CBE 30357

11/28/17

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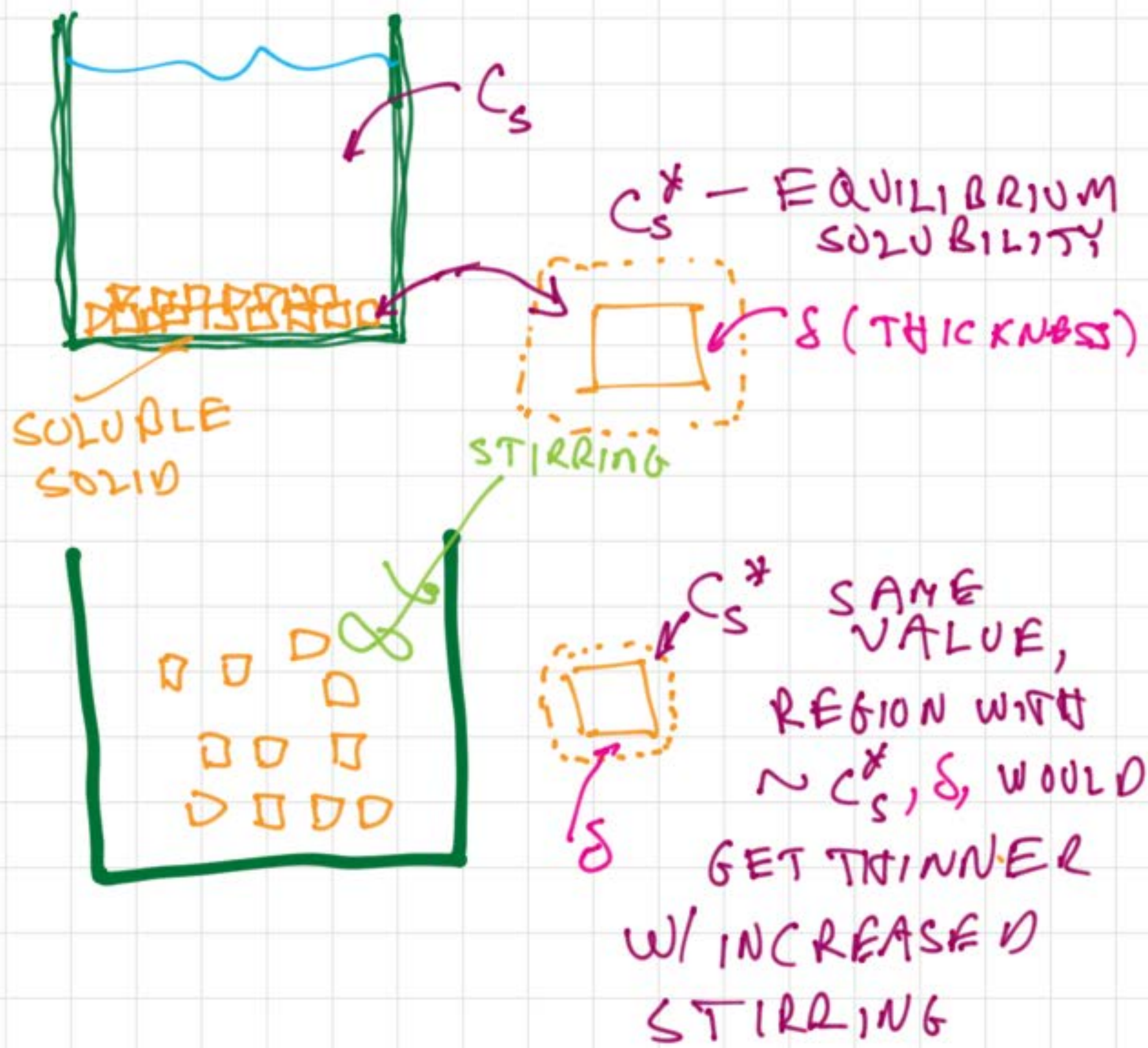
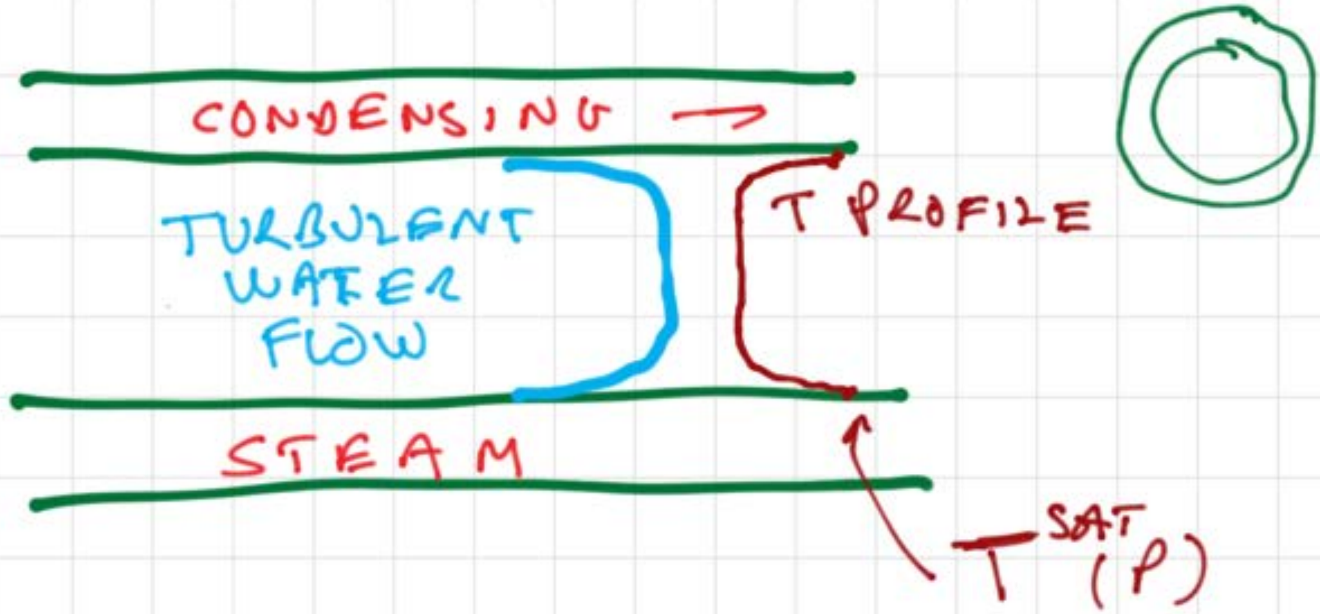
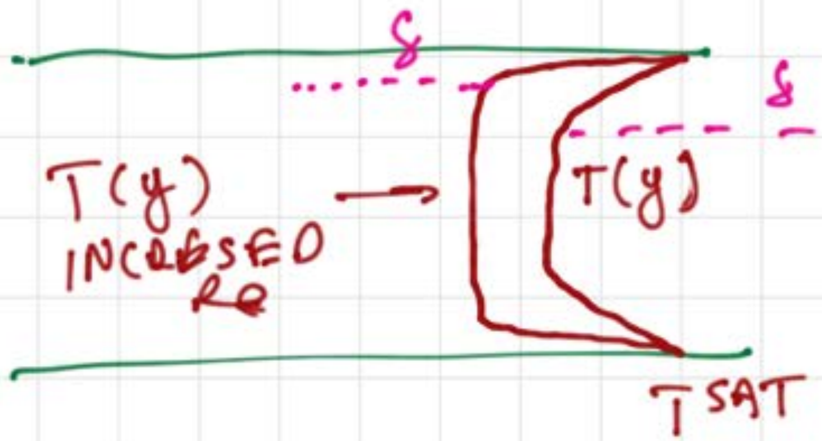




WE WOULD EXPECT THAT THE WALL TEMPERATURE WOULD STAY CLOSE TO T^{SAT} AS WE INCREASED WATER FLOW



δ THICKNESS OVER WHICH MOST OF TEMPERATURE CHANGE OCCURS

- FOR SOLID DISSOLVING AND WATER HEATING, MOST OF TEMPERATURE CHANGE OCCURS IN A THIN (COMPARED TO MACRO-REGION DIMENSION OF SYSTEM)

