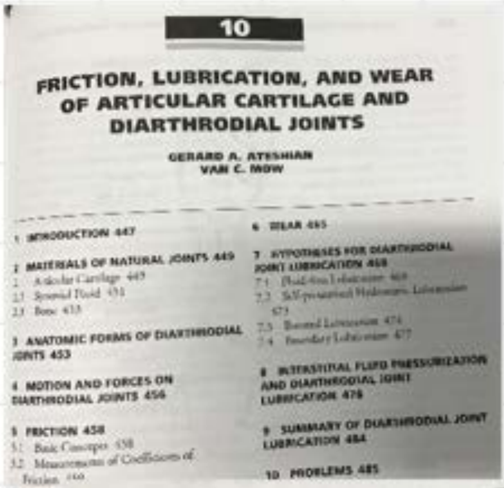
CBE 30357

12/7/17

HYDRODYNAMIC LUBRICATION THEORY APPLIED TO:

DIARTHROIDAL JOINTS

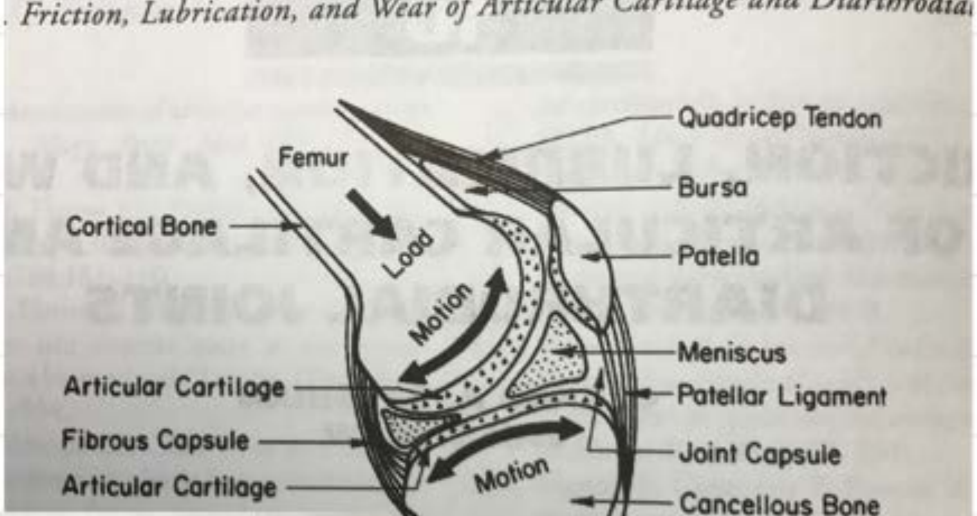


10

FRICTION, LUBRICATION, AND WEAR OF ARTICULAR CARTILAGE AND DIARTHRODIAL JOINTS

GERARD A. ATESHIAN
VAN C. MOW

Friction, Lubrication, and Wear of Articular Cartilage and Diarthrodial



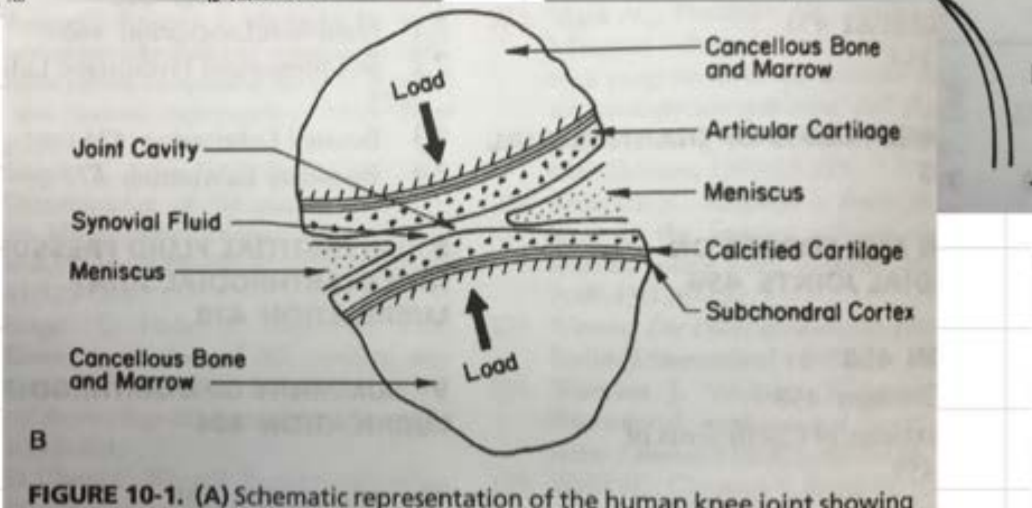
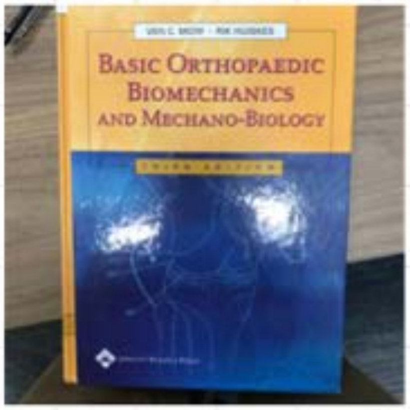


