CBE 30357
9/14/17

TOPICS FOR TODAY

1) REVIEW OF PRESSURE DRIVEN FLOW

2) REVIEW OF GRAVITY DRIVEN FLOW

3) FLOWS IN A CYLINDRICAL GEOMETRY
   - HYDROSTATICS

4) BOUNDARY CONDITIONS

5) SOME LIMITS...

PRESSURE DRIVEN FLOW

$P_1 > P_2$

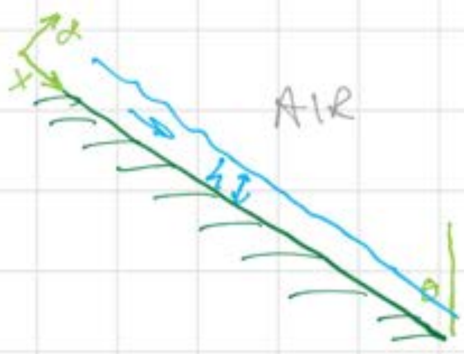
NO CHANGE W/ Z

$$0 = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 V_x}{\partial y^2}$$

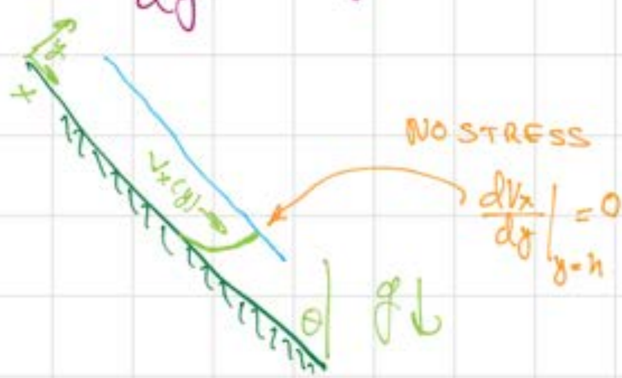
SOLUTION FOR V_x IS QUADRATIC IN y

$$V_x(y) = -\frac{1}{\mu}\frac{dP}{dx}\frac{h^2}{2}\left(1 - \frac{y^2}{h^2}\right)$$

GRAVITY DRIVEN FLOW



$$0 = \mu \frac{d^2 V_x}{dy^2} + \rho g \cos\theta$$



$$V_x(y) = \frac{\rho g}{\mu}\cos\theta\left(\frac{y^2}{2} - yh\right)$$

The mechanism of exercise-induced asthma is . . .

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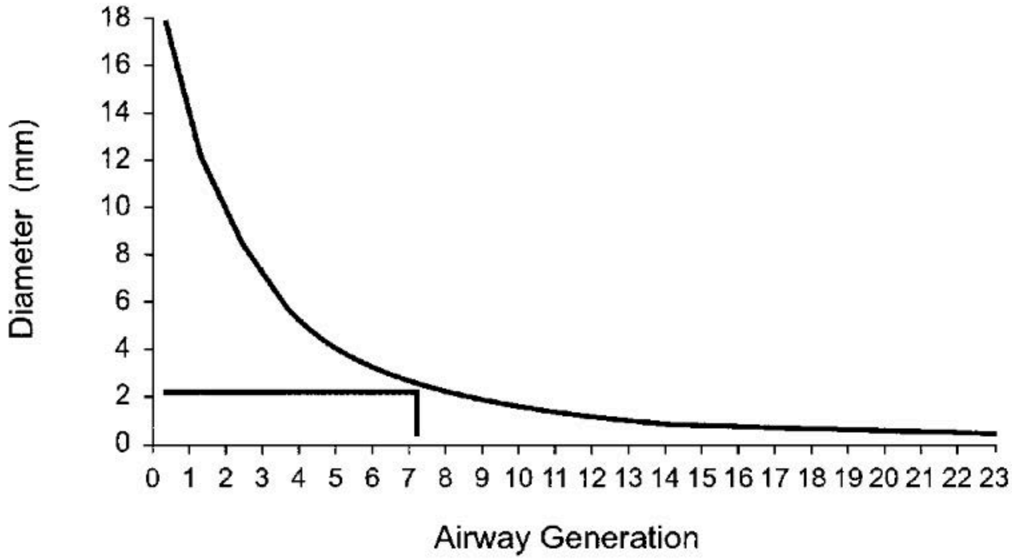
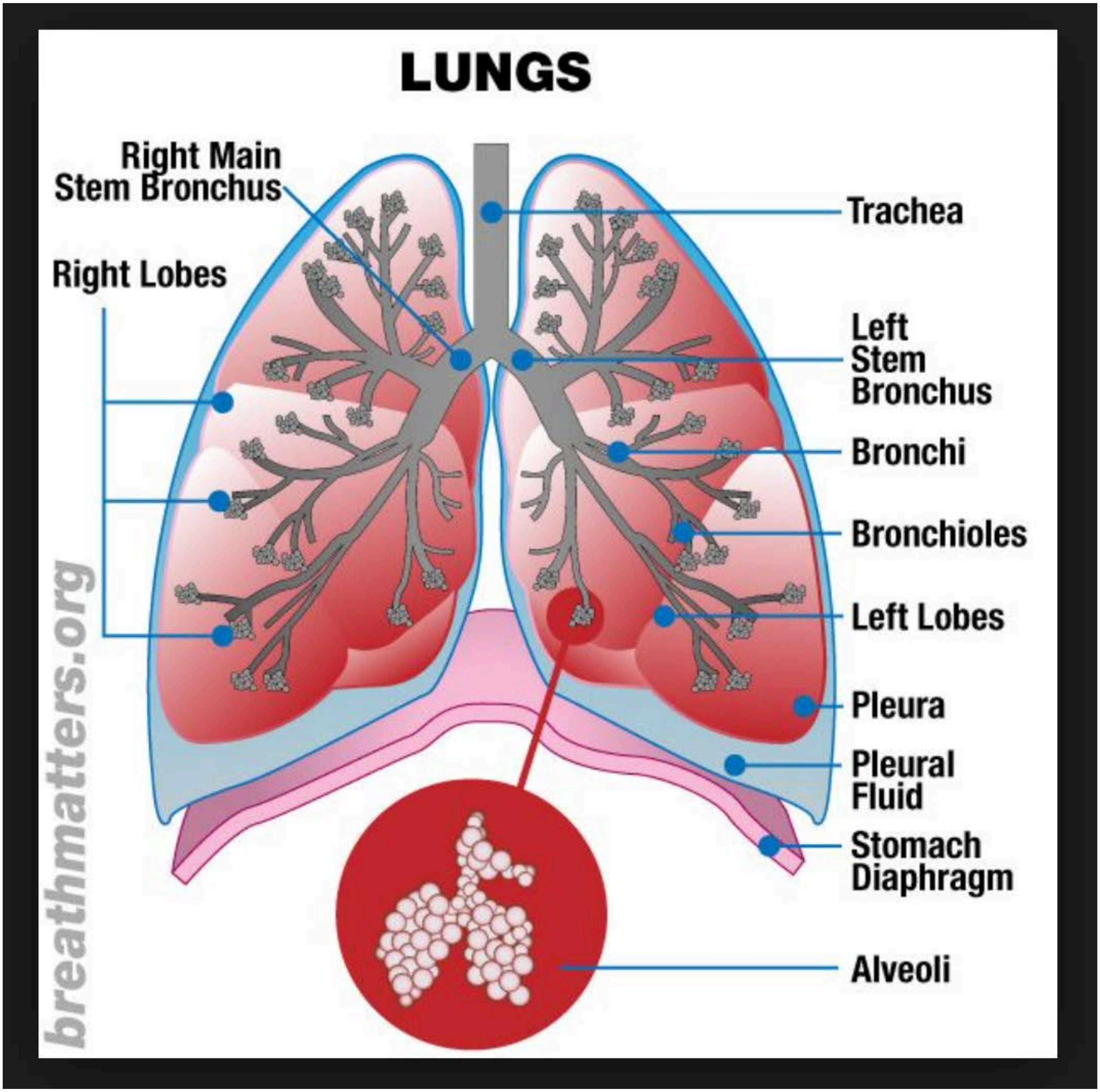