- AT THE BOUNDARY "EQUILIBRIUM VALUE" PERSISTS EVEN THOUGH REGION GETS THINNER AS STIRRING IS INCREASED

- DIFFUSION/CONDUCTION IS OCCURING, IT MIGHT NOT BE OBVIOUS, BUT CONVECTION IS ALSO IMPORTANT IN REGION

- WE CALL THE THIN LAYER A "BOUNDARY-LAYER"

CONVECTION/DIFFUSION
CONVECTION/CONDUCTION
INERTIA/VISCOUS

} SAME MAGNITUDE

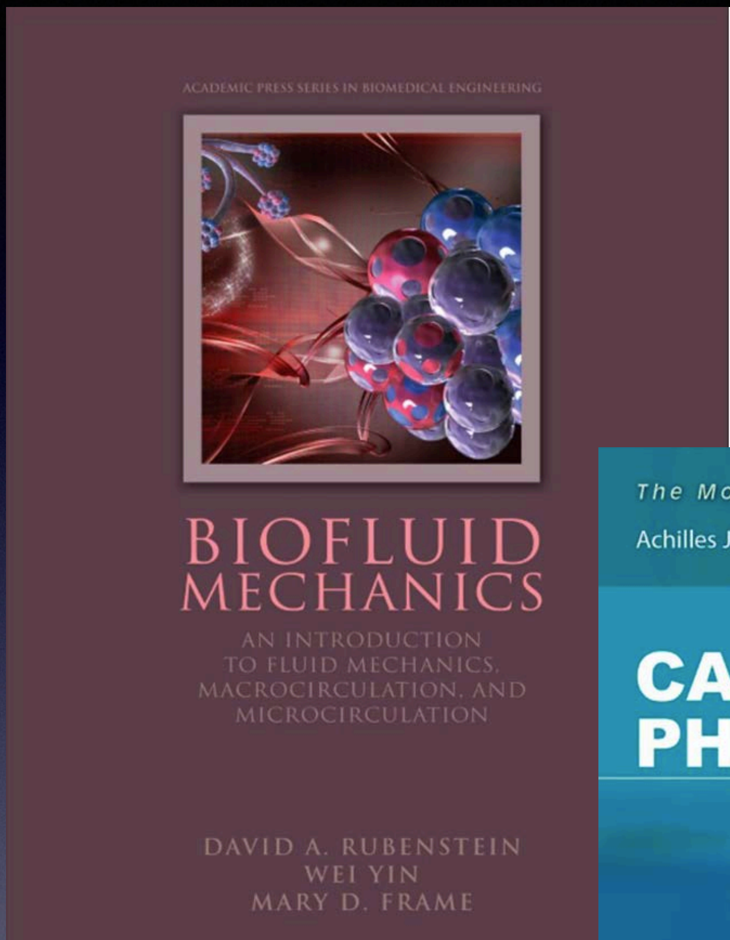
Relevance of developing, transient and *Boundary-Layer* Flows!

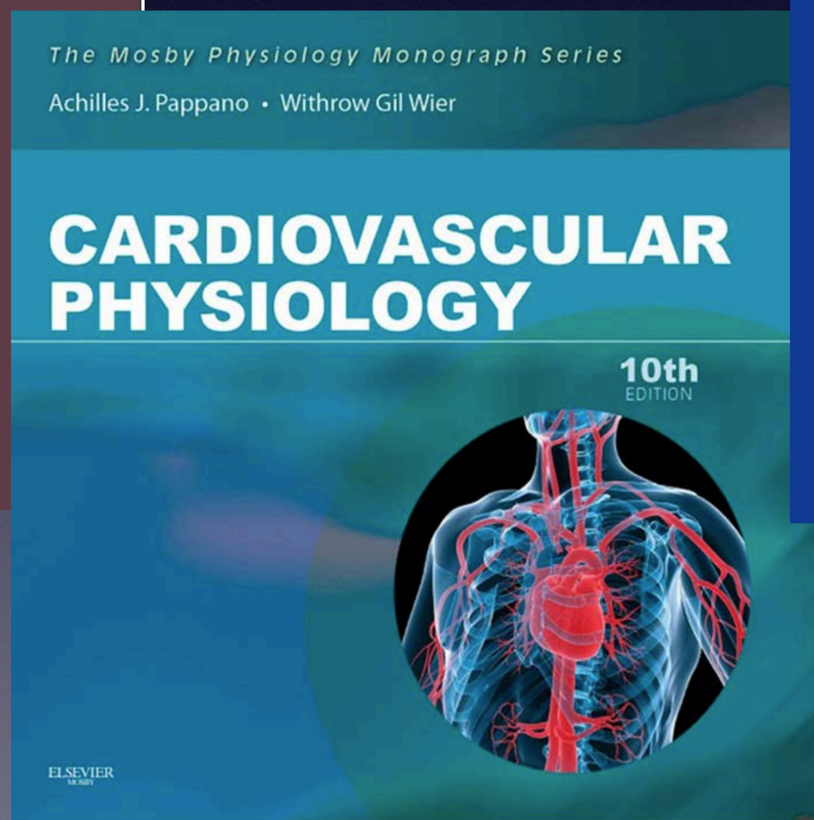
Thanks to Milton van Dyke, Robert Treybal and Bob Brodkey and cardiology literature

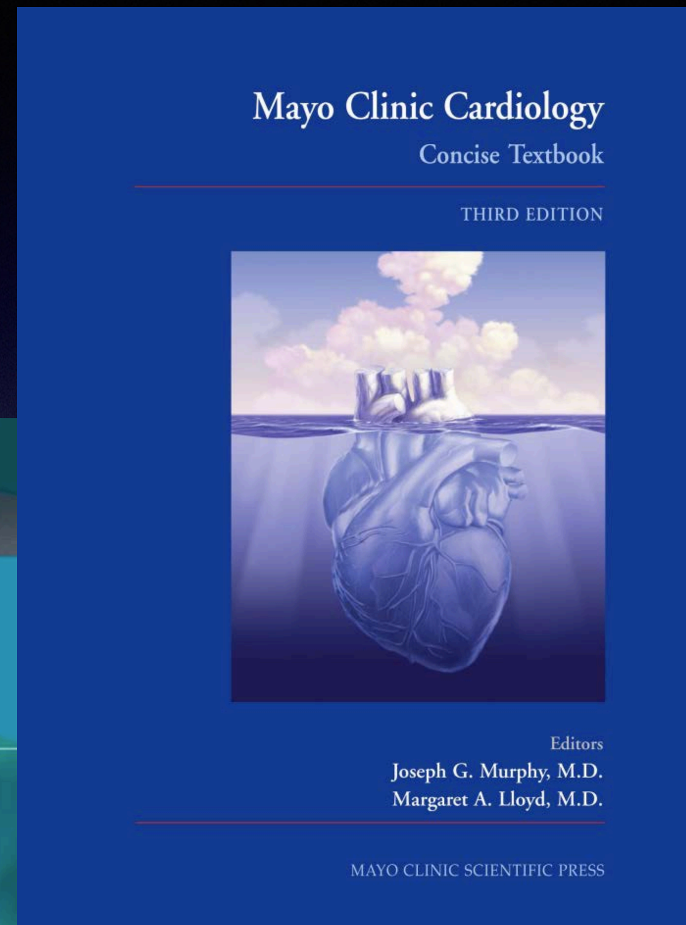
Boundary-layer flows

- *Boundary layer* flows are an important topic for the first Transport course because their essential "physics" is the next logical topic intellectually, there are many important engineering examples and the heat and mass transfer analogies are central to understanding chemical engineering
- Yet, I considered it almost a personal failing that I did not have much in the way of physiological motivation for this topic…

“Boundary-Layer” not found







Maybe I don’t have to feel so bad…

“boundary-layer”

- For flows with Reynolds number >> 1
  - inertia forces dominate viscous forces over (most) of the flow field
  - however because of the no-slip condition, the velocity will be very much slower close to the solid surface
  - the resulting velocity gradient is large, but confined to a thin region (small comported to macroscopic dimensions of the channel or object in the flow) close to the surface
  - in the thin, high gradient region viscous forces and inertia forces are about the same order magnitude.
  - This region is called a “boundary-layer”

Historical origins

- Aerodynamics
  - “Lift” can be calculated from “inviscid” flow theory (no viscosity, hence fluid is assumed to slip at wing surface)
  - “drag” requires knowledge of shear stress on the wing surface, thus viscous effects and the no-slip condition
  - Prandtl (1904) is credited with first explaining the idea of a boundary-layer