FIGURE 10-1. (A) Schematic representation of the human knee joint showing important anatomical features for mechanical function [171]. (B) Enlargement of the load-bearing region in the knee, depicting a thin layer of synovial fluid (<50 μm) and two layers of articular cartilage (each <7 mm) [9,171]. Each layer of articular cartilage contains approximately 80% fluid.



THIS SITUATION IS NOT COMPLETELY UNDERSTOOD!!

I WILL GIVE MY THOUGHTS...

Mechanisms of lubrication of natural joints

This table (from GRH, AU) gives a list of possible mechanisms for joint lubrication.

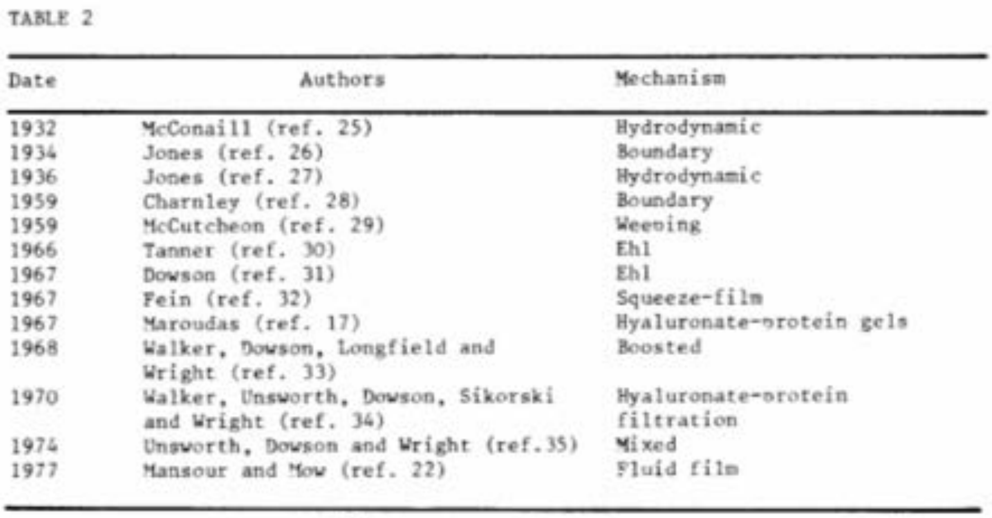


TABLE 2

Date	Authors	Mechanism
1932	McConaill (ref. 25)	Hydrodynamic
1934	Jones (ref. 26)	Boundary
1936	Jones (ref. 27)	Hydrodynamic
1959	Charnley (ref. 28)	Boundary
1959	McCutcheon (ref. 29)	Weeping
1966	Tanner (ref. 30)	Ehl
1967	Dowson (ref. 31)	Ehl
1967	Fein (ref. 32)	Squeeze-film
1967	Maroudas (ref. 17)	Hyaluronate-protein gels
1968	Walker, Dowson, Longfield and Wright (ref. 33)	Boosted
1970	Walker, Unsworth, Dowson, Sikorski and Wright (ref. 34)	Hyaluronate-protein filtration
1974	Unsworth, Dowson and Wright (ref.35)	Mixed
1977	Mansour and Mow (ref. 22)	Fluid film