Figure 1. *Airway diameters (mm) drawn against airway generations. (Adapted with permission from Ref. 6.)*

MECHANISMS OF REMODELLING

perhaps best illustrated by irritant-induced asthma. Certain characteristic changes, such as growth of ASM, seem likely to contribute to airways hyperresponsiveness (AHR), but the consequences of other changes, such as subepithelial fibrosis, are not so intuitively obvious. Even the link between growth of ASM and AHR is deduced largely on the basis of modelling studies. One of the major challenges facing the field is to devise strategies that link structure and function directly. For

Airway smooth
muscle (remodeling)


LUNGS
Right Main Stem Bronchus
Right Lobes
Trachea
Left Stem Bronchus
Bronchi
Bronchioles
Left Lobes
Pleura
Pleural Fluid
Stomach Diaphragm
Alveoli
breathmatters.org

Suppose we want to consider the flow in a trachea.

The geometry is a cylinder.

We need the Navier-Stokes equations in cylindrical coordinates.

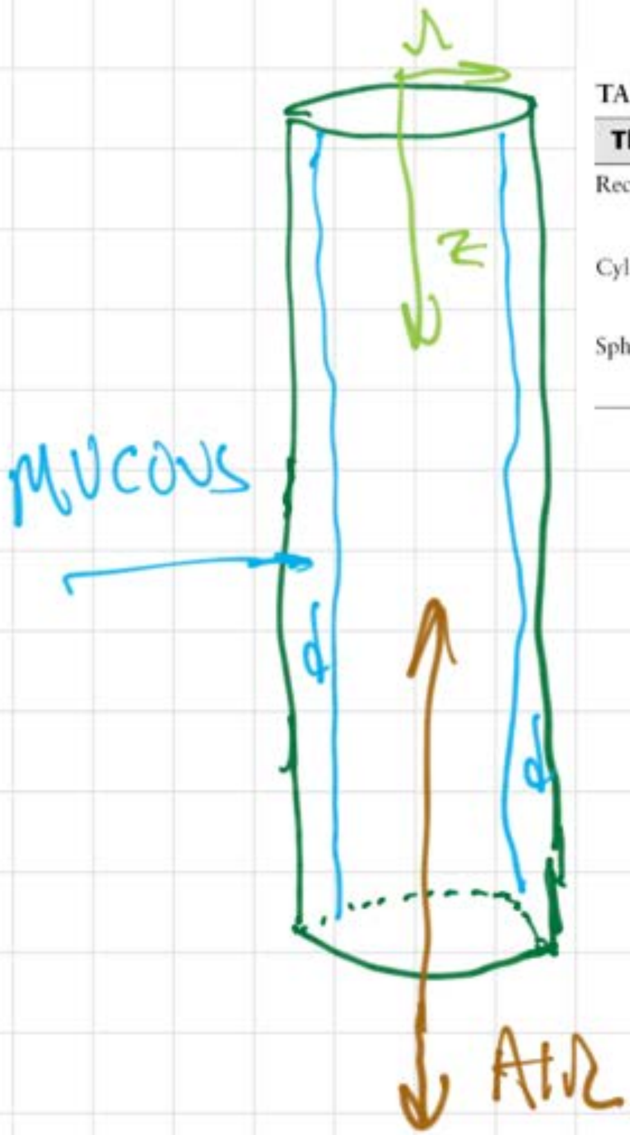


TABLE 3.1

The Conservation of Mass (Continuity Equation)	
Rectangular coordinates (x, y, z)	$\frac{\partial \rho}{\partial t} = -\left(\frac{\partial \rho v_x}{\partial x} + \frac{\partial \rho v_y}{\partial y} + \frac{\partial \rho v_z}{\partial z}\right)$
Cylindrical coordinates (r, θ, z)	$\frac{\partial \rho}{\partial t} = -\left(\frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z}\right)$
Spherical coordinates (r, θ, ϕ)	$\frac{\partial \rho}{\partial t} = -\left(\frac{1}{r^2}\frac{\partial(\rho r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_\phi)}{\partial \phi}\right)$

$$\frac{\partial v_z}{\partial z} = 0$$

$$v_z = v_z(r)$$

ONLY

Cylindrical coordinates

r direction

$$\rho\left(\cancel{\frac{\partial v_r}{\partial t}} + \cancel{v_r \frac{\partial v_r}{\partial r}} + \cancel{\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}} - \cancel{\frac{v_\theta^2}{r}} + \cancel{v_z \frac{\partial v_r}{\partial z}}\right) = \cancelto{0}{-\frac{\partial p}{\partial r}} + \mu\left[\cancel{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial r}\right)} + \cancel{\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}} - \cancel{\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}} + \cancel{\frac{\partial^2 v_r}{\partial z^2}}\right] + \cancel{\rho g_r} \tag{3.3.27a}$$