Prosaic interest

- If you want to dissolve sugar in your iced-tea, you feel the overwhelming, natural urge to stir it!
- It is obvious that the weather is changing and soon we will be talking about the dreaded words..
  - “Wind chill”

Physiology

Coronary Artery Wall Shear Stress Is Associated With Progression and Transformation of Atherosclerotic Plaque and Arterial Remodeling in Patients With Coronary Artery Disease

Habib Samady, MD; Parham Eshtehardi, MD; Michael C. McDaniel, MD; Jin Suo, PhD; Saurabh S. Dhawan, MD; Charles Maynard, PhD; Lucas H. Timmins, PhD; Arshed A. Quyyumi, MD; Don P. Giddens, PhD

Background—Experimental studies suggest that low wall shear stress (WSS) promotes plaque development and high WSS is associated with plaque destabilization. We hypothesized that low-WSS segments in patients with coronary artery disease develop plaque progression and high-WSS segments develop necrotic core progression with fibrous tis

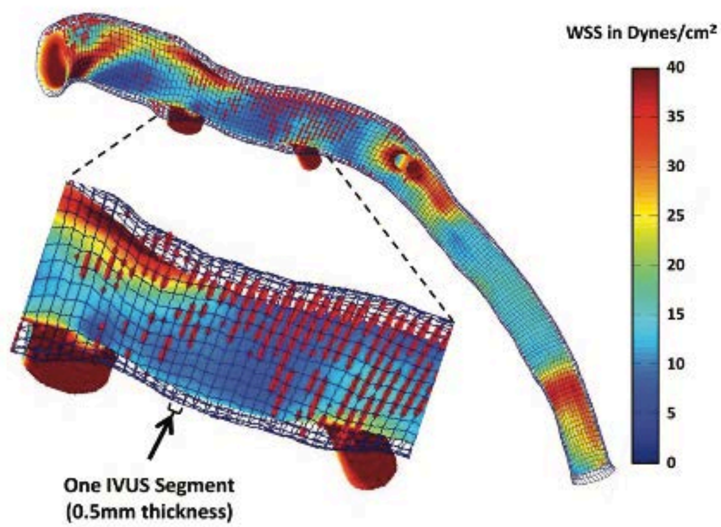


Figure 2. Example of a wall shear stress (WSS) profile of the left anterior descending coronary artery from a patient demonstrating lumen and external elastic membrane boundaries, superimposed virtual histology intravascular ultrasound (IVUS)–derived necrotic core data (red dots), and areas of variable WSS. The magnified segment of the vessel demonstrates the high-resolution spatial location of the IVUS images (thickness=0.5 mm) superimposed on the WSS profile. Time-averaged WSS values were circumferentially averaged for each IVUS segment to provide quantitative hemodynamic data to correlate with plaque progression data.

EDITORIAL

Role of Shear Stress in Atherosclerosis and Restenosis After Coronary Stent Implantation

Rosaire Mongrain[a] and Josep Rodés-Cabau[b]

[a]Department of Mechanical Engineering, McGill University, Montreal, Canada.
[b]Quebec Heart Institute, Laval Hospital, Quebec, Canada.

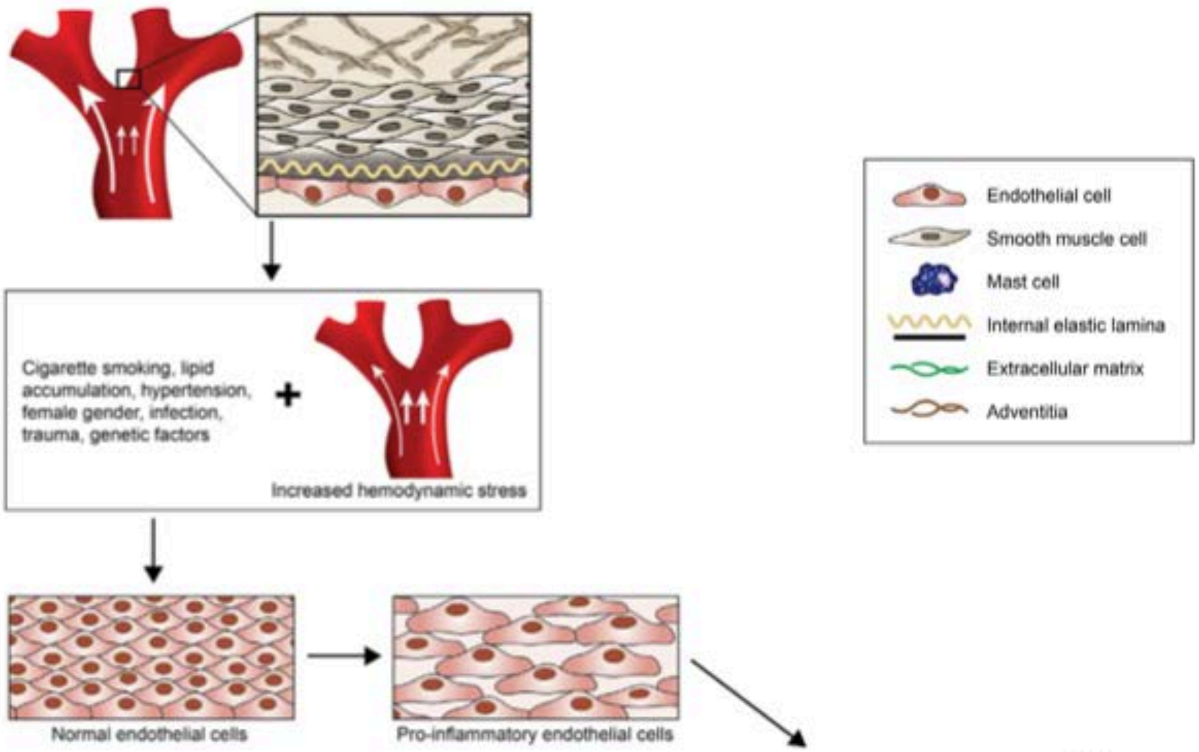
and progression of different plaques. In the carotid artery, for instance, there is a well-known predisposition for atheroma plaques to develop in the outer wall of the vessel at the level of the bifurcation of the common carotid artery,[1,2] and in the coronary arteries it has also been shown that there is a preferential progression of plaques in

segments with bifurcations, as well as along the inner wall of curves in the coronary arteries.[3,4] On the other hand, it is unclear why some plaques remain quiescent for many years, while others progress rapidly. These findings

