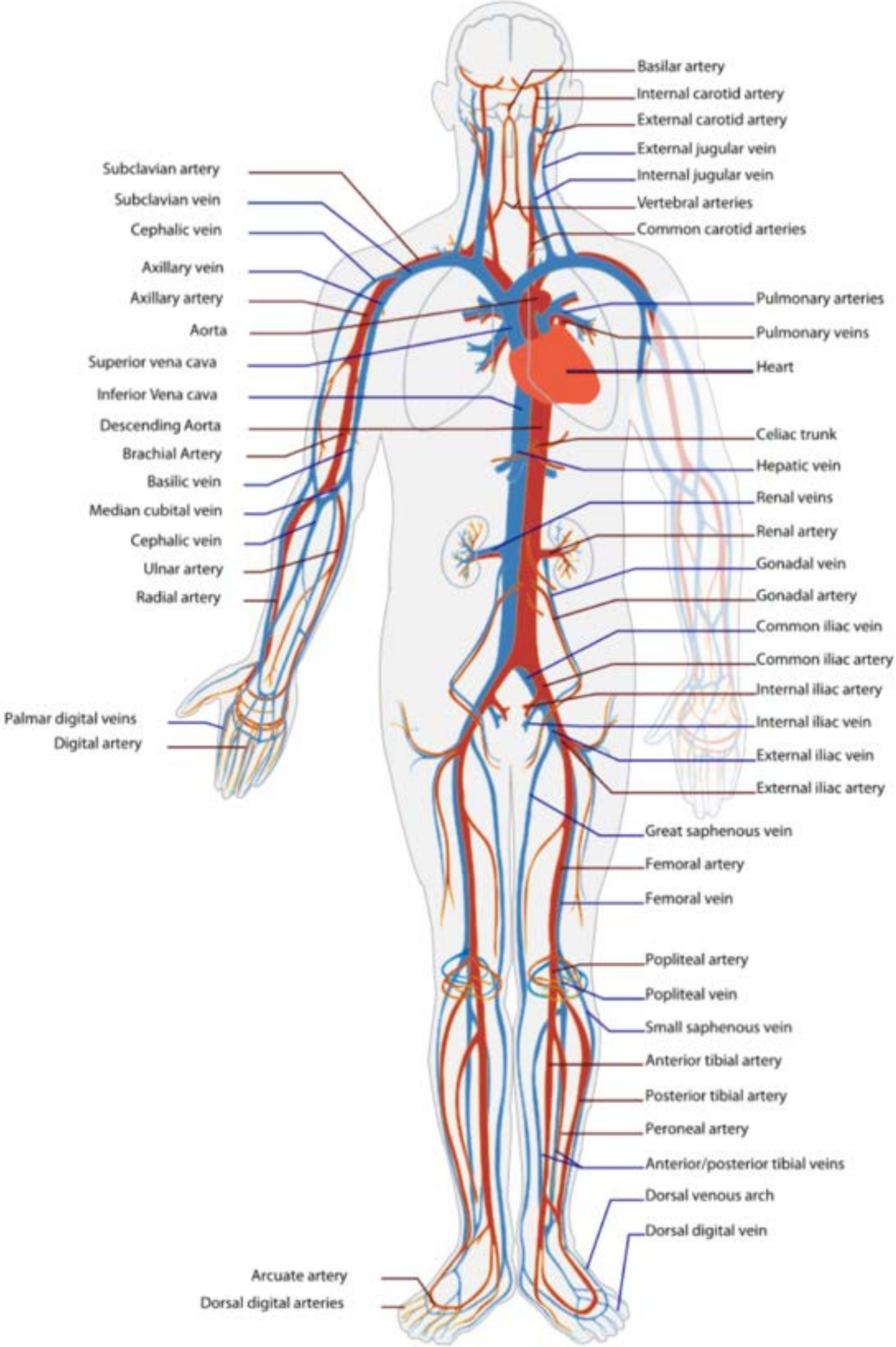


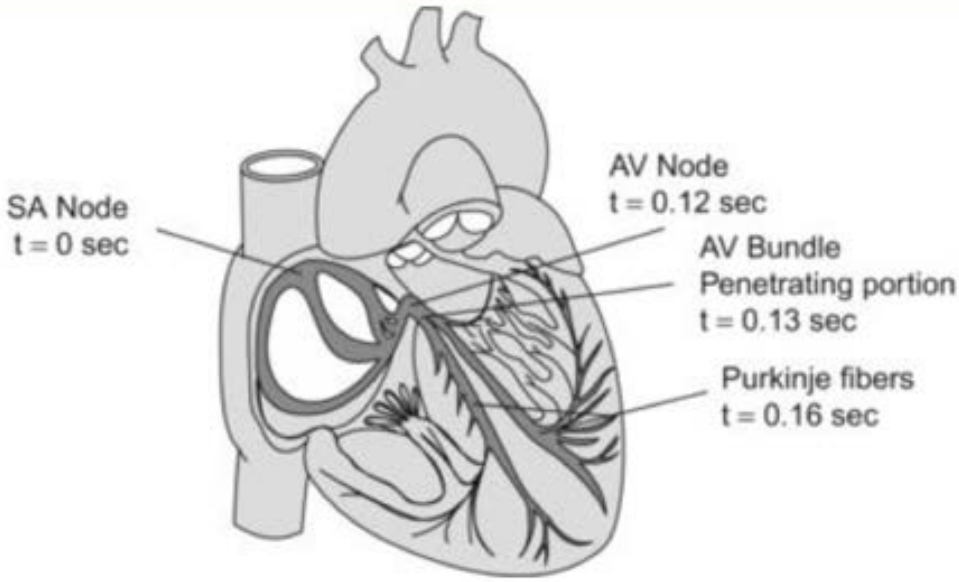
BIOFLUID MECHANICS

AN INTRODUCTION TO FLUID