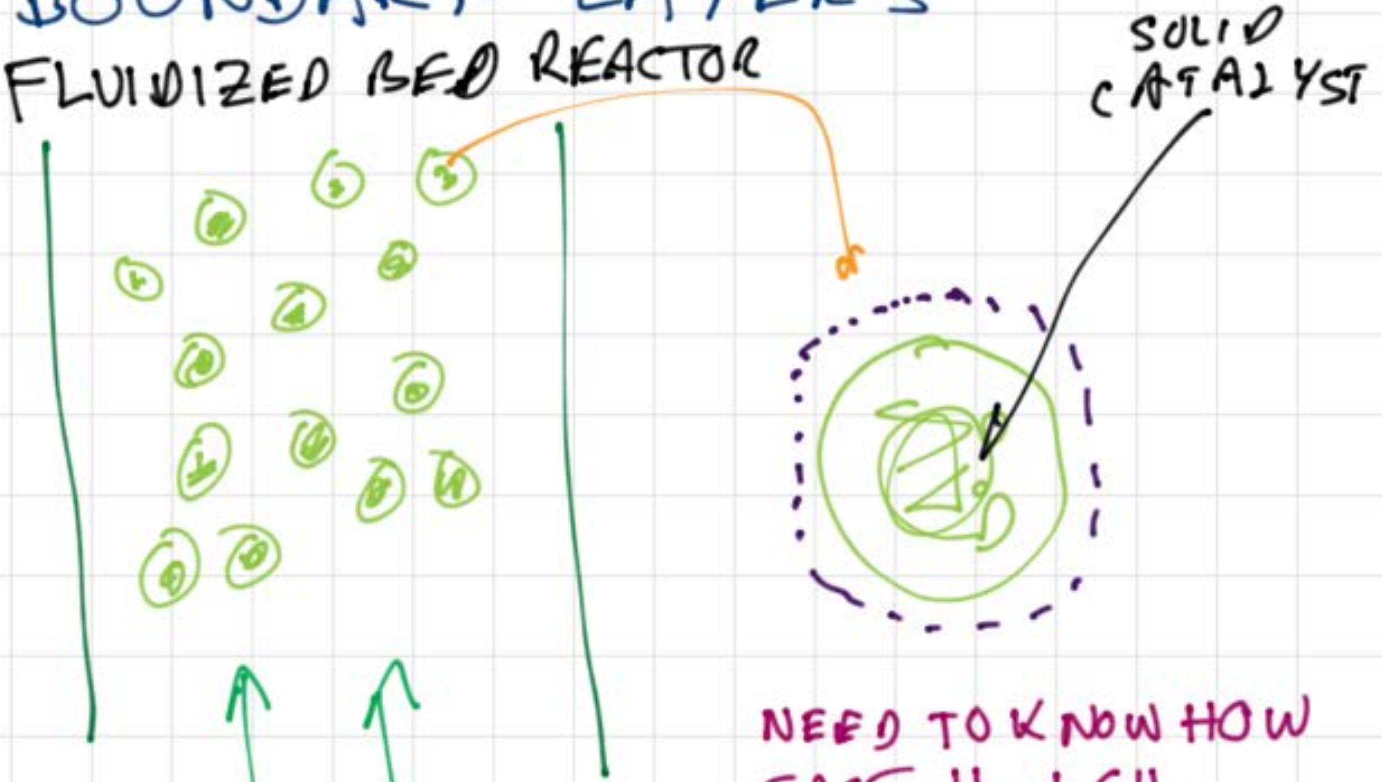
CBE 30357

11/30/17

CONTINUED DISCUSSION OF BOUNDARY-LAYERS

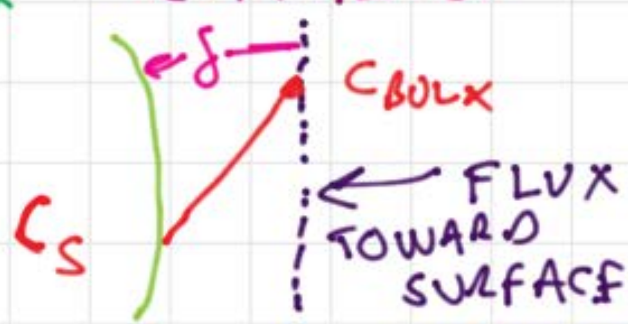
FLUIDIZED BED REACTOR



REACTANT FLOW

H_2, C_nH_{2n-x}

NEED TO KNOW HOW FAST H_2 + C_nH_m GET FROM BULK GAS TO SURFACE OF CATALYST



$$\text{FLUX} \equiv D \frac{(C_B - C_s)}{\delta} = k(C_B - C_s)$$

MOLECULAR DIFFUSIVITY

KEY QUESTION: VALUE OF δ

6.	Past single spheres	$Sc = 0.6\text{–}3200$ $Re'' Sc^{0.5} = 1.8\text{–}600\,000$	$Sh = Sh_0 + 0.347(Re'' Sc^{0.5})^{0.62}$ $Sh_0 = \begin{cases} 2.0 + 0.569(Gr_D Sc)^{0.250} & Gr_D Sc < 10^8 \\ 2.0 + 0.0254(Gr_D Sc)^{0.333} Sc^{0.244} & Gr_D Sc > 10^8 \end{cases}$	55