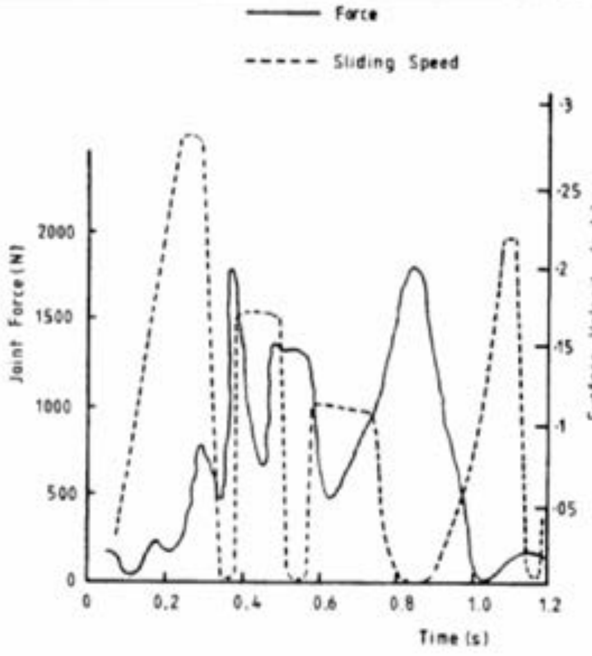


Fig.2 Knee joint forces and surface velocities at different parts of the walking cycle (after Seedhom).

WHAT KIND OF PERFORANCE
IS NEEDED?

POWER CALCULATION: $\Rightarrow$ HEAT
THROUGH KNEE: DISSIPATION

$$\left(80\,kg \times 9.8\,\frac{m}{s^2}\right)\frac{.04\,m}{.3\,s} = 100\,W$$

OR A HIGH PERFORMANCE ATHLETE

$\sim$ 400 W

MIGHT ARGUE FOR AS MUCH
AS 150 W THROUGH A KNEE

HOW MUCH 'FRICTION' COULD
THERE BE?

IN TERMS OF A FRICTION COEFFICIENT

$$f_T = \mu f_N$$

IF $\mu \sim .1$, 10 - 15 W

INEFFICIENT / IMPLAUSIBLE:
(THINK OF A RACE HORSE)

IF $\mu \sim .01$ 1 - 1.5 W

CAN HEAT BE CONDUCTED AWAY THROUGH BONE?

$$\dot{Q} = k \frac{\Delta T}{\Delta L} A$$

IF $k \sim .2 \frac{W}{m C^\circ}$ BONE

$$\frac{\Delta T}{L} = \frac{1.5\ W}{(20\ cm^2)(.2 \frac{W}{m C^\circ})} = 2500 \frac{C}{m}$$

$$= 25 C/cm$$

NOPE!!

IF WE CONSIDER HEAT LOST IN ALL DIRECTIONS

$k_W \sim .6 \, \frac{W}{mK}$

$4\pi R^2 \sim 4\pi (2.54)(3\,in)^2$

$= 700\,cm^2$

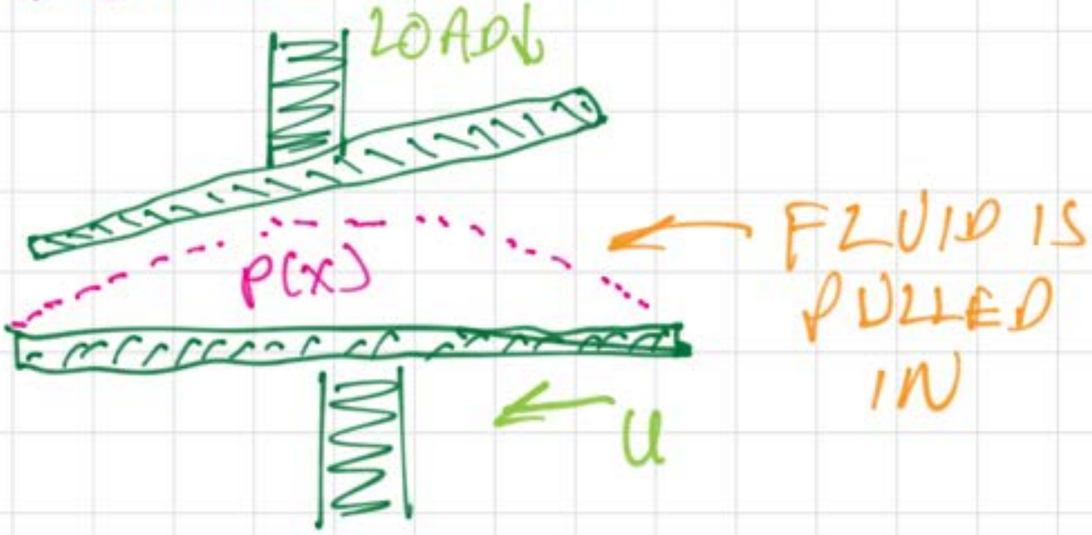
$\frac{1.5W}{700\,cm^2} \frac{1}{(.6W/mK)} \Rightarrow 36 \frac{K}{m}$