MECHANICS, MACROCIRCULATION, AND MICROCIRCULATION

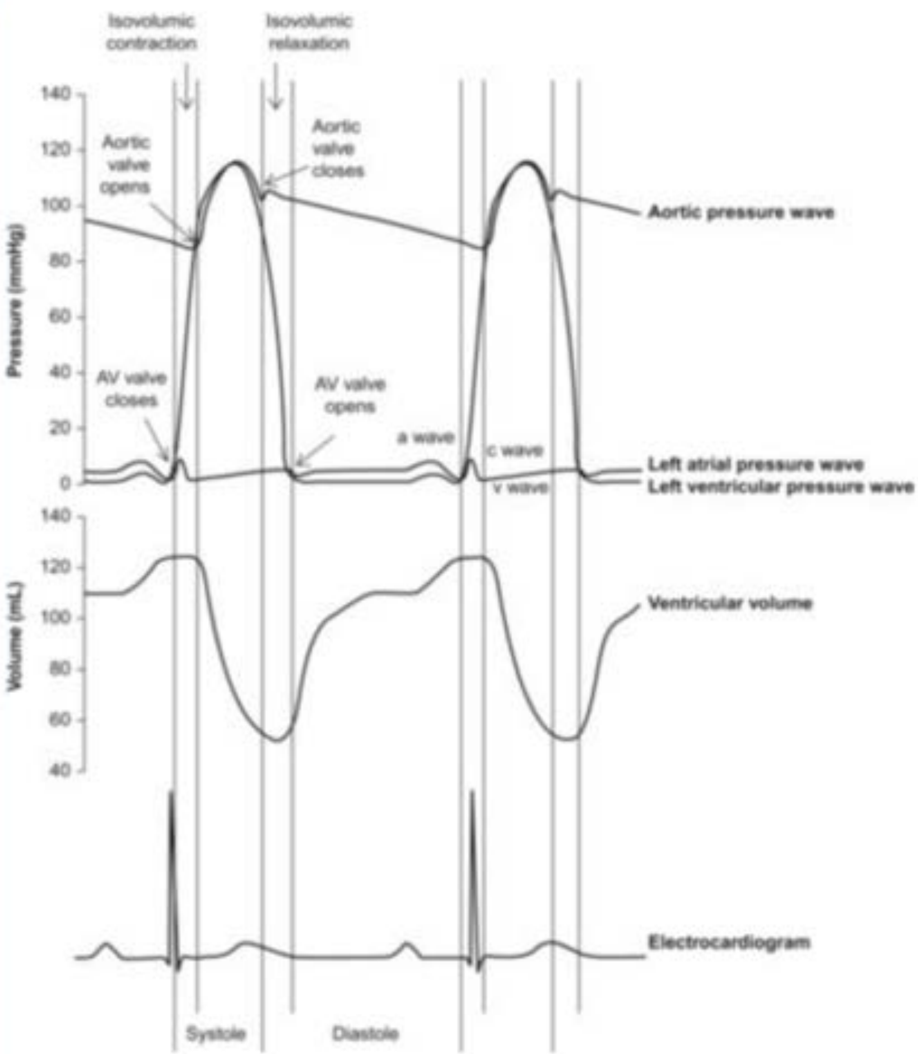
DAVID A. RUBENSTEIN
WEI YIN
MARY D. FRAME