$$Sh \equiv \frac{kd}{D}$$ PARTICLE DIAMETER

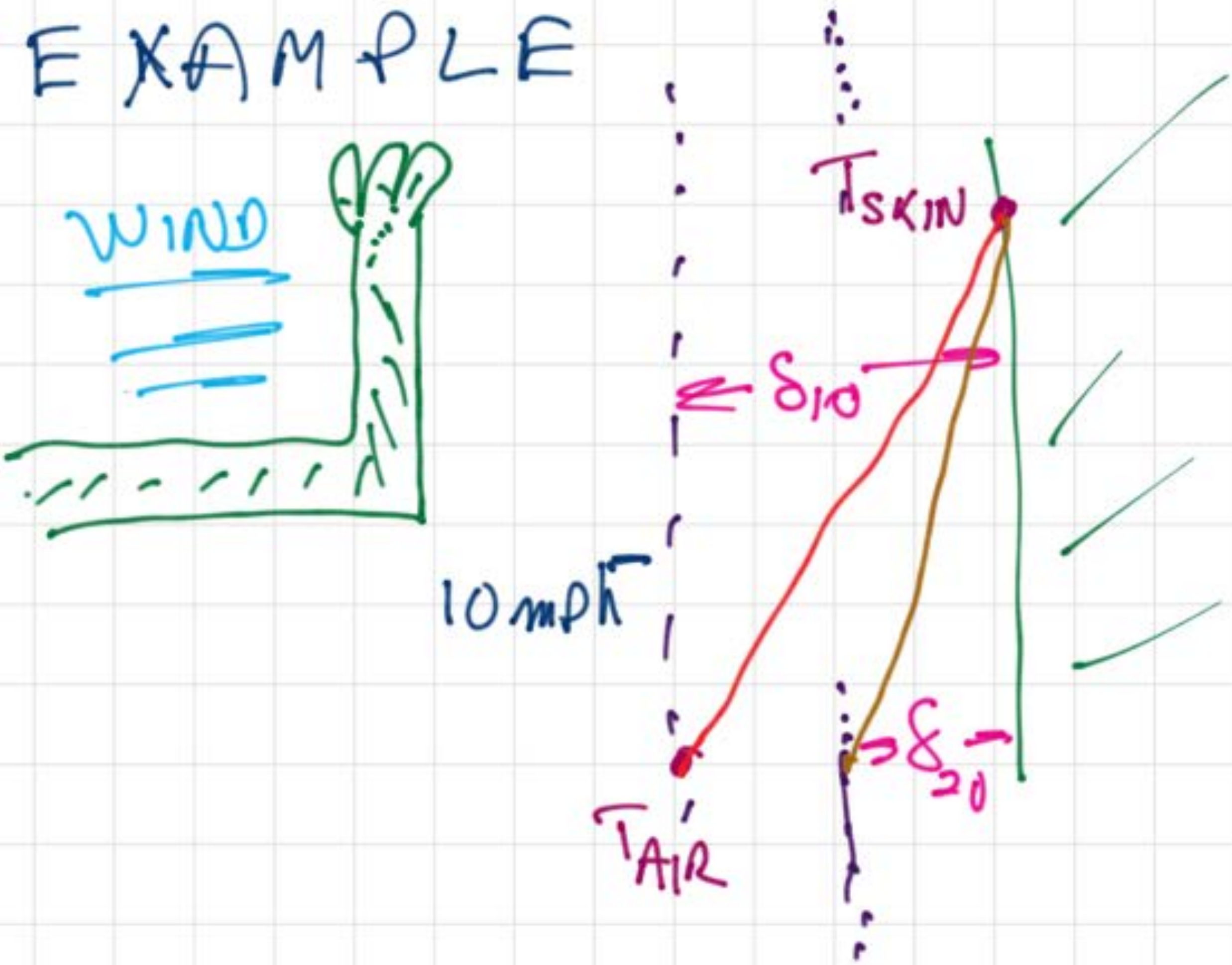
$$Sh = 2 + .347\, Re^{.62}\, Sc^{.3}$$

FIND Re, $Sc = \frac{\nu}{D}$

CALCULATE Sh,
GET k

$$\delta = \frac{D}{k}$$

WIND CHILL IS ANOTHER EXAMPLE



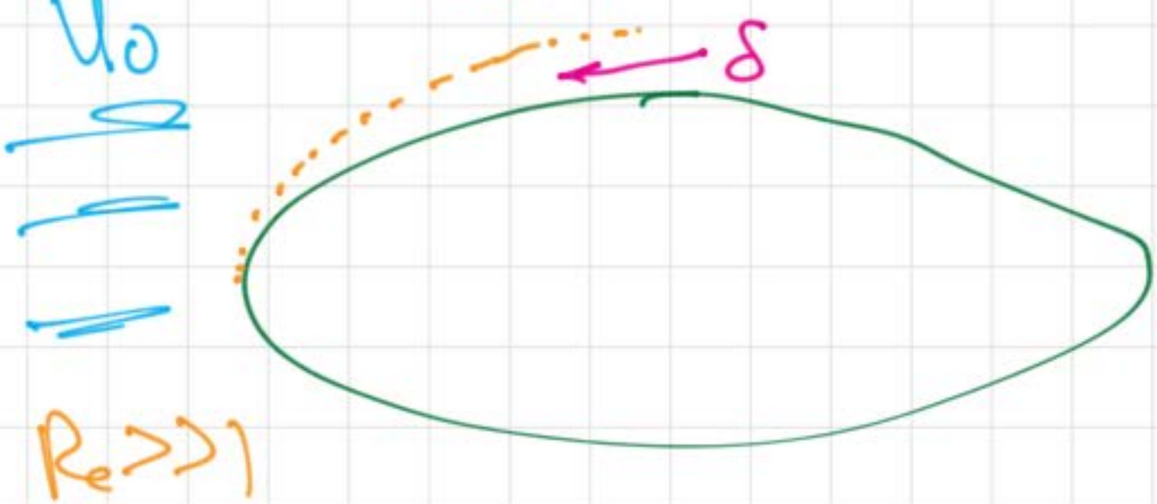
$$\text{FLUX} \hat{=} \alpha \frac{(T_{SKIN} - T_{AIR})}{\delta}$$

THERMAL DIFFUSIVITY $\quad Nu = \frac{hd}{\alpha}, \; h = \frac{\alpha}{\delta}$

5.	Perpendicular to single cylinders	Re = 400–25 000 Sc = 0.6–2.6	$\frac{k_G p_t}{G_M} Sc^{0.56} = 0.281\, Re'^{0.4}$	5
		Re′ = 0.1–10^5 Pr = 0.7–1500	$Nu = (0.35 + 0.34\, Re'^{0.5} + 0.15\, Re'^{0.58})\, Pr^{0.3}$	16, 21, 42

ANALYSIS OF NAVIER-STOKES EQUATIONS FOR A BOUNDARY-LAYER FLOW:

CAN WE LEARN ANYTHING WITHOUT ACTUALLY SOLVING THEY EQUATIONS?