$= .36\,K/cm$ OK

"MY" CONCLUSION IS THAT FRICTION COEFFICIENT BETTER BE < 0.01 FOR FEASIBLE HEAT REMOVAL

NONE OF THE CANDIDATE MECHANISMS WHERE LARGE MOLECULES TOUCH "SURFACES" GET THIS LOW:

SO CONSIDER (ELASTO) HYDRODYNAMIC LUBRICATION



PRESSURE PROFILE IS RESULT OF LIQUID BEING FORCED THROUGH SMALL GAP !!

WHAT WOULD PRESSURE "DROP" BE FOR KNEE DIMENSIONS . . .

$$\frac{\Delta P}{2 L} \frac{D}{\rho U^2} = \frac{6}{Re}$$

$$\Delta P = \frac{12 \mu L U}{D^2}$$

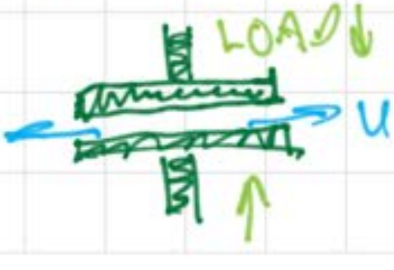
$\mu \sim 1$ g/cm·s $L \sim 4$ cm

$U \sim 10$ cm/s , $D \sim .01$ cm

$\Delta P = 5 \times 10^6$ g/cms²

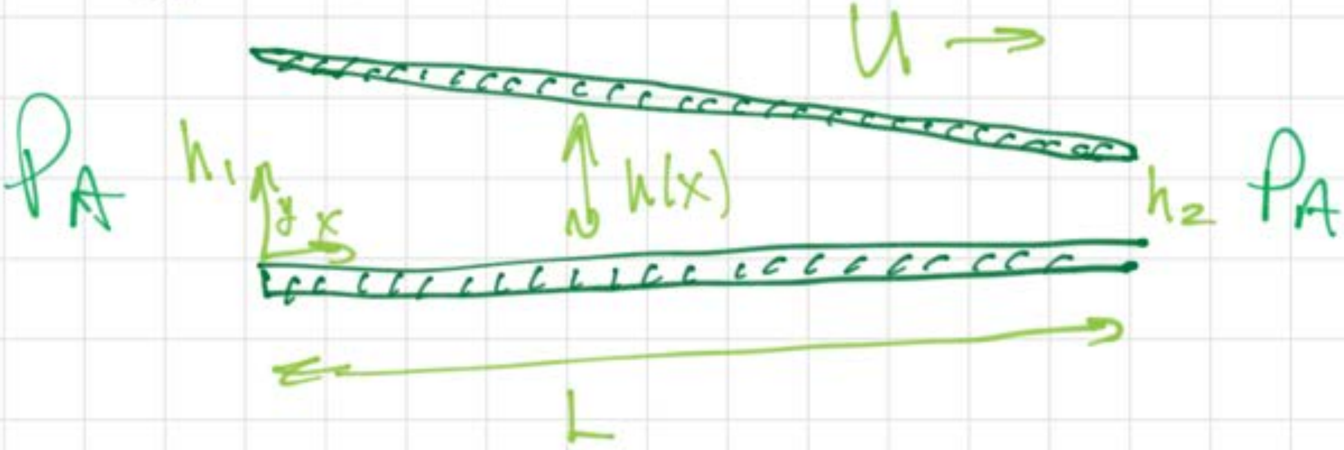
$= 5$ ATM !!