Aorta
Pulmonary artery
Lungs
Superior vena cava
Pulmonary vein
Right atrium
Left atrium
Right ventricle
Left ventricle
Inferior vena cava

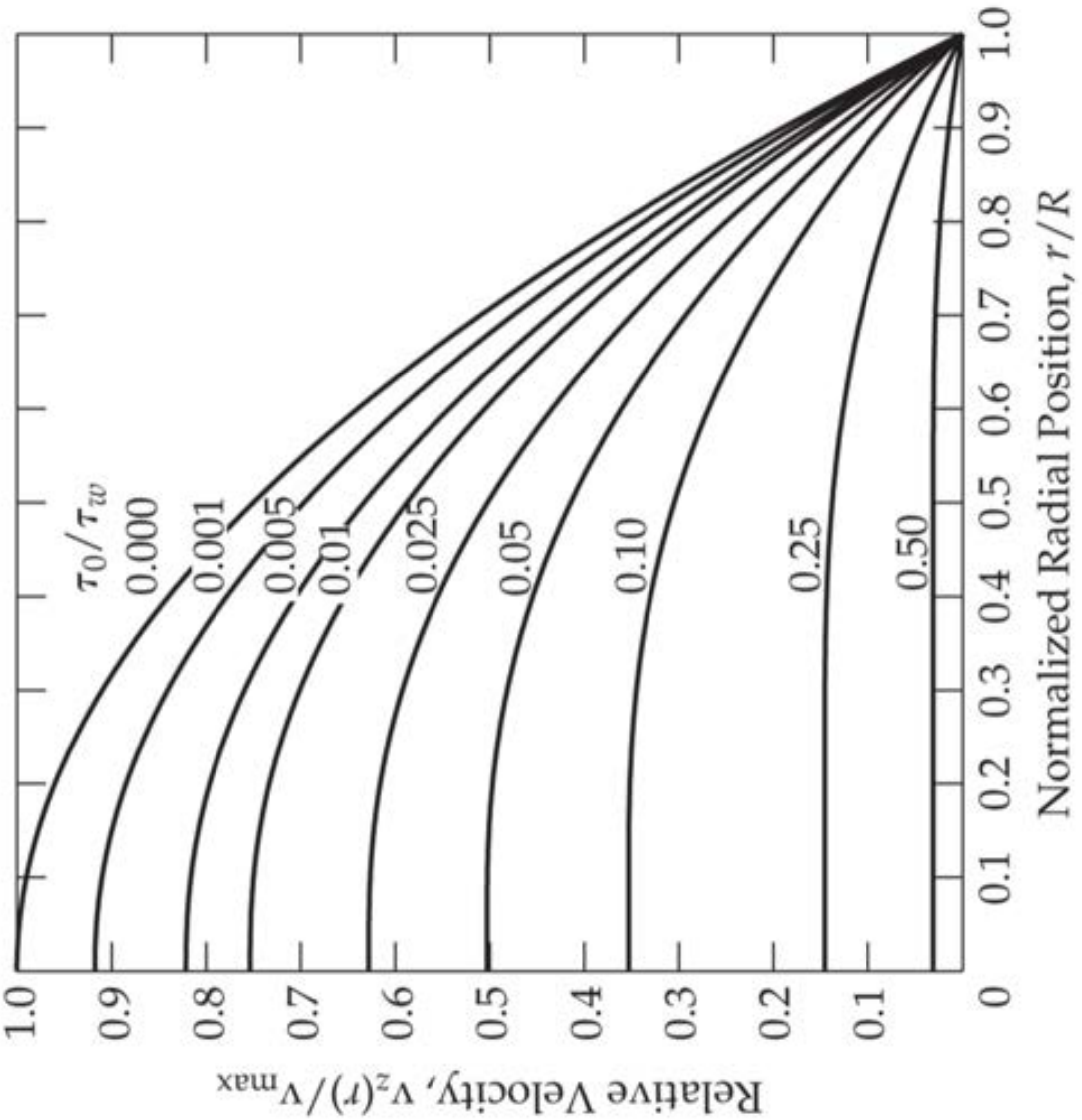

Basilar artery
Internal carotid artery
External carotid artery
External jugular vein
Internal jugular vein
Vertebral arteries
Common carotid arteries
Subclavian artery
Subclavian vein
Cephalic vein
Axillary vein
Axillary artery
Aorta
Superior vena cava
Inferior Vena cava
Descending Aorta
Brachial Artery
Basilic vein
Median cubital vein
Cephalic vein
Ulnar artery
Radial artery
Palmar digital veins
Digital artery
Pulmonary arteries
Pulmonary veins
Heart
Celiac trunk
Hepatic vein
Renal veins
Renal artery
Gonadal vein
Gonadal artery
Common iliac vein
Common iliac artery
Internal iliac artery
Internal iliac vein
External iliac vein
External iliac artery
Great saphenous vein
Femoral artery
Femoral vein
Popliteal artery
Popliteal vein
Small saphenous vein
Anterior tibial artery
Posterior tibial artery
Peroneal artery
Anterior/posterior tibial veins
Dorsal venous arch
Dorsal digital vein
Arcuate artery
Dorsal digital arteries



SA Node
t = 0 sec
AV Node
t = 0.12 sec
AV Bundle
Penetrating portion
t = 0.13 sec
Purkinje fibers
t = 0.16 sec



Isovolumic contraction
Isovolumic relaxation
Aortic valve opens
Aortic valve closes
Aortic pressure wave
AV valve closes
AV valve opens
a wave
c wave
v wave
Left atrial pressure wave
Left ventricular pressure wave
Pressure (mmHg)
Volume (mL)
Ventricular volume
Electrocardiogram
Systole
Diastole



SHAPE CHANGES WITH τ

SOME DATA ABOUT BLOOD FLOW IN DOGS ...