CONSIDER FLOW WITHIN B.L.

PHYSICAL PRINCIPLE: INERTIA & VISCOUS TERMS ARE BOTH IMPORTANT

NONDIMENSIONALIZE NAVIER STOKES EQUATIONS

$$v_x^* \equiv \frac{v_x}{U_0} \quad , \quad v_y^* \equiv \frac{v_y}{U_0} \quad \ldots$$

$$x^* \equiv \frac{x}{L} \quad , \quad y^* \equiv \frac{y}{L}$$

$$p^* \equiv \frac{p}{\rho U_0^2} \quad \ldots$$

$$\frac{\partial \bar{v}^*}{\partial t^*} + \bar{v}^* \cdot \bar{\nabla}^* \bar{v}^* = -\bar{\nabla}^* p^* + \frac{1}{Re} \nabla^{*2} \bar{v}^*$$

AS $Re \to \infty$

$$\frac{\partial \bar{v}^*}{\partial t^*} + \bar{v}^* \cdot \bar{\nabla}^* \bar{v}^* = -\bar{\nabla}^* p^*$$

WE HAVE TO MAKE AN ALTERATION IN OUR ANALYSIS TO KEEP VISCOUS TERMS !!

CHANGE EQUATION TO:

$$x^* = \frac{x}{L} \qquad v_x^* = \frac{v_x}{U_0}$$ 👋

$$y^* = \frac{y}{\delta} \qquad v_y^* = \frac{v_y}{V}$$ 👋

ADDITIONAL LENGTH SCALE

ADDITIONAL VELOCITY SCALE..

SUBSTITUTE INTO CONTINUITY

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$\frac{U_0}{L} \underbrace{\frac{\partial v_x^*}{\partial x^*}}_{O(1)} + \frac{V}{\delta} \underbrace{\frac{\partial v_y^*}{\partial y^*}}_{O(1)} = 0$$

TO MAKE THIS WORK

THUS: $V \sim \frac{\delta}{L} U_0$

WE EXPECT A "WEAK" FLOW NORMAL TO SURFACE, $v_y \sim v_x \frac{\delta}{L}$

NOW:

RE-NONDIMENSIONALIZE NAVIER-STOKES EQUATIONS WITH U_0, L, V, δ . . .

X- N.S EQUATION:

$$\rho\left(U_0 V_x^* \frac{U_0}{L}\frac{\partial V_x^*}{\partial x^*} + V\, V_y^* \frac{U_0}{\delta}\frac{\partial V_x^*}{\partial y^*}\right) =$$

$$-\frac{\rho U_0^2}{L}\frac{\partial P^*}{\partial x^*} + \mu\left(\frac{U_0}{L^2}\frac{\partial^2 V_x^*}{\partial x^{*2}} + \frac{U_0}{\delta^2}\frac{\partial^2 V_x^*}{\partial y^{*2}}\right)$$

$$\lll$$

CLEAR VARIABLES TO LEFT

$$\frac{\rho \delta^2 U_0}{\mu L}\left(V_x^*\frac{\partial V_x^*}{\partial x} + V_y^*\frac{\partial V_x^*}{\partial y}\right) = -\frac{\partial P^*}{\partial x^*} + \frac{\partial^2 V_x^*}{\partial y^{*2}}$$

FOR OUR ANALYSIS TO WORK WE REQUIRE:

$$\frac{\rho \delta^2 U_0}{\mu L} = O(1)$$

THUS: $\delta^2 = \dfrac{\mu L}{\rho U_0}$

$$Re = \frac{L U_0 \rho}{\mu}$$

KEY RESULT!!