θ direction

$$\rho\left(\cancel{\frac{\partial v_\theta}{\partial t}} + \cancel{v_r \frac{\partial v_\theta}{\partial r}} + \cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}} + \cancel{\frac{v_r v_\theta}{r}} + \cancel{v_z \frac{\partial v_\theta}{\partial z}}\right) = -\frac{1}{r}\overset{=0}{\frac{\partial p}{\partial \theta}} + \mu\left[\cancel{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right)} + \cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}} + \cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}} + \cancel{\frac{\partial^2 v_\theta}{\partial z^2}}\right] + \cancel{\rho g_\theta} \tag{3.3.27b}$$

z direction

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_r \frac{\partial v_z}{\partial r}} + \cancel{\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}} + \cancel{v_z \frac{\partial v_z}{\partial z}}\right) = \cancel{-\frac{\partial p}{\partial z}} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right] + \rho g_z \tag{3.3.27c}$$

$$0 = \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \rho g_z$$

FOR LIQUID FILM

$r = R \quad v_z(R) = 0$

$r = R-h \quad \left.\frac{\partial v_z}{\partial r}\right|_{r=(R-h)} = 0$

ASIDE: IF $\frac{h}{R} \ll 1$ SHOULD APPROACH "FLAT PLATE" SOLUTION FROM ABOVE.

$$0 = \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \rho g_z$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = -\frac{\rho g_z}{\mu}$$

$$\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = -\frac{\rho g_z}{\mu} r$$

$$\int d\left(r\frac{dv_z}{dr}\right) = \int \frac{\rho g_z}{\mu} r\, dr$$

$$r\frac{dv_z}{dr} = -\frac{\rho g_z}{\mu}\frac{r^2}{2} + C_1$$

$$\frac{dv_z}{dr} = -\frac{\rho g_z}{\mu}\frac{r}{2} + \frac{C_1}{r}$$

$$\frac{dv_z}{dr} = \frac{-\rho g_z}{\mu}\frac{r}{2} + \frac{C_1}{r}$$

$$\int dv_z = \int \left(\frac{-\rho g_z}{\mu}\frac{r}{2} + \frac{C_1}{r} \right) dr$$

$$v_z = \frac{-\rho g_z}{4\mu} r^2 + C_1 \ln r + C_2$$

FIT BC'S

$$v_z(r) = \frac{\rho g_z}{4\mu}\left((R^2 - r^2) + 2(h-R)^2 \ln\left(\frac{r}{R}\right) \right)$$

IF FLOW DOMAIN INCLUDES $r=0$, THE "ln" TERM MUST BE EXCLUDED.

USING MATHEMATICA, THE AVERAGE VELOCITY IS EASILY OBTAINED

Find the average velocity

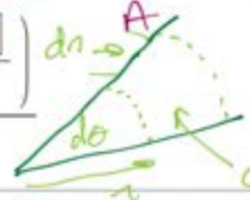
```
In[9]:= Integrate[vzee r, {r, R, R - h}, Assumptions → {h > 0, h < R}] /
  Integrate[ r, {r, R, R - h}, Assumptions → {h > 0, h < R}]
```

Out[9]= $$\frac{g z \rho \left(-\frac{1}{4} h (h-2R)\left(3h^2-6hR+2R^2\right)+(h-R)^4 \mathrm{Log}\left[1-\frac{h}{R}\right]\right)}{4\left(-\frac{R^2}{2}+\frac{1}{2}(-h+R)^2\right)\mu}$$

```
In[10]:= FullSimplify[%]
```

$$Q = \int v_z(r,\theta)\, dA$$

Out[10]= $$\frac{g z \rho \left(-3h^2+6hR-2R^2+\frac{4(h-R)^4 \mathrm{Log}\left[1-\frac{h}{R}\right]}{h(h-2R)}\right)}{8\mu}$$



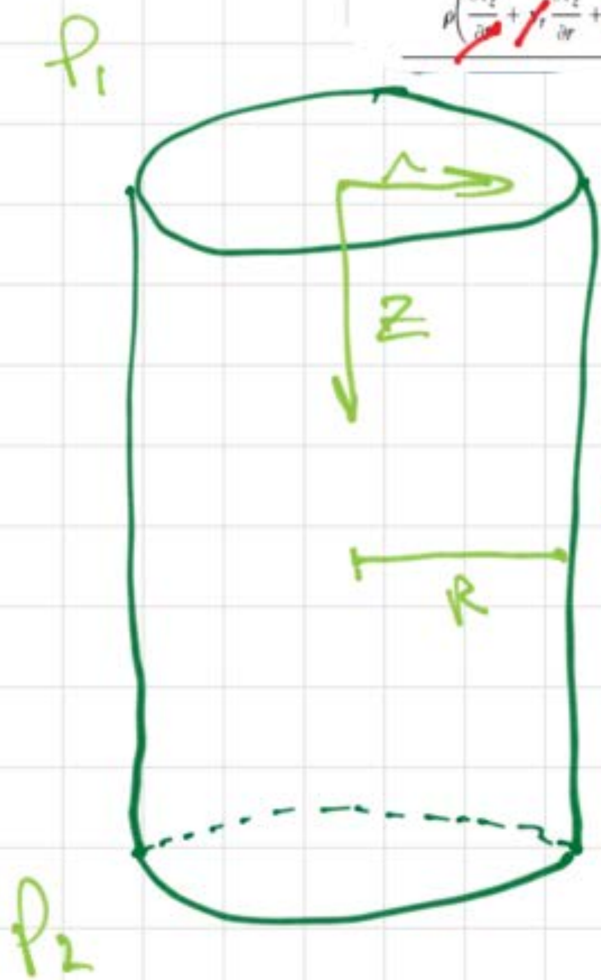
$$dA = (r\,d\theta)(dr)$$

$$\frac{\int_R^{R-h} v_z(r)\, r\, dr}{\int_R^{R-h} r\, dr} = \langle v_z \rangle$$

Suppose that we have a circular pipe oriented,vertically.

z direction

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_r}\frac{\partial v_z}{\partial r} + \frac{\cancel{v_\theta}}{r}\frac{\partial v_z}{\cancel{\partial \theta}} + v_z\frac{\partial v_z}{\cancel{\partial z}}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right] + \rho g_z$$



$g_z \downarrow$

$$0 = -\frac{\partial P}{\partial z} + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \rho g_z$$

BC's

$v_z(R) = 0$

$\left.\frac{\partial v_z}{\partial r}\right|_{r=0} = 0$

"SYMMETRY"