Review of Cerebral Aneurysm Formation, Growth, and Rupture

Nohra Chalouhi, MD; Brian L. Hoh, MD; David Hasan, MD

3614 *Stroke* **December 2013**

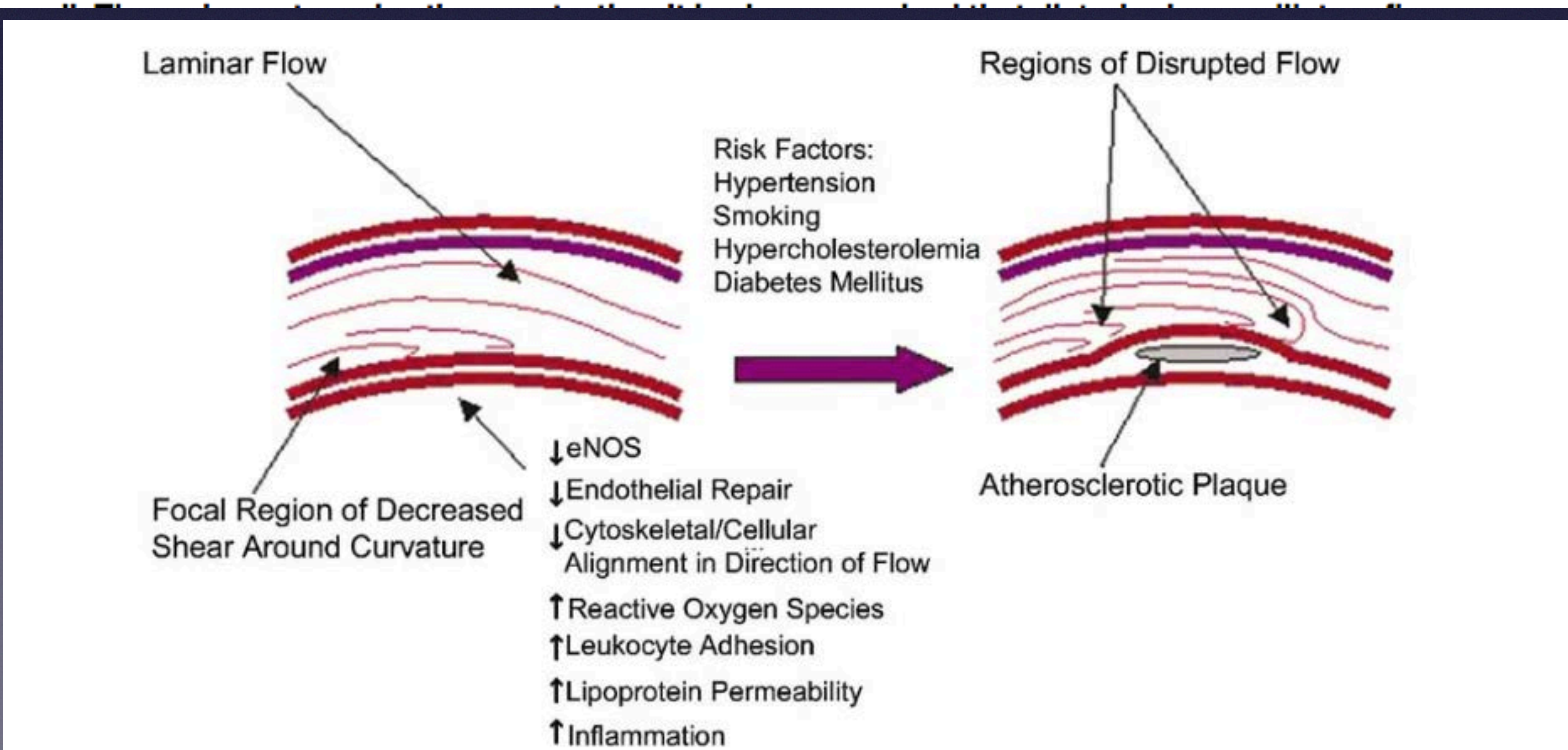


The role of shear stress in the pathogenesis of atherosclerosis

Kristopher S Cunningham[1,2] and Avrum I Gotlieb[1,2]

[1]*Department of Pathology, Toronto General Research Institute, University Health Network, Canada and*
[2]*Department of Laboratory Medicine and Pathobiology, University of Toronto, Toronto, Ontario, Canada*

Although the pathobiology of atherosclerosis is a complex multifactorial process, blood flow-induced shear stress has emerged as an essential feature of atherogenesis. This fluid drag force acting on the vessel wall is mechanotransduced into a biochemical signal that results in changes in vascular behavior. Maintenance of a physiologic, laminar shear stress is known to be crucial for normal vascular functioning, which includes the regulation of vascular caliber as well as inhibition of proliferation, thrombosis and inflammation of the vessel



Examples: Developing flows

- Recall that our fully-developed (i.e., not changing with distance) flows, don't have non-zero inertia terms.
- Thus, "boundary-layer" flows are spatially changing flows.
- Classic example, flow along a flat plate the boundary layer starts at the front edge and gets thicker along the plate in the flow direction

TABLE 3.4

Navier–Stokes Equation for an Incompressible Fluid

Rectangular coordinates

x direction

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x \qquad (3.3.26a)$$

y direction

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y \qquad (3.3.26b)$$

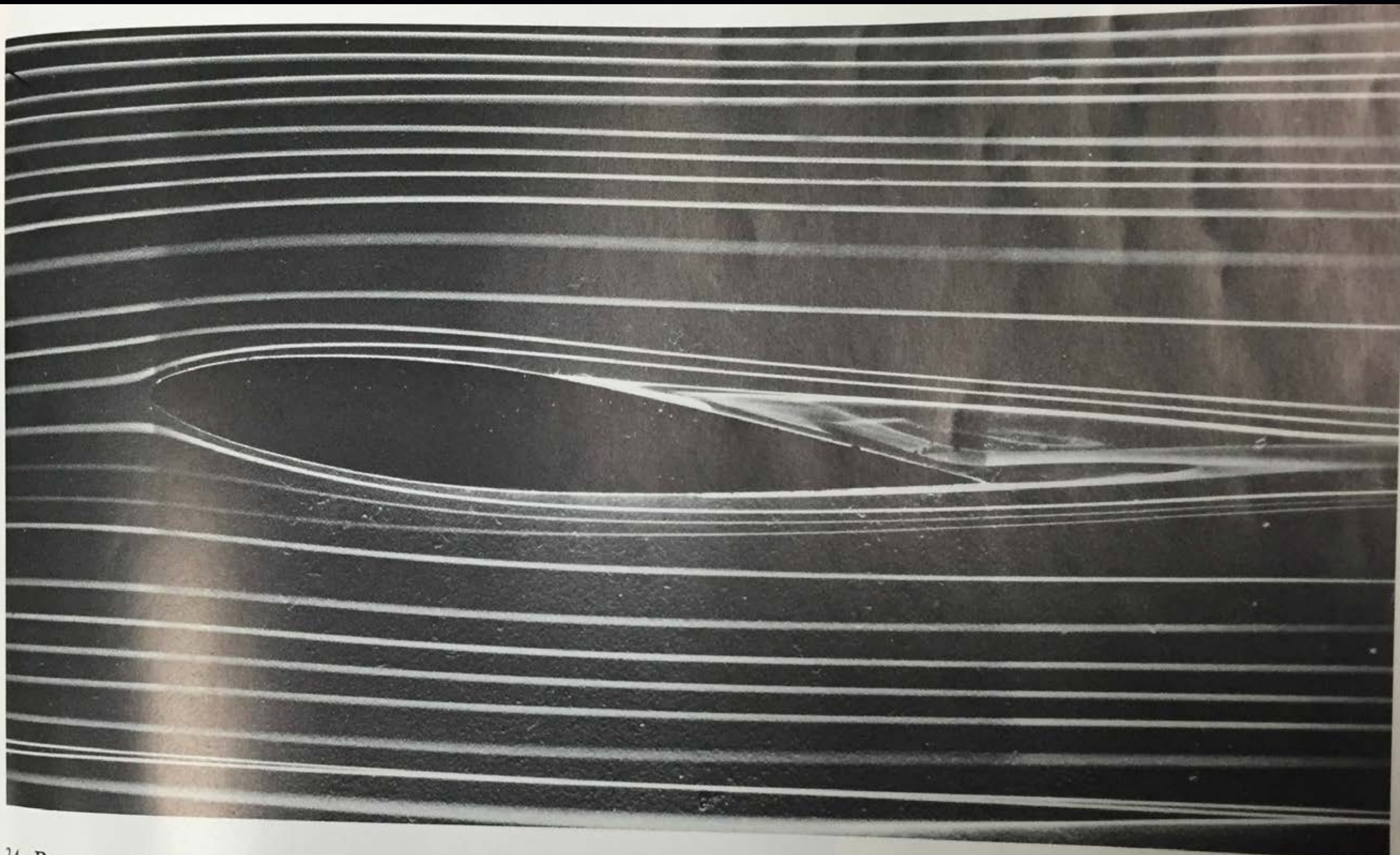
z direction

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \qquad (3.3.26c)$$

Boundary-Layers and Correlations!