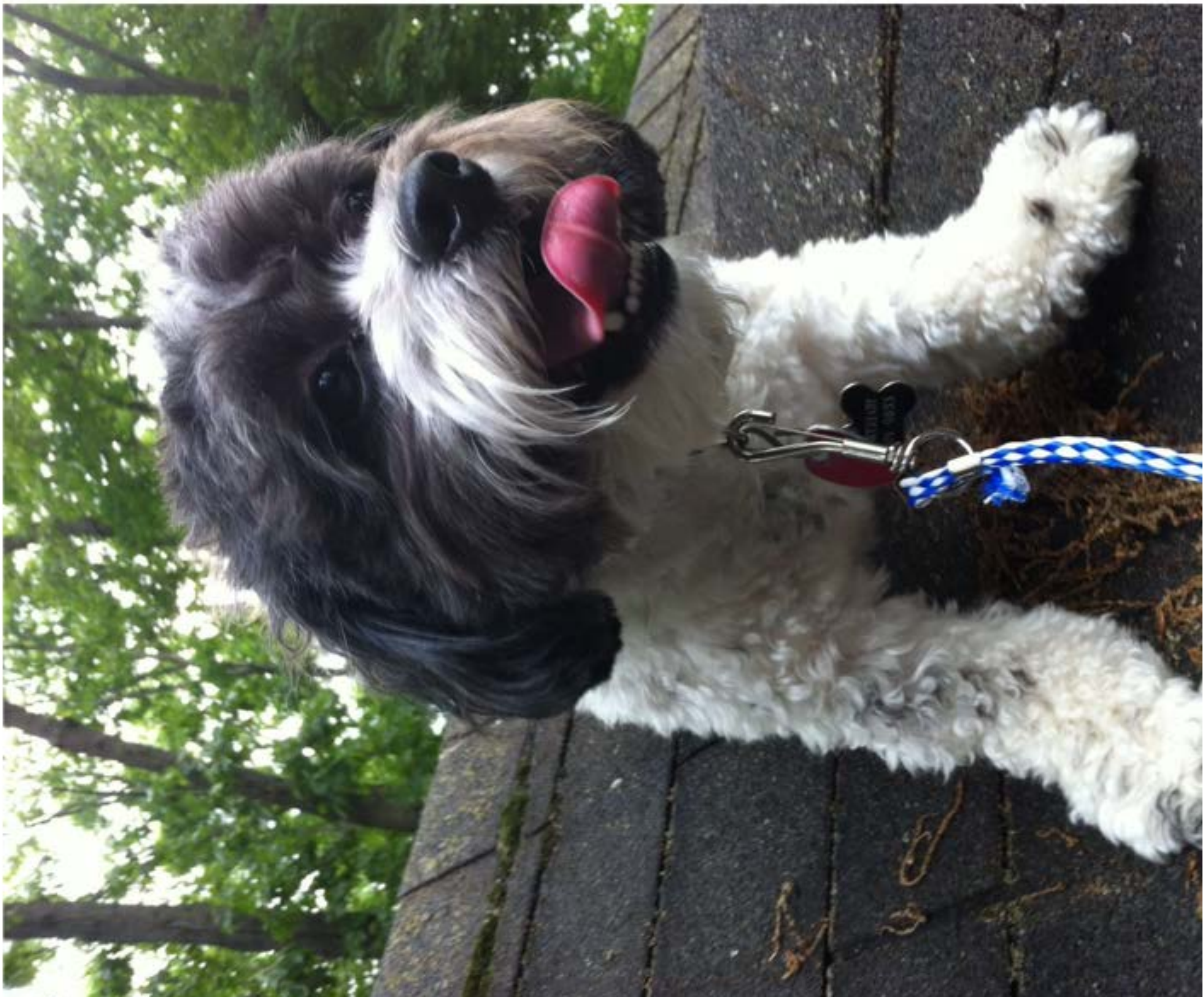
TABLE 2.4

Dimensions, Velocities, and Reynolds Numbers in the Canine Cardiovascular System

Vessel	Internal diameter (cm)	Length (cm)	Peak blood velocity (cm s^{-1})	Re_{peak}	Mean blood velocity (cm s^{-1})	Re_{mean}
Ascending Aorta	1.5	5	120	4,500	20	750
Descending Aorta	1.3	20	105	3,400	20	648
Abdominal Aorta	0.9	15	55	1,250	15	341
Femoral Artery	0.4	10	100	1,000	10	100
Arteriole	0.005	0.15	75	0.09	0.5–1	0.0006–0.0012
Capillary	0.0006	0.06	7	0.001	0.02–0.17	$2.86-24.3 \times 10^{-6}$
Venule	0.004	0.15	35	0.035	0.2–0.5	0.0002–0.0005
Inferior Vena Cava	1.0	30	25	700		
Main Pulmonary Artery	1.7	3.5	70	3,000	0.15	6.43

Source: Adapted from Ref. [19], with permission.

HOW MUCH STRESS IS BLOOD SUBJECT TO



Can we say anything about how the blood vessel branching might occur?

There is obviously a trade off between size and pressure drop

This is similar to capital versus operating costs for a chemical process.

We might suspect there are physiological constraints.

Perhaps there is a "sweet spot" in the amount of wall shear that this collection of components finds optimal.

Let's check some numbers....

AT LEAST FOR
NEWTONIAN MODEL

$R_e \longrightarrow f$

IF WE HAVE μ & D

WE WOULD GET $\frac{\Delta P}{L}$

$$\rho\left[\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right] = \rho g_z - \frac{\partial p}{\partial z} + \left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right]$$

Cylindrical coordinates

***r* component**

$$\rho\left[\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right] = \rho g_r - \frac{\partial p}{\partial r} + \left[\frac{1}{r}\frac{\partial(r\tau_{rr})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta r}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{zr}}{\partial z}\right]$$

***θ* component**

$$\rho\left[\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right] = \rho g_\theta - \frac{\partial p}{\partial \theta} + \left[\frac{1}{r^2}\frac{\partial(r^2\tau_{r\theta})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{z\theta}}{\partial z}\right]$$

***z* component**

$$\rho\left[\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right] = \rho g_z - \frac{\partial p}{\partial z} + \left[\frac{1}{r}\frac{\partial(r\tau_{rz})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}\right]$$

Spherical coordinates

***r* component**

$$\rho\left[\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right] = \rho g_r - \frac{\partial p}{\partial r} + \left[\frac{1}{r^2}\frac{\partial(r^2\tau_{rr})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\tau_{\theta r}\sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}\right]$$

$$\frac{1}{r}\frac{d}{dr}(r\tau_{rz}) = \frac{dP}{dz}$$

$$\int d(r\tau_{rz}) = \int \frac{dP}{dz} r\,dr$$

$$\boxed{\tau_{rz} = \frac{dP}{dz}\frac{r}{2}} + \cancel{\frac{C_1}{r}}\ 0$$

$$\tau_{rr} = 2\mu \cancel{\frac{\partial v_r}{\partial r}} \rightarrow 0$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left(r \cancel{\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)} + \frac{1}{r}\cancel{\frac{\partial v_r}{\partial \theta}} \right) \quad 0$$