$$\boxed{\frac{\delta}{L} = \frac{1}{\sqrt{Re}}}$$

BOUNDARY - LAYER THICKNESS

y - DIRECTION.
USE SAME ANALYSIS!

$$\frac{\partial P}{\partial y} = O(\delta) \approx 0$$

BOUNDARY-LAYER EQUATIONS

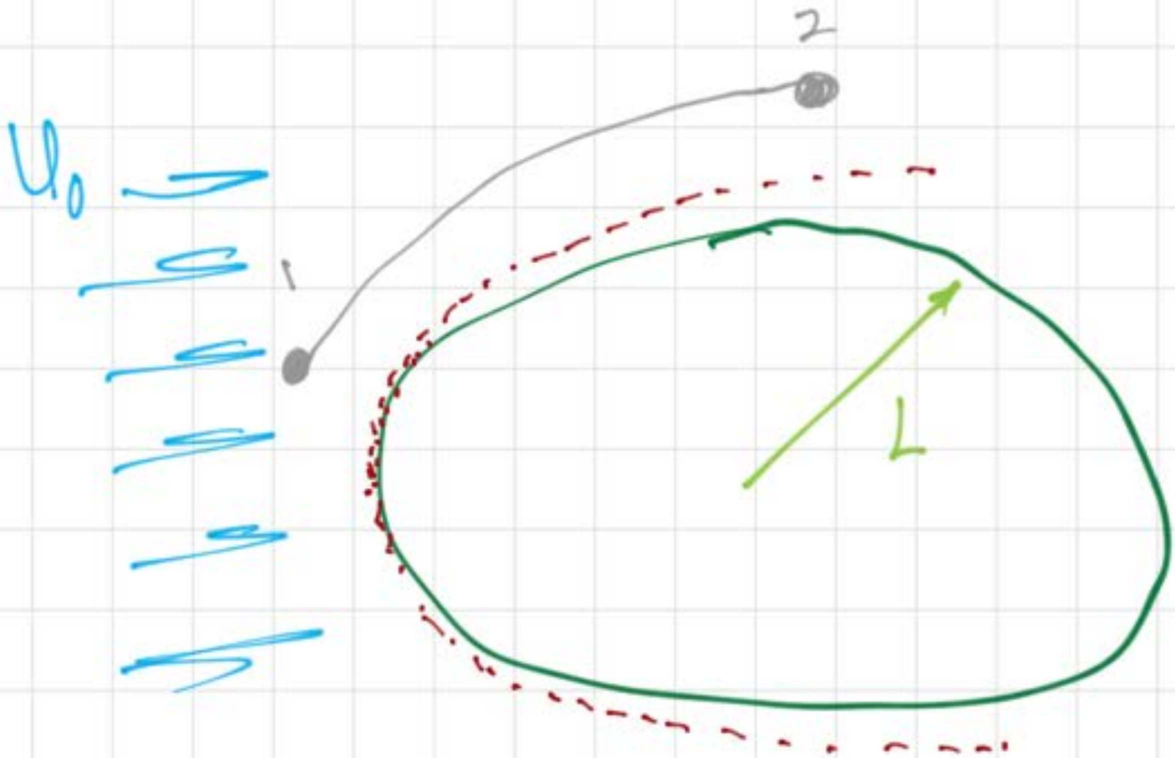
$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