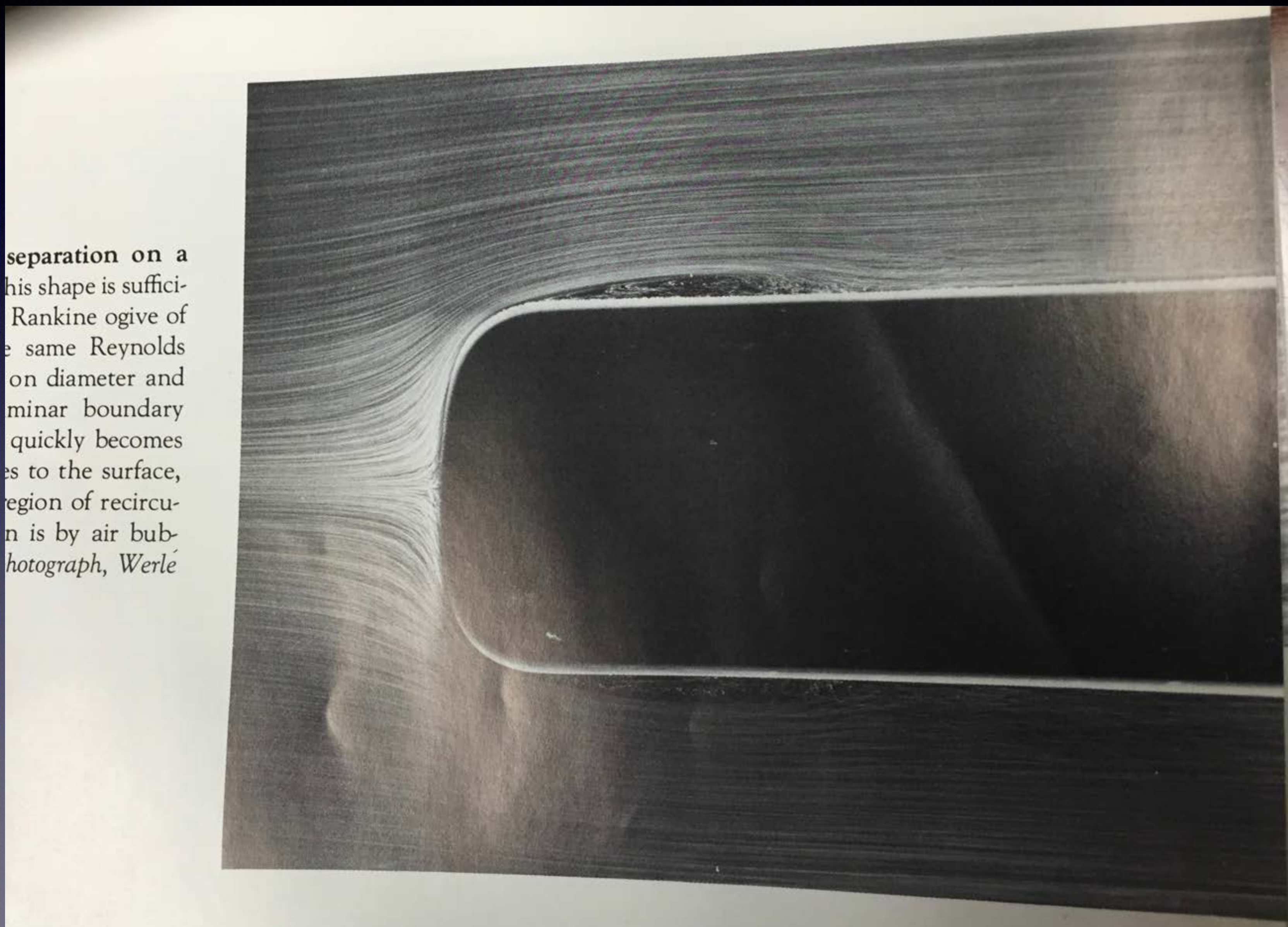
Thanks to Milton van Dyke, Robert Treybal and Bob Brodkey

Boundary-Layer must be very thin and viscous effects confined to region close to surface



34. **Boundary-layer separation on an inclined airfoil.** When the NACA 64A015 airfoil of figure 23 is raised to 5° incidence the laminar boundary layer separates from the rear half of the upper surface. The flow remains attached to the lower surface, from which it leaves tangentially at the trailing edge. Streamlines are shown by colored fluid filaments in water. *ONERA photograph, Werlé 1974*

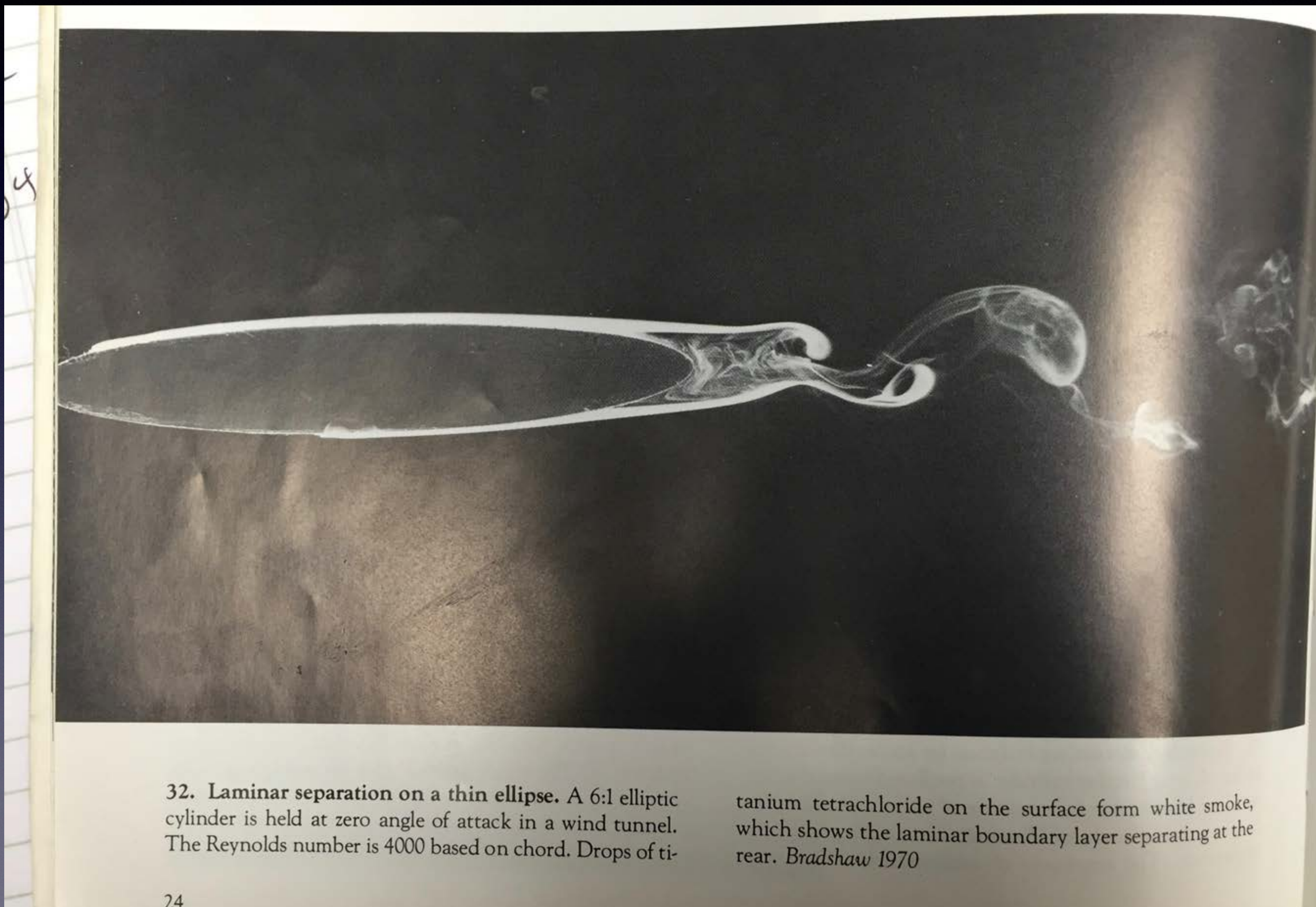
Flow is forced around object (hence acceleration of free stream) but boundary layer is small compared to object size



boundary-layer separation and "reattachment"

35. Leading-edge separation on a plate with laminar reattachment. A flat plate 2 per cent thick with beveled edges is inclined at 2.5° to the stream. The laminar boundary layer separates at the leading edge over the upper surface. At this Reynolds number of 10,000 based on length it then reattaches while still laminar, enclosing a long leading-edge "bubble" of recirculating fluid. Visualization is by air bubbles in water. *ONERA photograph, Werlé 1974*