2 CASES:

1) OPEN AT BOTH ENDS $P_1 = P_2$

2) CLOSE BOTTOM: NO FLOW

LET'S DO CASE #2

$$0 = -\frac{\partial P}{\partial z} + \rho g_z$$

$$\frac{dP}{dz} = \rho g_z$$

$$\int dP = \int \rho g_z dz$$

$$P = \rho g_z z + C_0$$

$P(0) = P_1$

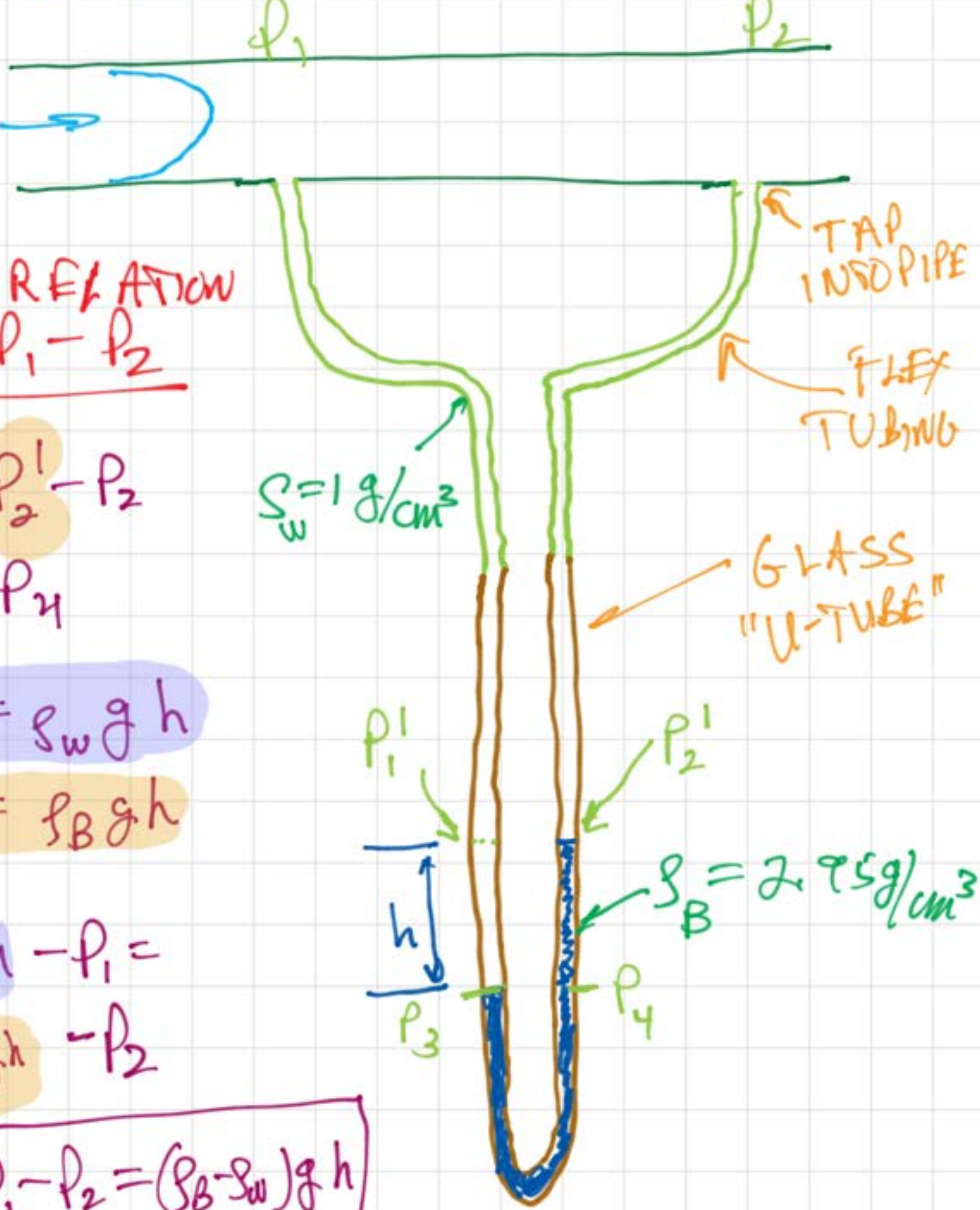
$$\therefore \quad P = P_0 + \rho g z$$

Z INCREASES DOWNWARD

P IS LINEAR IN Z.

BASIC EQUATION OF HYDROSTATICS

EXAMPLE: MANOMETER



FIND A RELATION FOR $P_1 - P_2$

$$P_1' - P_1 = P_2' - P_2$$

$$P_3 = P_4$$

$$P_3 - P_1' = \rho_w g h$$

$$P_4 - P_2' = \rho_B g h$$

SUBS:

$$P_3 - \rho_w g h - P_1 =$$

$$P_4 - \rho_B g h - P_2$$

$$\therefore \quad \boxed{P_1 - P_2 = (\rho_B - \rho_w) g h}$$

NOW CONSIDER CASE 1:

FLOW BY GRAVITY

$$0 = \frac{\mu}{r}\frac{\partial}{\partial r} r \frac{\partial v_z}{\partial r} + \rho g_z$$

$$\frac{\mu}{r}\frac{d}{dr} r \frac{dv_z}{dr} = -\rho g_z$$

$$\frac{d}{dr} r \frac{dv_z}{dr} = \frac{-\rho}{\mu} g_z r$$

$$\int d\left(r\frac{dv_z}{dr}\right) = \frac{-\rho}{\mu} g_z \int r\,dr$$

$$r\frac{dv_z}{dr} = \frac{-\rho}{\mu} g_z \frac{r^2}{2} + c_1$$

$$\int dv_z = \int \left(\frac{-\rho}{\mu} g_z \frac{r}{2} + \frac{c_1}{r}\right) dr$$

$$v_z = -\frac{\rho}{\mu} g_z \frac{r^2}{4} + c_1 \ln r + c_2$$

BUT $C_1 = 0$

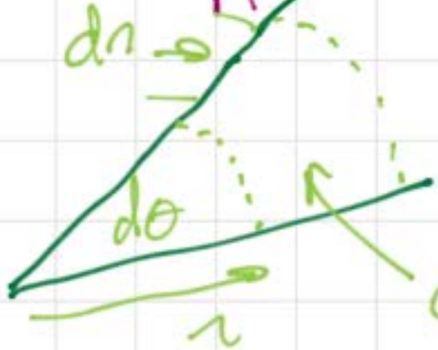
AT $v_z(R) = 0$

$$v_z(r) = \frac{\rho g_z}{4\mu}\left(R^2 - r^2\right)$$ PARABOLIC

WE WOULD LIKE AVERAGE $\langle v_z \rangle$

NEED VOLUMETRIC FLOW

$$Q = \int_A v_z(r,\theta)\, dA$$



$$dA = (r\,d\theta)(dr)$$

$$\langle v_z \rangle = \frac{\int_0^{2\pi}\int_0^R v_z(r,\theta)\, r\, dr\, d\theta}{\int_0^{2\pi}\int_0^R r\, dr\, d\theta}$$