$$\tau_{zr} = \tau_{rz} = \mu\left(\cancel{\frac{\partial v_r}{\partial z}} + \frac{\partial v_z}{\partial r} \right)$$

$$\tau_{rz} = \mu \frac{\partial V_z}{\partial r}$$

$$\tau_{\theta\theta} = 2\mu\left(\frac{1}{r}\cancel{\frac{\partial v_r}{\partial \theta}} + \cancel{\frac{v_r}{r}} \right)$$

$$\tau_{z\theta} = \tau_{\theta z} = \mu\left(\cancel{\frac{\partial v_\theta}{\partial z}} + \frac{1}{r}\cancel{\frac{\partial v_z}{\partial \theta}} \right)$$

$$\tau_{zz} = 2\mu \cancel{\frac{\partial v_z}{\partial z}}$$

Spherical coordinates

TO GET A NOMINAL VALUE FOR WALL SHEAR STRESS WE WILL USE NEWTONIAN FLOW

FROM f – Re RELATION

$$\langle v_z \rangle = \frac{1}{2} \frac{\Delta P}{4 L \mu} R^2$$

$$\frac{\Delta P}{L} = \frac{8 \langle v_z \rangle \mu}{R^2}$$

$\mu \sim .03\ g/cm\text{-}s$

$$\tau_w = \frac{4 \langle v_z \rangle \mu}{R}$$

CAN THE DATA GIVE WALL SHEAR STRESS?