$$V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 V_x}{\partial y^2}$$

$$\frac{\partial P}{\partial y} = 0$$

SO THAT WE CAN GET PRESSURE INSIDE BOUNDARY-LAYER FROM BERNOULLI EQUATION JUST ABOVE SURFACE

$$\Delta\left(\frac{U^2}{2} + gh + \frac{P}{\rho}\right) = 0$$



FOR 1 → 2 $\quad \Delta\left(\frac{V^2}{2} + gh + \frac{P}{\rho}\right) = 0$

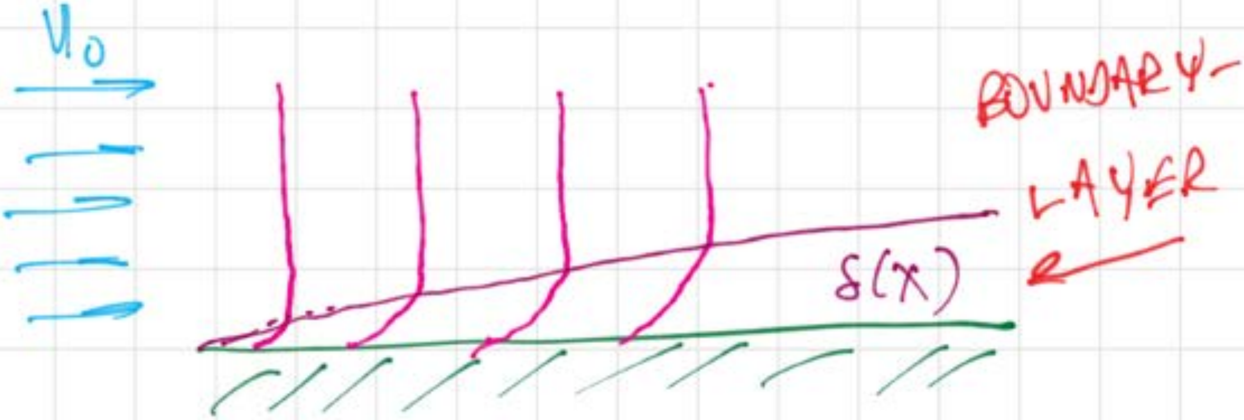
BERNOULLI

INSIDE BOUNDARY-LAYER

BOUNDARY-LAYER EQUATIONS

$$\boxed{\begin{gathered} \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \\ V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 V_x}{\partial y^2} \\ \frac{\partial P}{\partial y} = 0 \end{gathered}}$$

WE WANT TO ANALYZE:



BOUNDARY-LAYER EQUATIONS

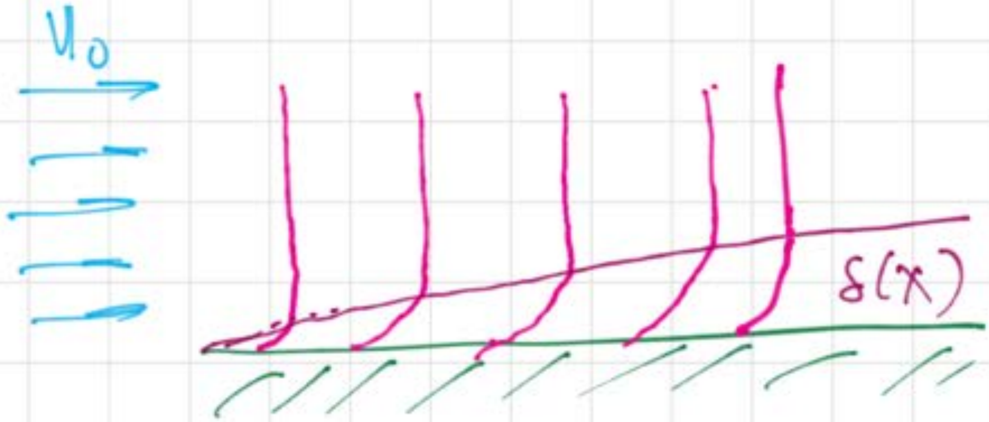
$$\boxed{\begin{gathered}\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}=0\\ v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 v_x}{\partial y^2}\\ \frac{\partial p}{\partial y}=0\end{gathered}}$$

CONSIDER "FLAT PLATE"

$$\Delta\left(\frac{u^2}{2}+gh+\frac{p}{\rho}\right)=0$$

$$u_1=u_2 \quad p_1=p_2 \quad \therefore \frac{\partial p}{\partial x}=0$$

STILL HAVE COUPLED, NONLINEAR PDE'S ... CAN WE SIMPLIFY?