SQUEEZE FILM IS SIMILAR:



LUBRICATION ANALYSIS
(SECTION 4.7 IN TEXT)

CONSIDER THE "SLIDER" AS A SIMPLE EXAMPLE



$h << L$

"SLIGHTLY" NON-PARALLEL FLOW

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

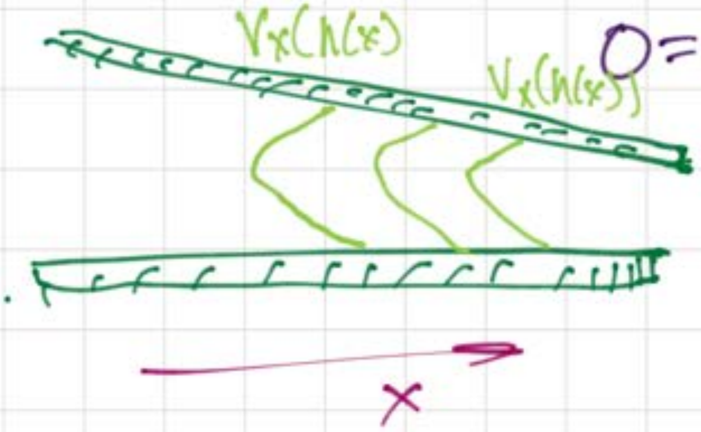
$$\frac{U}{L}\frac{\partial v_x^*}{\partial x^*} + \frac{V}{h}\frac{\partial v_y^*}{\partial y^*} \Rightarrow V \sim \frac{h}{L} U$$

SAME RESULT AS BOUNDARY-LAYER

WE COULD SHOW THROUGH ADDITIONAL SCALING ANALYSIS THAT IF $\frac{h}{L}\frac{\rho U h}{\mu} \ll 1$

INERTIA CAN BE NEGLECTED WITHIN GAP AND VELOCITY IS ALWAYS "LOCALLY" A FULLY-DEVELOPED LAMINAR PROFILE

h(x) ←



$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 v_x}{\partial y^2}$$

FOR SLIDER

$$V_x(0) = 0 \quad V_x(h) = U$$

$$V_x = \frac{1}{2\mu}\frac{dP}{dx}\left[y(y-h)\right] + U\frac{y}{h}$$

FROM GEOMETRY!
ADJUST U,
SEE h,
μ = FIXED

WHAT ABOUT $\frac{dP}{dx}$??

WE USE THE INTEGRAL CONSTRAINT

$$Q_x = \int_0^{h(x)} V_x(y)\,dy \quad = \text{CONSTANT}$$

$$\Rightarrow Q_x = -\frac{h^3}{12\mu}\frac{dP}{dx} + \frac{Uh}{2}$$

SOLVE FOR $\frac{dP}{dx}$

$$\frac{dP}{dx} = \frac{-Q_x - Uh/2}{h^3/12\mu}$$

$$\frac{dP}{dx} = 6\mu\left(\frac{U}{h^2} - \frac{2Q_x}{h^3}\right) \quad h=h(x)$$

NOW PICK $h(x)$ TO BE A
FLAT BLOCK: "SLIDER"

$$h = h_1 - \alpha x \qquad \alpha \equiv \frac{h_1 - h_2}{L}$$

$$dh = -\alpha\, dx$$

ELIMINATE dx

$$dP = \frac{-6\mu}{\alpha}\left(\frac{U}{h^2} - \frac{2Q_x}{h^3}\right)dh$$

$$\int_{P_a}^{P(x)} dP = \frac{-6\mu}{\alpha}\int_{h_1}^{h(x)}\left(\frac{U}{h^2} - \frac{2Q_x}{h^3}\right)dh$$

$$P(x) - P_a = \frac{6\mu}{\alpha}\left[U\left(\frac{1}{h} - \frac{1}{h_1}\right) - Q_x\left(\frac{1}{h^2} - \frac{1}{h_1^2}\right)\right]$$

NOTE THAT $P(h_2) - P(h_1) = 0 \quad \therefore$

$$0 = \frac{6\mu}{\alpha}\left[U\left(\frac{1}{h_2}-\frac{1}{h_1}\right) - Q_x\left(\frac{1}{h_2^2}-\frac{1}{h_1^2}\right)\right]$$