$$\langle v_z \rangle = \frac{1}{2} \frac{\Delta P}{4L\mu} R^2$$

$$\frac{\Delta P}{L} = \frac{8 \langle v_z \rangle \mu}{R^2}$$

$$\mu \sim .03\ g/cm \cdot s$$

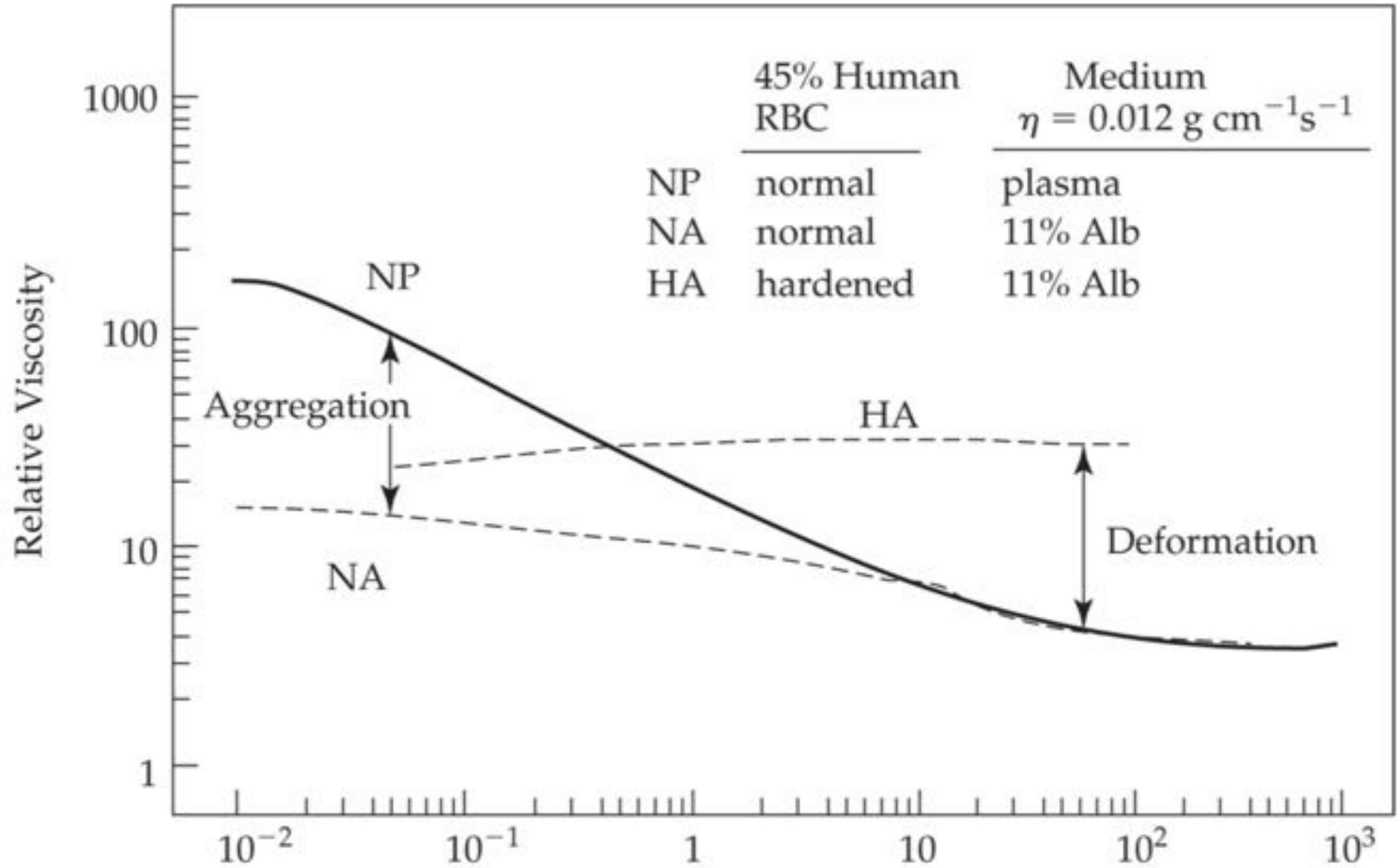


TABLE 2.5

Properties of Whole Blood

Parameters	Values	Solute	meq L^{-1}
pH	7.35 – 7.40	Na^+	144
$\mu/\mu H_2O$ (37°C)	~3.0	K^+	5
$\rho/\rho H_2O$ (25°C/4°C)	1.0546	Ca^{++}	2.5
Surface tension	~75 dyn cm^{-1}	Mg^{++}	1.5
Venous hematocrit	Male 0.47	Cl^-	107
	Female 0.42	$HCO3^-$	27
Whole blood volume	~78 mL kg^{-1} body wt.	HPO_4^-	2
Plasma or serum colloid osmotic pressure	~330 mmH_2O	$H_2PO_4^-$	2
		SO_4^-	0.5