Boundary layer separates



Flow along a “flat” plate

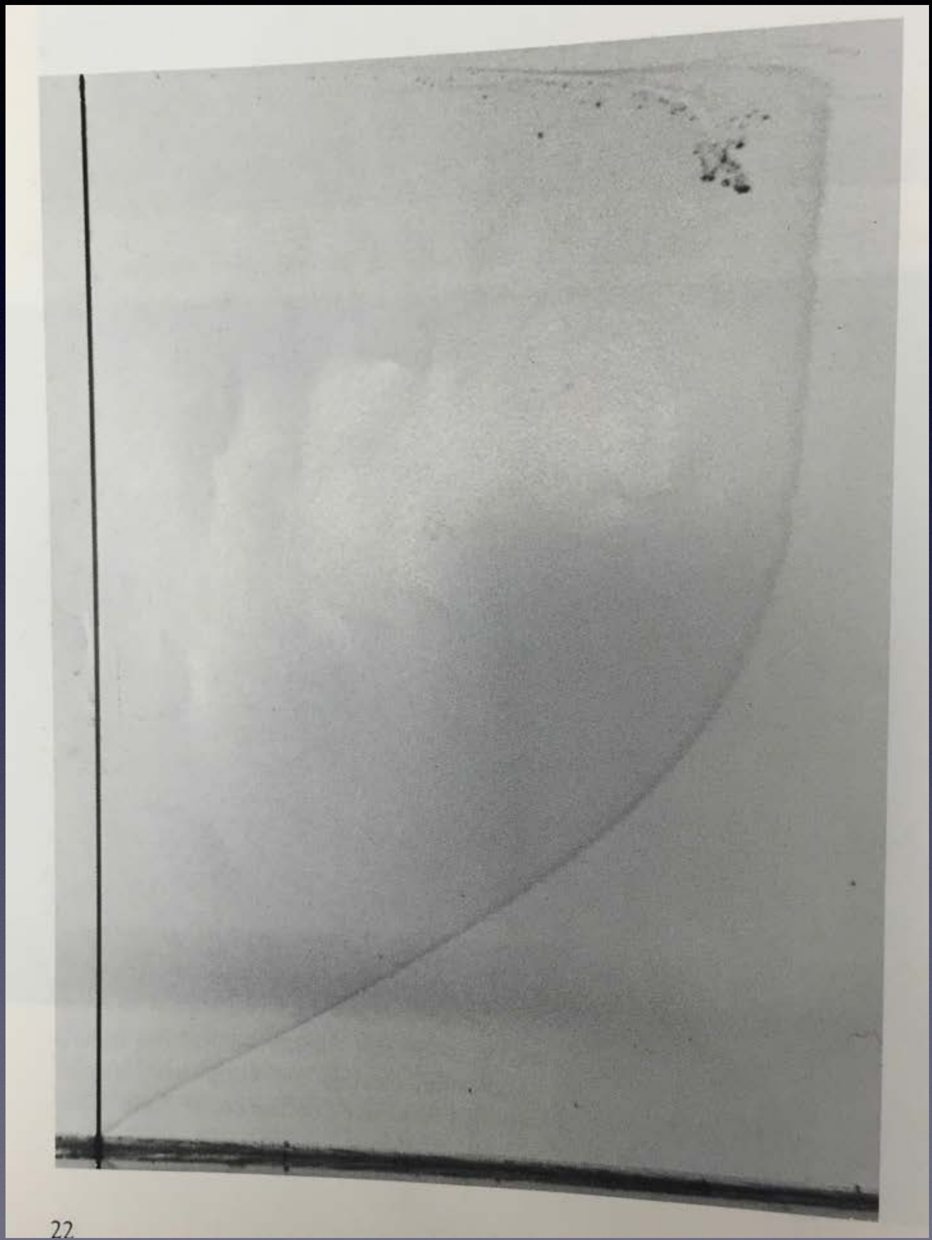


Table 3.3 Mass transfer† for simple situations

Fluid motion	Range of conditions	Equation	Ref.
1. Inside circular pipes	Re = 4000–60 000 Sc = 0.6–3000	$j_D = 0.023\ \mathrm{Re}^{-0.17}$ $\mathrm{Sh} = 0.023\ \mathrm{Re}^{0.83}\ \mathrm{Sc}^{1/3}$	41, 52
	Re = 10 000 – 400 000 Sc > 100	$j_D = 0.0149\ \mathrm{Re}^{-0.12}$ $\mathrm{Sh} = 0.0149\ \mathrm{Re}^{0.88}\ \mathrm{Sc}^{1/3}$	44
2. Unconfined flow parallel to flat plates‡	Transfer begins at leading edge $\mathrm{Re}_x < 50\,000$	$j_D = 0.664\ \mathrm{Re}_x^{-0.5}$	32
	$\mathrm{Re}_x = 5 \times 10^5 – 3 \times 10^7$ Pr = 0.7–380	$\mathrm{Nu} = 0.037\ \mathrm{Re}_x^{0.8}\ \mathrm{Pr}_0^{0.43}\left(\frac{\mathrm{Pr}_0}{\mathrm{Pr}_i}\right)^{0.25}$	65
	$\mathrm{Re}_x = 2 \times 10^4 – 5 \times 10^5$ Pr = 0.7–380	Between above and $\mathrm{Nu} = 0.0027\ \mathrm{Re}_x\ \mathrm{Pr}_0^{0.43}\left(\frac{\mathrm{Pr}_0}{\mathrm{Pr}_i}\right)^{0.25}$	
3. Confined gas flow parallel to a flat plate in a duct	$\mathrm{Re}_e = 2600–22\,000$	$j_D = 0.11\ \mathrm{Re}_e^{-0.29}$	46
4. Liquid film in wetted-wall tower, transfer between liquid and gas	$\frac{4\Gamma}{\mu} = 0–1200$, ripples suppressed	Eqs. (3.18)–(3.22)	20, 37
	$\frac{4\Gamma}{\mu} = 1300–8300$	$\mathrm{Sh} = (1.76 \times 10^{-5})\left(\frac{4\Gamma}{\mu}\right)^{1.506} \mathrm{Sc}^{0.5}$	

5.	Perpendicular to single cylinders	Re = 400–25 000 Sc = 0.6–2.6	$\frac{k_G p_t}{G_M} \mathrm{Sc}^{0.56} = 0.281\ \mathrm{Re}'^{0.4}$	5
		$\mathrm{Re}' = 0.1\text{–}10^5$ Pr = 0.7–1500	$\mathrm{Nu} = (0.35 + 0.34\ \mathrm{Re}'^{0.5} + 0.15\ \mathrm{Re}'^{0.58})\ \mathrm{Pr}^{0.3}$	16, 21, 42
6.	Past single spheres	Sc = 0.6–3200 $\mathrm{Re}''\ \mathrm{Sc}^{0.5} = 1.8\text{–}600\ 000$	$\mathrm{Sh} = \mathrm{Sh}_0 + 0.347(\mathrm{Re}''\ \mathrm{Sc}^{0.5})^{0.62}$ $\mathrm{Sh}_0 = \begin{cases} 2.0 + 0.569(\mathrm{Gr}_D\ \mathrm{Sc})^{0.250} & \mathrm{Gr}_D\ \mathrm{Sc} < 10^8 \\ 2.0 + 0.0254(\mathrm{Gr}_D\ \mathrm{Sc})^{0.333}\ \mathrm{Sc}^{0.244} & \mathrm{Gr}_D\ \mathrm{Sc} > 10^8 \end{cases}$	55
7.	Through fixed beds of pellets§	Re″ = 90–4000 Sc = 0.6	$j_D = j_H = \frac{2.06}{\varepsilon} \mathrm{Re}''^{-0.575}$	4, 23, 64
		Re″ = 5000–10 300 Sc = 0.6	$j_D = 0.95 j_H = \frac{20.4}{\varepsilon} \mathrm{Re}''^{-0.815}$	
		Re″ = 0.0016–55 Sc = 168–70 600	$j_D = \frac{1.09}{\varepsilon} \mathrm{Re}''^{-2/3}$	
		Re″ = 5–1500 Sc = 168–70 600	$j_D = \frac{0.250}{\varepsilon} \mathrm{Re}''^{-0.31}$	

† Average mass-transfer coefficients throughout, for constant solute concentrations at the phase surface. Generally, fluid properties are evaluated at the average conditions between the phase surface and the bulk fluid. The heat-mass-transfer analogy is valid throughout.

‡ Mass-transfer data for this case scatter badly but are reasonably well represented by setting $j_D = j_H$.