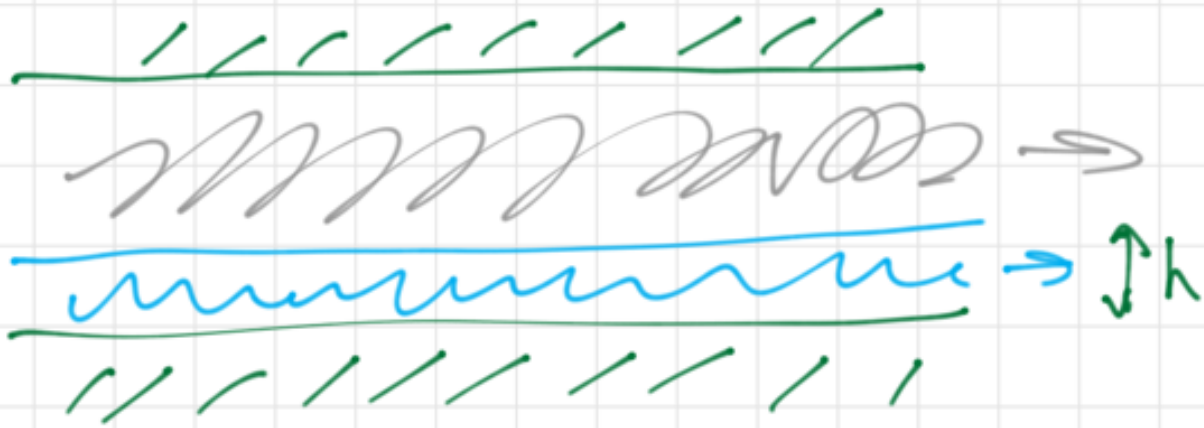
$$\langle v_z \rangle = \frac{\rho g_z R^2}{8\mu} = \frac{1}{2} v_{MAX}$$

$$Q = \langle V_z \rangle \pi R^2$$

$$= \frac{2\pi \rho g_z R^4}{8\mu}$$

VERY SENSITIVE TO THIS VALUE

SUPPOSE 2 IMMISCIBLE OR SEPARATE LAYERS ARE FLOWING TOGETHER



2 SEPARATE EQUATIONS

$$0 = -\frac{\partial P}{\partial x} + \mu^{I} \frac{\partial^2 V_x^{I}}{\partial y^2} \qquad V_x^{I}(h) = V_x^{II}(h)$$

$$0 = -\frac{\partial P}{\partial x} + \mu^{II} \frac{\partial^2 V_x^{II}}{\partial y^2} \qquad \mu^{I} \left.\frac{\partial V_x^{I}}{\partial y}\right|_h = \mu^{II} \frac{\partial V_x^{II}}{\partial y}$$

Boundary conditions:

1. Fluid sticks to solid surfaces $V_x(0) = 0$
2. Fluid sticks to another immiscible fluid $V_x^{I}(h) = V_x^{II}(h)$
3. The shear stress is continuous across the fluid-fluid interface

$$\tau_{yx}^{I}(h) = \tau_{yx}^{II}(h)$$

$$\mu^{I} \left.\frac{\partial V_x^{I}}{\partial y}\right|_h = \mu^{II} \frac{\partial V_x^{II}}{\partial y}$$

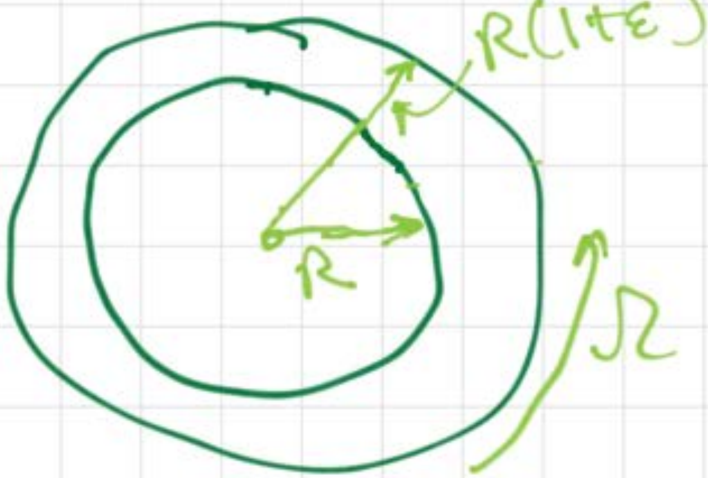
4. For liquid flowing next to quiescent air, the stress can be considered approximately 0.

$$\tau_{yx}(h) \approx 0$$

$$\frac{\partial V_x}{\partial y}(h) \approx 0$$

"FALLING FILM"

ROTATING FLOW



Cylindrical coordinates

r direction

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho g_r \qquad (3.3.27a)$$

θ direction

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho g_\theta \qquad (3.3.27b)$$

z direction

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \qquad (3.3.27c)$$

$$0 = \mu \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r V_\theta)}{\partial r}\right)$$

$$V_\theta(R) = 0$$

$$V_\theta(R(1+\epsilon)) = R(1+\epsilon)\Omega$$

SOLVE

$$\frac{1}{r}\frac{d}{dr}(r V_\theta) = C_1$$

$$\frac{d}{dr}(r V_\theta) = \frac{C_1}{r}$$

$$\boxed{V_\theta = \frac{C_1 r}{2} + \frac{C_2}{r}}$$

NO r=0
SO
BOTH
TERMS
ARE OK.