Water content	~93% by volume	Amino acids	2
Cellular components	cells mL^{-1} whole blood	Creatinine	0.2
Erythrocyes	Male 5.4×10^9	Lactate	1.2
	Female 4.8×10^9	Glucose	5.6
Leukocytes	~7.4×10^6	Protein	1.2
Platelets	~2.8×10^8	Urea	1

ASCENDING AORTA

PEAK VELOCITY. . .

$$\frac{\Delta P}{\Delta L} = \frac{8\,(120)\cdot .03}{(.75)^2}$$

$$= 51.2\ \frac{\text{DYNES}}{\text{cm}^3}$$

$$\tau_w = -51.2\ \frac{.75}{2}$$

$$\boxed{\tau_w = 19.2\ \text{DYNES}/\text{cm}^2}$$

$$\tau_w = \frac{-4 \langle v_z \rangle \mu}{R}$$

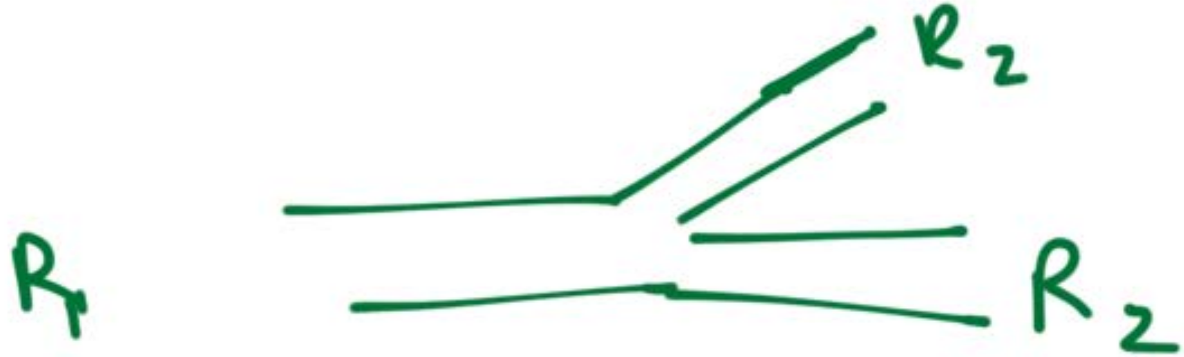
FOR MEAN VELOCITY

$$\tau_w = 3.2 \text{ DYNE}/\text{cm}^2$$

WHAT ABOUT OTHER VESSELS...