§ For fixed beds, the relation between ε and d_p is $a = 6(1 - \varepsilon)/d_p$, where a is the specific solid surface, surface per volume of bed. For mixed sizes [58]

$$d_p = \frac{\sum_{i=1}^{n} n_i d_{pi}^3}{\sum_{i=1}^{n} n_i d_{pi}^2}$$

wall roughness conditions. The modern form [M3, S6] of the Dittus–Bo[elter]
correlation [D3], which is based on Eq. (11.65), is

$$N_{\mathrm{Nu,mb}} = \bar{h} d_i / k_{\mathrm{mb}} = 0.023 (N_{\mathrm{Re,mb}})^{0.8} (N_{\mathrm{Pr,mb}})^{n} \quad (11$$

$$0.7 \le N_{\mathrm{Pr,mb}} \le 100$$

$$10\,000 \le N_{\mathrm{Re,mb}} \le 120\,000$$

$$L/d_i \ge 60 \quad \text{(smooth tubes)}$$

where n is 0.4 for heating ($T_w > T_b$) and 0.3 for cooling. Note that
conditions listed below Eq. (11.66) are the range of data used in

For large ΔT, another equation by Sieder and Tate [S4] is recommended:[1]

$$N_{\mathrm{Nu,mb}} = 0.027 (N_{\mathrm{Re,mb}})^{0.8} (N_{\mathrm{Pr,mb}})^{1/3} (\mu_{\mathrm{mb}}/\mu_w)^{0.14} \quad (11.67)$$

$$0.7 < N_{\mathrm{Pr,mb}} \le 160$$

$$N_{\mathrm{Re,mb}} \ge 10\,000$$

$$L/d_i \ge 60 \quad \text{(smooth tubes)}$$

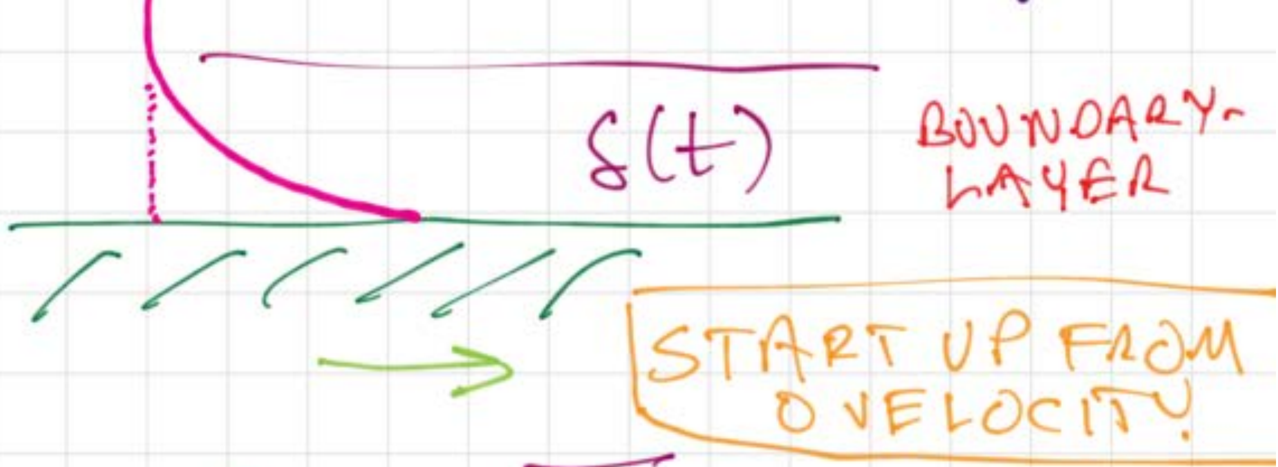
Friend–Metzner analogy. The Friend–Metzner analogy uses an equation of substantially different form in order to correlate data over wide ranges of N_{Pr} and N_{Sc} [F3]. Their correlation for heat transfer is

$$N_{\mathrm{Nu,mb}} = \frac{N_{\mathrm{Re,mb}} N_{\mathrm{Pr,mb}} (f/2) (\mu_{\mathrm{mb}}/\mu_w)^{0.14}}{1.20 + (11.8)(f/2)^{1/2} (N_{\mathrm{Pr,mb}} - 1)(N_{\mathrm{Pr,mb}})^{-1/3}} \quad (11.83)$$

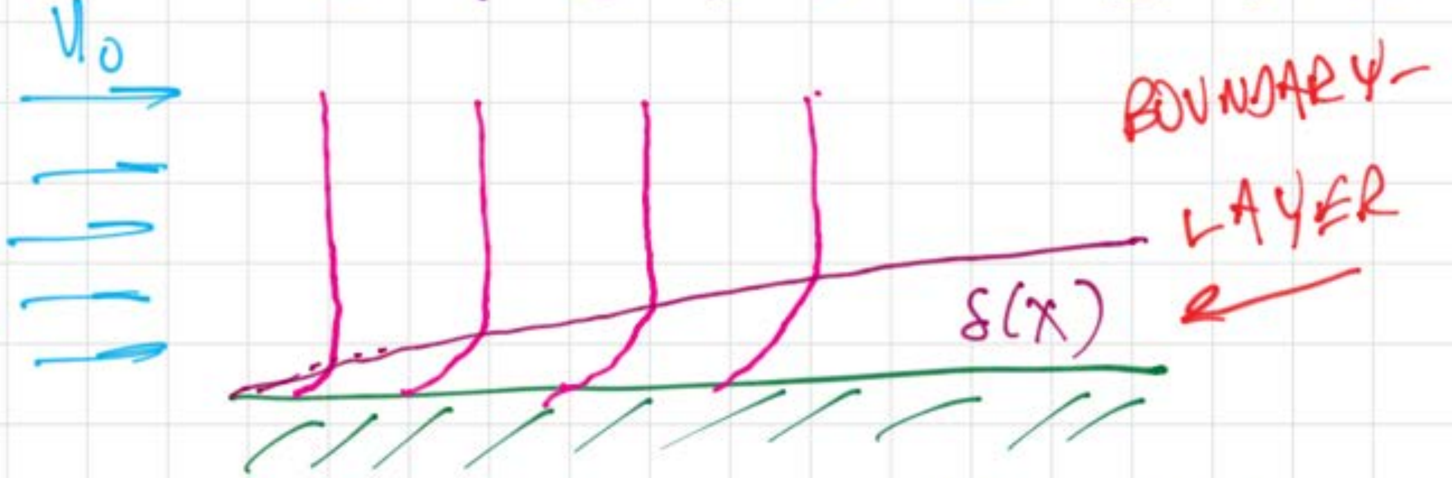
$$0.5 \le N_{\mathrm{Pr,mb}} \le 600 \qquad N_{\mathrm{Re,mb}} \ge 10\,000$$

SIMPLE PROBLEM TO CONSIDER:

$$\frac{\partial v_x}{\partial t} = \nu \frac{\partial^2 v_x}{\partial y^2}$$



$$\delta \sim \sqrt{\nu t}$$



$$\rho\left(v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right)$$

$$\rho\left(v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2}\right)$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

BOUNDARY LAYER ANALYSIS

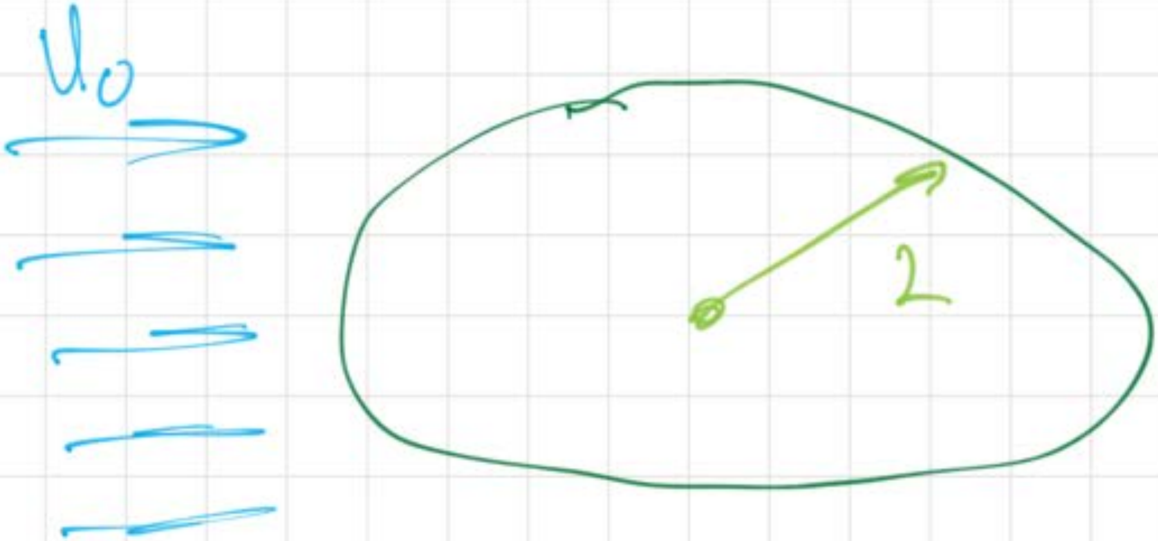
HIGH Re FLOW, $Re \gg 1$

WE ARE NOT ADDRESSING ISSUE OF TURBULENCE

FOR ERASER THROUGH AIR...