FIT B.C.'S

$$0 = \frac{C_1 R}{2} + \frac{C_2}{R}$$

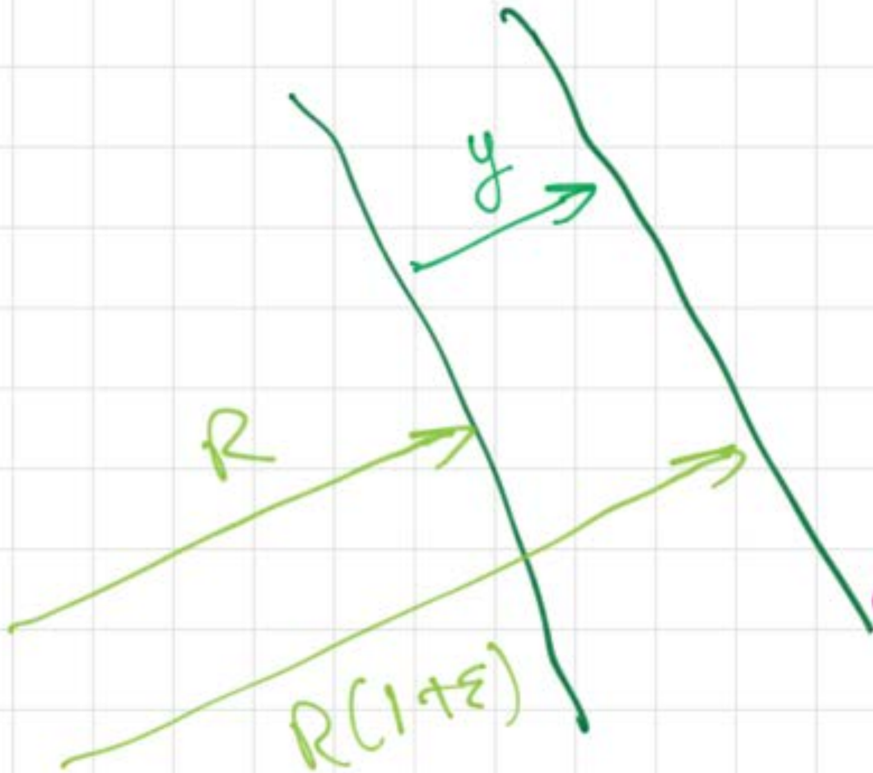
$$C_2 = -\frac{C_1 R^2}{2}$$

$$R(1+\epsilon)\Omega = C_1 \frac{R(1+\epsilon)}{2} - \frac{C_1 R^2}{2R(1+\epsilon)}$$

$$1 = \frac{C_1}{2\Omega}\left(1 - \frac{1}{(1+\epsilon)^2}\right)$$

$$C_1 = \frac{2\Omega}{\left(1 - \frac{1}{(1+\epsilon)^2}\right)}$$

$$V_\theta(r) = \frac{\Omega}{\left(1 - \frac{1}{(1+\varepsilon)^2}\right)}\left(r - \frac{R^2}{r}\right)$$



$0 \leq y \leq 1$

$r = R + \varepsilon R y$

OR: $y = \frac{r - R}{\varepsilon R}$

SERIES EXPANSION IN ε

$$V_\theta(y) = R\Omega y + \frac{\varepsilon}{2}\left(\frac{3Ry\Omega}{2} - \frac{1}{2}Ry^2\Omega\right)$$

AS $\varepsilon \to 0$ LOOKS LIKE
PARALLEL PLATE

USING WOLFRAM ALPHA TO GET SERIES EXPANSION

7:37 AM

WolframAlpha

substitute r=R+R*eps*y into (r-R)*(r+R)*(1+eps)^2*omega/r/eps/(2+eps)

Input interpretation

$$(r-R)(r+R)(1+\epsilon)^2 \times \frac{\frac{\frac{\omega}{r}}{\epsilon}}{2+\epsilon} \text{ where } r = R + R\,y\,\epsilon$$

Result

$$\frac{R\,y\,\omega(\epsilon+1)^2(R\,y\,\epsilon+2\,R)}{(\epsilon+2)(R\,y\,\epsilon+R)}$$

Step-by-step solution

WolframAlpha

taylor series R*y*omega*(eps+1)^2*(R*y*eps+2*R)/(eps+2)/(R*y*eps+R) in eps

Input interpretation

series	$R\,y\,\omega(\epsilon+1)^2 \times \frac{\frac{R\,y\,\epsilon+2\,R}{\epsilon+2}}{R\,y\,\epsilon+R}$	point	$\epsilon=0$

Series expansion at ϵ=0

$$R\,y\,\omega - \frac{1}{2}\epsilon(R(y-3)\,y\,\omega) + \frac{1}{4}R\,y\left(2\,y^2-3\,y+1\right)\omega\,\epsilon^2 - \frac{1}{8}\epsilon^3\left(R\,y\left(4\,y^3-6\,y^2+y+1\right)\omega\right) + \frac{1}{16}R\,y\left(8\,y^4-12\,y^3+2\,y^2+y+1\right)\omega\,\epsilon^4 - \frac{1}{32}\epsilon^5\left(R\,y\left(16\,y^5-24\,y^4+4\,y^3+2\,y^2+y+1\right)\omega\right) + O\left(\epsilon^6\right)$$

(Taylor series)

$$\lim_{\epsilon\to 0} V = y(R\omega) \quad 0 \le y \le 1$$

CATHETER FLOW

8:00 AM

WolframAlpha

dsolve(mu/r*(v'(r)+r*v''(r))-G=0,v(R)=0,v(lambda*R)=0)

Input

$$\left\{\frac{\mu}{r}\left(v'(r)+r\,v''(r)\right)-G=0,\ v(R)=0,\ v(\lambda R)=0\right\}$$

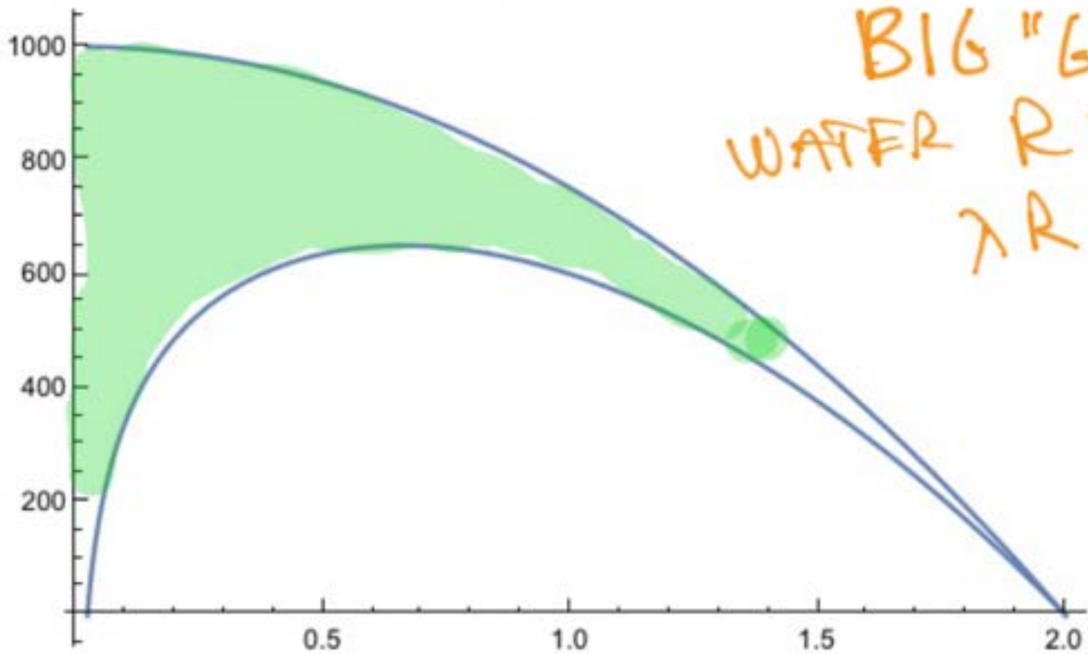
Differential equation solution

$$v(r)=\frac{G\left(\log(R)\left(r^2-\lambda^2R^2\right)+\left(R^2-r^2\right)\log(\lambda R)+\left(\lambda^2-1\right)R^2\log(r)\right)}{4\,\mu\left(\log(R)-\log(\lambda R)\right)}$$