SUPPOSE THAT THIS CARTOON HAS SOME REAL INSIGHT

- NO GEOMETRIC LENGTH SCALE FOR BOUNDARY - LAYER
- COULD THE PROFILES BE SELF SIMILAR?

IF THE PROFILES ARE SELF-SIMILAR THEN AS WE PROGRESS IN DISTANCE THERE IS A RELATIONSHIP BETWEEN X + Y AS:

$\eta \Rightarrow$ SIMILARITY VARIABLE

$$\eta \equiv \frac{y}{\sqrt{\nu x / u_0}}$$

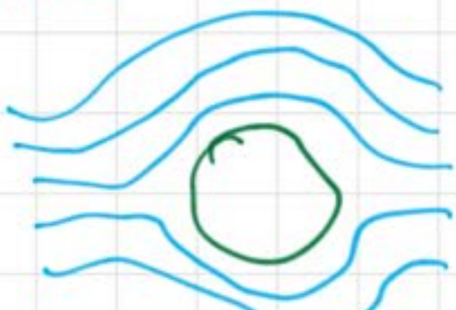
ONLY LENGTH SCALE TO MAKE

APPARENT CHANGE IN δ W/ DISTANCE

WE WILL ALSO USE THE "STREAM FUNCTION", ψ

DEFINED FROM CONTINUITY

$$v_x \equiv \frac{\partial \psi}{\partial y}, \quad v_y \equiv -\frac{\partial \psi}{\partial x}$$



LINES OF ψ = CONSTANT

$$\psi(x,y) \equiv f(\eta)\sqrt{\nu x u_0}$$

FROM THIS:

$$V_x = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y}$$

$$V_y = -\frac{\partial \psi}{\partial x} = -\frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial x} \quad \ldots$$

$$f'''(\eta) + \frac{1}{2} f(\eta) f''(\eta) = 0$$

BC's

$$f(\eta) = 0 \quad V_y(y=0, x) = 0$$

$$f'(\eta) = 0 \quad V_x(y=0, x) = 0$$

$$f'(\infty) = 1 \quad \begin{cases} V_x(x=0, y) = U_0 \\ V_x(x, y=\infty) = U_0 \end{cases}$$

SOLVE BY SHOOTING METHOD

BOUNDARY-LAYER THICKNESS IS ABOUT $\eta = 5$

SO, WHAT IS y VALUE OF δ ?

$$\frac{y}{\sqrt{\frac{x\nu}{U_0}}} = 5 \quad \therefore \quad \delta = 5\sqrt{\frac{x\nu}{U_0}}$$

$$\delta = 5x\sqrt{\frac{\nu}{xU_0}}$$

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}} \qquad Re_x \equiv \frac{xU_0}{\nu}$$

WHAT IS WALL SHEAR STRESS?

$$\tau_w = \mu\frac{\partial V_x}{\partial y} = \mu U_0\left(\frac{U_0}{x\nu}\right)^{1/2} f''(0)$$

$$\tau_w = .332\,\mu U_0\left(\frac{U_0}{\nu x}\right)^{1/2}$$

COMPARE TO:

$$\tau_w = \mu \frac{U_0}{\delta} \ \ldots$$

$$\tau_w = \mu \frac{U_0}{5x\, Re_x^{-1/2}}$$

$$= .2 \mu U_0 \left(\frac{U_0}{\nu x} \right)^{1/2}$$

SO, .2 COMPARED TO .33 IS PRETTY CLOSE!!

WHAT IS EFFECT OF PRESSURE GRADIENT ON BOUNDARY-LAYER?