	PEAK	AVE	
D.A.	19.4	3.7	
A.A	14.7	4	6
A	3600!!	24	

NONE OF THESE ARE
COMPLETELY CONSTANT
BUT CONSIDER A
BRANCHING FLOW
WHERE WE WANT TO
KEEP STRESS CONSTANT



FLOWRATE IS 1/2 ORIGINAL IN EACH "DAUGHTER" VESSEL

$$Q_1 = \pi R_1^2 \langle v_{z_1} \rangle$$

$$Q_2 = \pi R_2^2 \langle v_{z_2} \rangle$$

$$Q_2 = \tfrac{1}{2} Q_1$$

WE WANT

$$\tau_{w_1} = \tau_{w_2} \Rightarrow \frac{4 \langle v_{z_1} \rangle \mu}{R_1}$$

OR 2

$$\frac{4 \langle v_{z_1} \rangle \mu}{R_1} = \frac{4 \langle v_{z_2} \rangle \mu}{R_2}$$

$$\langle v_{z_2} \rangle = \frac{\langle v_{z_1} \rangle R_2}{R_1}$$

$$\frac{1}{2} \pi R_1^2 \langle v_{z_1} \rangle = \pi R_2^2 \langle v_{z_2} \rangle$$

$$\frac{1}{2} \pi R_1^2 \langle v_{z_1} \rangle = \pi R_2^2 \langle v_{z_1} \rangle \frac{R_2}{R_1}$$

$$R_2^3 = \frac{1}{2} R_1^3$$

Or

$$2R_2^3 = R_1^3$$

"MURRAY'S LAW"

CAN ALSO BE DERIVED BY AN OPTIMIZATION BETWEEN METABOLIC COST OF SUPPORTING BLOOD VESSELS + PUMPING COSTS!!

TABLE I

DATA OF F. P. MALL (1888)

Description of vessel	Number	Radius	Probable rank
		μm	
Superior mesenteric artery	1	1,500	0
Main branches of mesenteric art.	15	500	1
Final branches of mesenteric art.	45	300	2
Short intestinal arteries (s.i.a.)	1,440	40	3
Long intestinal arteries (l.i.a.)	459	96	3
Last branches of s.i.a.	8,640	25	4
Last branches of l.i.a.	18,000	26.5	4
Branches to crypts	4,000,000	4	5, 6, 7
Branches to villi	328,500	15.5	5
Arteries of the villi	1,051,000	11.25	6
Capillaries of the villi (upper 2/3)	31,536,000	4	7
Capillaries of the villi (lower 1/3)	15,768,000	2.5	x
Veins at base of villi	2,102,400	13.25	6′
Veins between villi & submucosa	131,400	37.5	5′
Last branches of submucosal veins	18,000	64	4′
Anastomoses of submucosal veins	2,500,000	16	xx
Last branches of s.i.v.	28,800	32	4′
Long intestinal veins	459	220	3′
Short intestinal veins (s.i.v.)	1,440	56	3′
Last branches of mesenteric veins	45	750	2′
Branches of mesenteric vein	15	1,200	1′
Mesenteric vein	1	3,000	0′
Muscle layers			
Direct muscle arteries	1,800	15	3, 4, 5, 6
Indirect muscle arteries	3,600	20	3, 4, 5, 6
Capillaries of circular muscle	27,000,000	1.5	7
Capillaries of longitudinal muscle	9,000,000	1.5	7
Veins	3,600	56	3′, 4′, 5′, 6′
Peritoneum			
Arteries	360	24	3, 4, 5, 6
Capillaries	36,000	9	7
Veins	360	40	3′, 4′, 5′, 6′

TABLE II

VESSELS IN TABLE I GROUPED ACCORDING TO RANK

Vessel rank	Σr^2	Σr^3	Σr^4
	mm^2	mm^3	mm^4
0	2.2	3.4	5.1
1	3.8	1.9	0.94
2	4.0	1.2	0.36
3	8.6	0.54	0.043
4	20	0.51	0.013
5	140	1.5	0.021
6	200	1.8	0.019
7	650	2.4	0.0095
6′	380	5.5	0.10
5′	200	7.6	0.30
4′	120	6.3	0.37
3′	39	5.8	1.1
2′	25	19	14
1′	22	26	31
0′	9	27	81

The vessels of Table I have been grouped according to rank and the sums of r^2, r^3, and r^4 have been calculated for each rank.

R^3 WORKS BEST