Step-by-step solution

Related Wolfram|Alpha queries

```
Plot[{veez, vezee} /. {dpdz → -10, μ → .01, R → 2, λ → .01}, {r, .02, 2}]
```



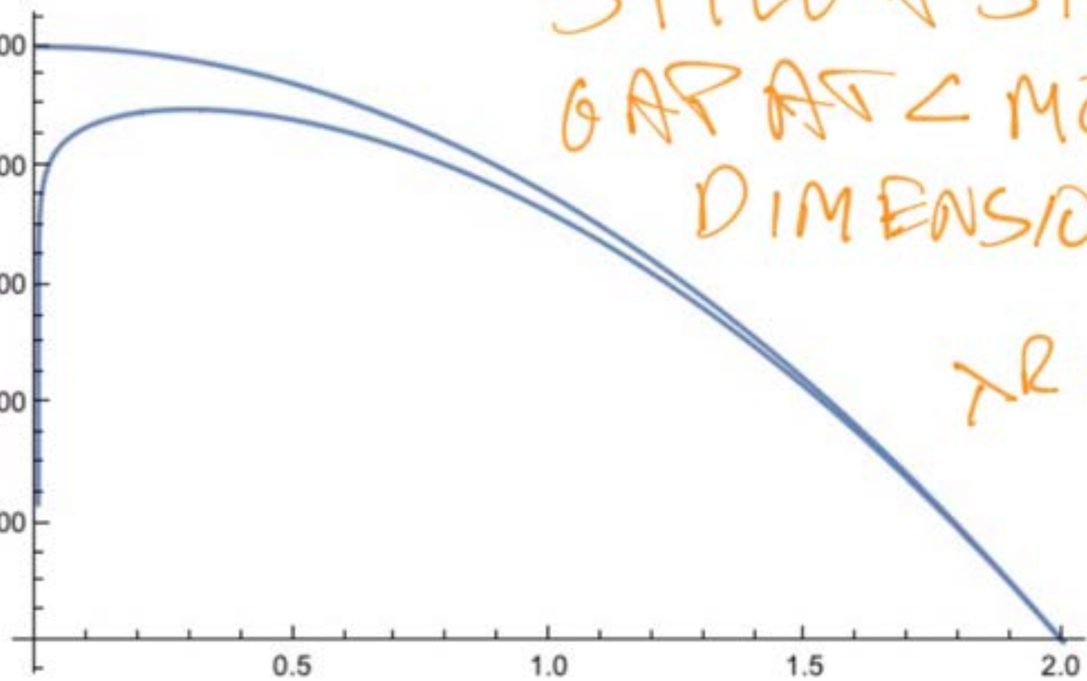
BIG "GAP" FOR WATER $R = 2\,cm$

$\lambda R = .02\,cm$

```
Plot[{veez, vezee} /. {dpdz → -10, μ → .01, R → 2, λ → .0000000001},
 {r, .0000000002, 2}]
```

STILL A SIGNIFICANT GAP AT < MOLECULAR DIMENSION !!

$\lambda R = 2\times10^{-10}$ cm



LIMIT OF OPEN CHANNEL EXISTS, BUT NOT RELEVANT PHYSICALLY

In[36]:= `Series[veez, {λ, 0, 2}]`

Out[36]=
$$\frac{1}{4\,\mu\,\mathrm{Log}[\lambda]}\left(\mathrm{dpdz}\,R^2\,\mathrm{Log}[r]-\mathrm{dpdz}\,R^2\,\mathrm{Log}[R]+\mathrm{dpdz}\,r^2\,\mathrm{Log}[\lambda]-\mathrm{dpdz}\,R^2\,\mathrm{Log}[\lambda]\right)+\frac{\left(-\mathrm{dpdz}\,R^2\,\mathrm{Log}[r]+\mathrm{dpdz}\,R^2\,\mathrm{Log}[R]\right)\lambda^2}{4\,\mu\,\mathrm{Log}[\lambda]}+O[\lambda]^3$$