$$Re = \frac{(1\ kg/m^3)(2\ \frac{m}{s})(.5\ m)}{1.5\times10^{-5}\ kg/m\cdot s}$$

$$= 60000$$



NONDIMENSIONALIZE NAVIER STOKES EQUATIONS

$$v_x^* \equiv \frac{v_x}{U_0}, \quad v_y^* \equiv \frac{v_y}{U_0} \quad \ldots$$

$$x^* \equiv \frac{x}{L}, \quad y^* \equiv \frac{y}{L}$$

$$p^* \equiv \frac{p}{\rho U_0^2} \quad \ldots$$

$$\frac{\partial \bar{v}^*}{\partial t^*} + \bar{v}^* \cdot \bar{\nabla}^* \bar{v}^* = -\bar{\nabla}^* p^* + \frac{1}{Re} \nabla^{*2} \bar{v}^*$$

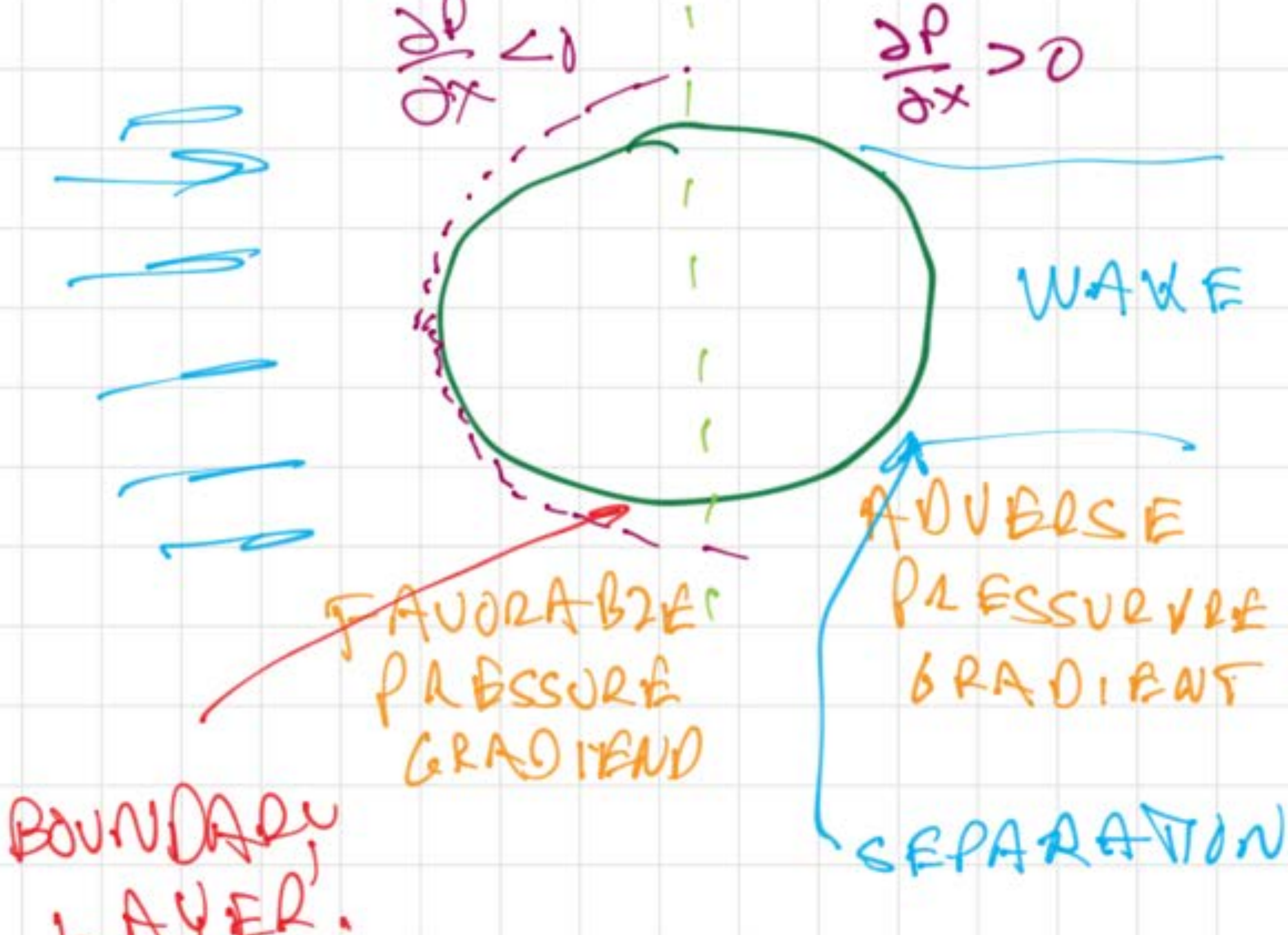
AS $Re \to \infty$

$$\frac{\partial \bar{v}^*}{\partial t^*} + \bar{v}^* \cdot \bar{\nabla}^* \bar{v}^* = -\bar{\nabla}^* p^*$$

IF $\quad \bar{v} = -\bar{\nabla}\phi$

THEN $\Delta\left(-\frac{\partial \phi}{\partial t} + \frac{v^2}{2} + \frac{p}{\rho} + gh\right) = 0$

"BERNOULLI" EQ.



MORE EQUATION SCALING

$$x^* = \frac{x}{L} \qquad v_x^* = \frac{v_x}{U_0}$$

$$y^* = \frac{y}{\delta} \qquad v_y^* = \frac{v_y}{V}$$

SUBSTITUTE INTO CONTINUITY

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$\frac{U_0}{L} \underbrace{\frac{\partial v_x^*}{\partial x^*}}_{O(1)} + \frac{V}{\delta} \underbrace{\frac{\partial v_y^*}{\partial y^*}}_{O(1)} = 0$$

$$V \sim \delta \left(\frac{U_0}{L}\right) = U\left(\frac{\delta}{L}\right)$$

GO TO NAVIER-STOKE EQ'.

WITH δ, V AS ABOVE

$$\rho \left(U_0 v_x^* \frac{U_0}{L} \frac{\partial v_x^*}{\partial x^*} + \frac{V v_y^*}{\delta} \frac{U_0 \partial v_x^*}{\partial y^*} \right) =$$

$$- \frac{\rho U_0^2}{L} \frac{\partial p^*}{\partial x^*} + \mu \left(\frac{U_0}{L^2} \frac{\partial^2 v_x^*}{\partial x^{*2}} + \frac{U_0}{\delta^2} \frac{\partial^2 v_x^*}{\partial y^{*2}} \right)$$

$$\ll$$

$$\frac{\rho \delta^2 U_0}{\mu L} \left(v_x^* \frac{\partial v_x^*}{\partial x^*} + v_y^* \frac{\partial v_x^*}{\partial y^*} \right) = - \frac{\partial p^*}{\partial x^*} + \frac{\partial^2 v_x^*}{\partial y^{*2}}$$

$$\frac{\rho \delta^2 U_0}{\mu L} = O(1)$$

$$\delta^2 = \frac{\mu L}{\rho U_0}$$

$$Re = \frac{L U_0 \rho}{\mu}$$

KEY RESULT!!

$$\boxed{\frac{\delta}{L} = \frac{1}{\sqrt{Re}}}$$

BOUNDARY - LAYER THICKNESS

CAN SHOW BY
SAME ARGUMENTS...

V-EQ $\frac{\partial P^*}{\partial y^*} \approx 0$

BOUNDARY-LAYER EQUATIONS

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 v_x}{\partial y^2}$$

$$\frac{\partial P}{\partial y} = 0$$

CAN MAKE SOME PROGRESS TO
A SOLUTION!!