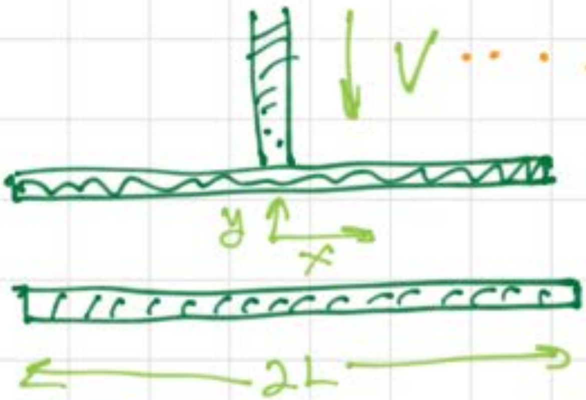
$$Q_x = \frac{U\, h_1 h_2}{(h_1 + h_2)}$$

4.7.9a $$P(x) - P_a = \frac{6\mu U}{\alpha(h_1+h_2)}\left[\frac{(h_2-h)(h-h_1)}{h^2}\right]$$

WE CAN SEE RESULTS IN

30357-16-LUBRICATION.nb

SQUEEZE FILM



USE REYNOLDS EQUATION: 4.7.7

$$\frac{1}{\mu}\frac{d}{dx}\left(h^3\frac{dP}{dx}\right)=6\left[h\frac{dU}{dx}-U\frac{dh}{dx}+2V\right]$$

$$h \neq h(x)$$

$$\frac{1}{\mu}\frac{d}{dx}\left(h^3\frac{dP}{dx}\right)= 12V$$

$$\frac{d}{dx}\left(\frac{dP}{dx}\right) = \frac{12\mu V}{h^3}$$

$$\frac{dP}{dx}= \frac{12\mu V}{h^3}x + C_1$$

@ $x=0$ $\frac{dP}{dx}=0$ BY SYMMETRY

$$\therefore \quad dP = \frac{12\mu V x}{h^3}\,dx$$

$$P(x) - P(L) = \frac{12\mu V}{h^3}\left(\frac{x^2}{2} - \frac{L^2}{2}\right)$$

$$P(L) = 0$$

$$\text{LOAD} = 2W\int_0^L P(x)\,dx = 2W\int_0^L \frac{12\mu V}{h^3}\left(\frac{x^2}{2} - \frac{L^2}{2}\right)dx$$

$$= \frac{24 W\mu V}{h^3}\left(\frac{x^3}{6} - \frac{xL^2}{2}\right)\Bigg|_0^L$$

$$= \frac{8\mu V L^3 W}{h^3} \longleftarrow$$

WHAT WE EXPECT?

$$V = \frac{dh}{dt}$$

CAN INTEGRATE IN TIME...

SEE PLOTS IN MATHEMATICA NOTEBOOK

WHAT IS COEFFICIENT OF FRICTION FOR "SLIDER"?

$$\beta \equiv \frac{h_2}{h_1} \qquad \frac{\text{TANGENTIAL FORCE}}{\text{NORMAL FORCE}} \qquad \text{pp. } 201\text{-}202$$

$$F_V = \int_0^L \mu \frac{\partial v_x}{\partial y}\bigg| dx = \frac{2\mu U}{\alpha} W \left(3\frac{\beta-1}{\beta+1} - \ln\beta\right)$$

TANGENTIAL PRESSURE:

$$F_p = \frac{6\mu U W}{\alpha}\left(\ln\beta - 2\frac{\beta-1}{\beta+1}\right)$$

$$F_T = F_V + F_P$$

$$F_T = \frac{2\mu U W}{\alpha}\left(2\ln\beta - 3\frac{(\beta-1)}{(\beta+1)}\right)$$

$$F_N = \frac{1}{(1+\alpha^2)^{1/2}}\left(\frac{6\mu U W}{\alpha^2}\right)\left(\ln\beta - 2\frac{(\beta-1)}{(\beta+1)}\right)$$

$\boxed{4.7.15}$

$$R = \frac{F_T}{F_N} = \frac{\alpha}{3}\left[\frac{2(\beta+1)\ln\beta - 3(\beta-1)}{(\beta+1)\ln\beta - 2(\beta-1)}\right]$$

FOR "MY" KNEE, $R < .002$