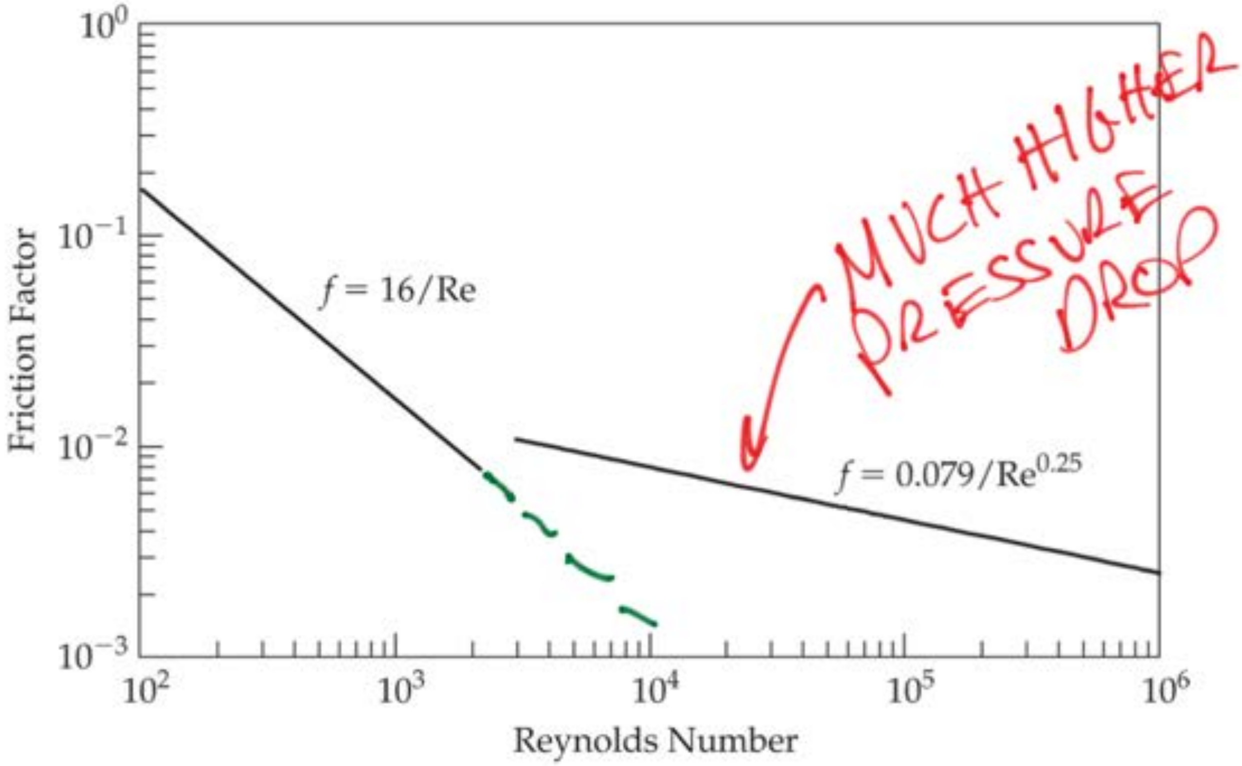
In[37]:= `Apart[Normal[%]]`

Out[37]=
$$-\frac{\mathrm{dpdz}\left(-r^2+R^2\right)}{4\,\mu}+\frac{1}{4\,\mu\,\mathrm{Log}[\lambda]}\left(\mathrm{dpdz}\,R^2\,\mathrm{Log}[r]-\mathrm{dpdz}\,R^2\,\lambda^2\,\mathrm{Log}[r]-\mathrm{dpdz}\,R^2\,\mathrm{Log}[R]+\mathrm{dpdz}\,R^2\,\lambda^2\,\mathrm{Log}[R]\right)$$

TRANSITION TO TURBULENT FLOW.

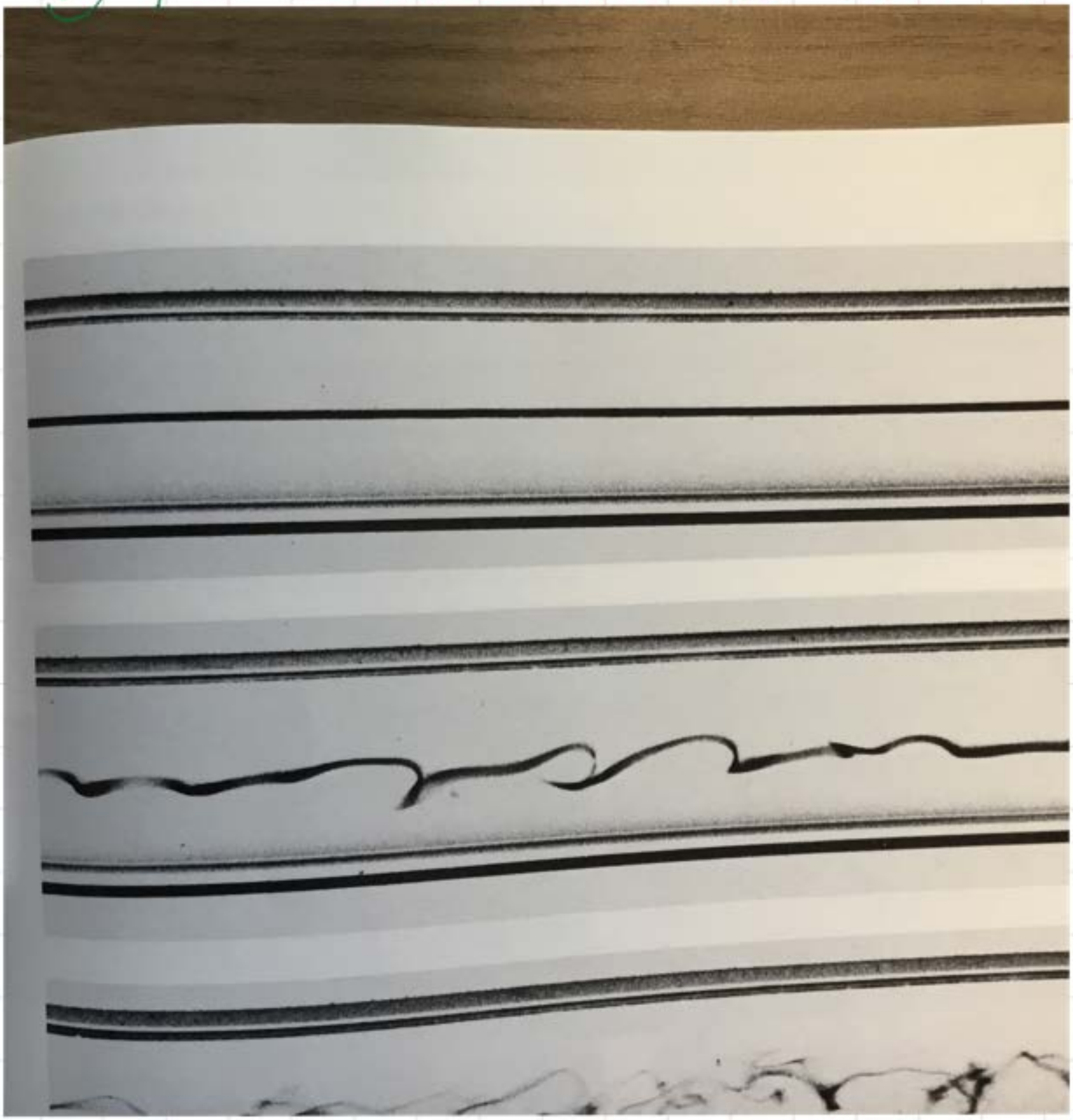
$$Re \equiv \frac{DV\rho}{\mu}$$

Figure 3.11 The Fanning friction factor versus the Reynolds number.



$$f \equiv \frac{\Delta P\, D}{2 L \rho v^2}$$

DYE STREAM IN PIPE FLOW



103. **Repetition of Reynolds' dye experiment.** Osborne Reynolds' celebrated 1883 investigation of stability of flow in a tube was documented by sketches rather than photography. However the original apparatus has survived at the University of Manchester. Using it a century later, N. H. Johannesen and C. Lowe have taken this sequence of photographs. In laminar flow a filament of colored water introduced at a bell-shaped entry extends undisturbed the whole length of the glass tube. Transition is seen in the second of the photographs as the speed is increased; and the last two photographs show fully turbulent flow. Modern traffic in the streets of Manchester made the critical Reynolds number lower than the value 13,000 found by Reynolds.

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AS Re ↑ MORE "MIXING"

STRONGER DISTURBANCES