BERNOULLI EQ.

$$\Delta\left(\frac{V^2}{2}+gh+\frac{p}{\rho}\right)=0$$

$$\mathcal{P}=p+\rho g h$$

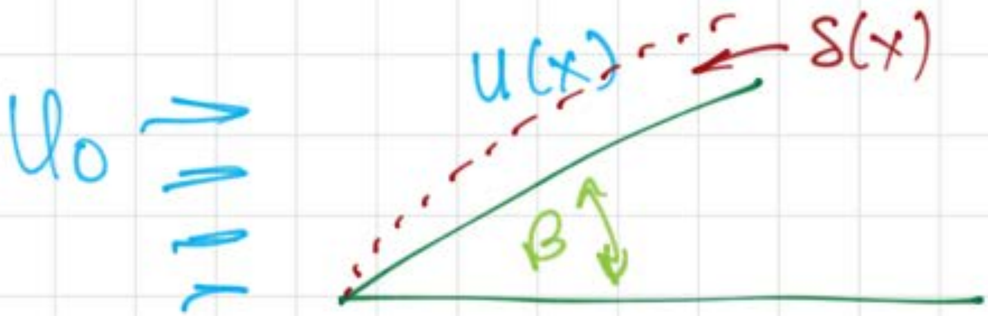
THEN DIFFERENTIATE:

$$\underbrace{V\frac{dV}{dx}}_{\text{CHANGE IN FARAWAY FLOW}}+\frac{1}{\rho}\frac{d\mathcal{P}}{dx}=0$$

REPLACE $\frac{d\mathcal{P}}{dx}$ IN BOUNDARY-LAYER EQ

$$v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}=\underbrace{U\frac{dU}{dx}}+\nu\frac{\partial^2 v_x}{\partial y^2}$$

FLOW PAST A WEDGE



$$U(x) = A x^m$$

$$m \equiv \frac{\beta}{(2\pi - \beta)}$$

INCLUDE THIS NEW TERM IN ANALYSIS:

$$f''' + \frac{m+1}{2} f f'' + m(1 - f'^2) = 0$$

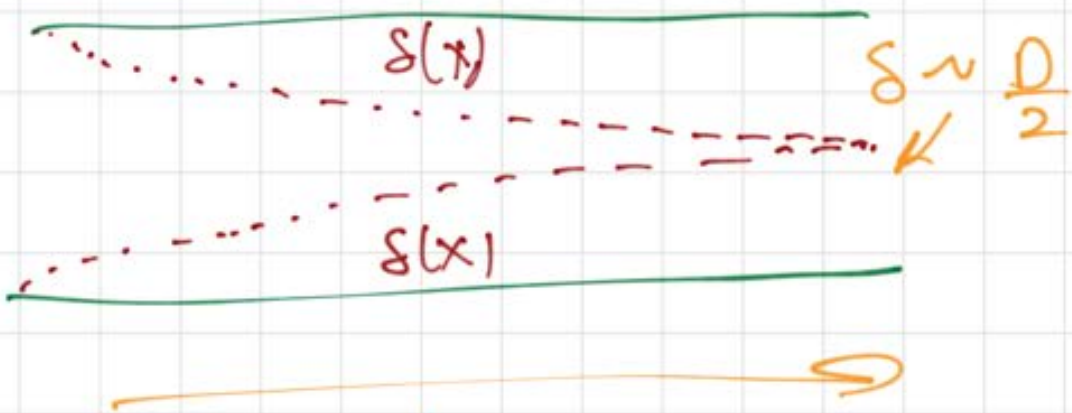
SYSTEM OF ODE'S:

$$y_1' = y_2$$
$$y_2' = y_3$$
$$y_3' = -\frac{m+1}{2} y_1 y_3 + m(1 - y_2^2)$$

SOLVE ....

SEE "BOUNDARYLAYER .. 2016"

LET US EXTEND THE DEVELOPING BOUNDARY-LAYER ON A PLATE TO DEVELOPING LAMINAR FLOW IN A PIPE (OR CHANNEL)



ENTRANCE LENGTH: L_e

$$\frac{D}{2} = \delta = 5x\sqrt{\frac{\nu}{x u_0}} \qquad x \to L_e$$

$$\frac{D^2}{4} = 25\,\frac{\nu L_e}{u_0}$$

$$\frac{L_e}{D} \sim \frac{1}{100}\,\frac{u_0 D}{\nu} = 0.01\,Re$$