CBE 30357

11/2/17

1) INTEGRAL FORMS OF MASS AND MOMENTUM EQUATIONS

2) USING THESE TO SOLVE FLOW PROBLEMS

- "FORCE"

MASS BALANCE

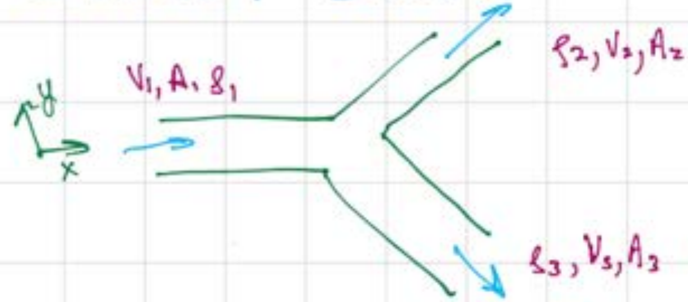
DIFFERENTIAL: $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{v} = 0$

INTEGRATE OVER CONTROL VOLUME: $\int_V \frac{\partial \rho}{\partial t} dV = -\int_V \vec{\nabla} \cdot \rho \vec{v} \, dV$

SOME MATH: $\frac{dm}{dt} = -\int_S \rho \, \vec{v} \cdot \vec{n} \, dS$

USEFUL FORM $\frac{dm}{dt} = \rho \sum_{\substack{i \\ IN}} \langle v \rangle_i A_i - \rho \sum_{\substack{j \\ OUT}} \langle v \rangle_j A_j$

STEADY-STATE



$0 = \rho_1 V_1 A_1 - \rho_2 V_2 A_2 - \rho_3 V_3 A_3$

MOMENTUM BALANCE

DIFFERENTIAL MOMENTUM EQ

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nabla\cdot\boldsymbol{\tau} + \rho\mathbf{g}.$$

INTEGRATED OVER CONTROL VOLUME:

$$\int_v \rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right)dV = \int_V (-\nabla p + \nabla\cdot\boldsymbol{\tau} + \rho\mathbf{g})dV.$$

WE DO SOME MATH...

$$\frac{\partial \int_V \rho\mathbf{v}dV}{\partial t} + \int_S \mathbf{v}\rho(\mathbf{n}\cdot\mathbf{v})dS = -\int_S p\mathbf{n}dS + \int_S \mathbf{n}\cdot\boldsymbol{\tau}dS + m\mathbf{g}.$$

SLIGHTLY MORE USEFUL TO BREAK OUT "$\bar{F}$"

$$\frac{d}{dt}\left(\int \rho\bar{v}\,dV\right) + \int_S \bar{v}\rho(\bar{n}\cdot\bar{v})dS =$$

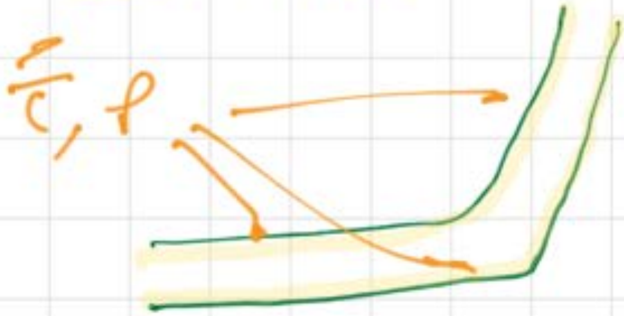
$$-\int_{\substack{\text{OPEN}\\\text{SURFACE}}} p\,\bar{n}\,dS + \int_{\substack{\text{OPEN}\\\text{SURFACE}}} (\bar{n}\cdot\bar{\tau})dS + m\bar{g} + \bar{F}$$

↑ ONLY RARELY WOULD THIS BE NEEDED

$$\bar{F} \equiv -\int_{\substack{\text{SOLID}\\\text{SURFACE}}} p\,\bar{n}\,dS + \int_{\substack{\text{SOLID}\\\text{SURFACE}}} \bar{n}\cdot\bar{\tau}\,dS$$

HOW TO INTERPRET P + $\bar{\tau}$ INTEGRALS

THEY REPRESENT FORCES ON SURFACE OF CONTROL VOLUME?



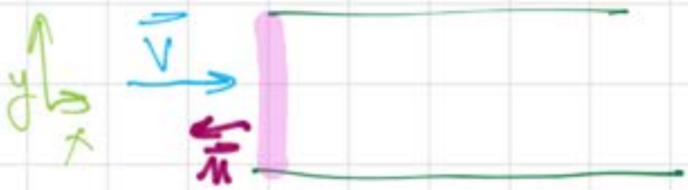
$$\int_{\text{SOLID SURFACES}} (\bar{n} \cdot \bar{\tau}) ds$$

$$- \int_{\text{SOLID SUFACE}} P \bar{n} ds$$

THIS DEFINES $\bar{F}$

$$\bar{F} \equiv - \int_{\substack{\text{SOLID}\\ \text{SURFACE}}} p \bar{n} ds + \int_{\substack{\text{SOLID}\\ \text{SURFACE}}} \bar{n} \cdot \bar{\tau} ds$$

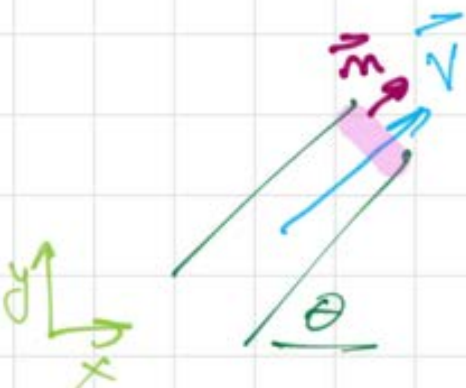
$\bar{F}$ Is the total vector action of the pressure and shear on the solid surfaces of our pipe, device or control volume. We usually get this from the other terms. (Or we could do an experiment to measure it and use this to get some other term in the equation.)



$$\int \vec{V} \rho (\vec{V} \cdot \vec{n}) dS = -\int \rho \vec{V} \; V_x \, dS$$

$\vec{V}$ IS ALSO ONLY V_x ∴

$$-\int \rho \, V_x V_x \, dS = -\rho \langle V_x V_x \rangle_1 A_1$$



$$X \rightarrow \int \rho \vec{V} \; \vec{V} \cdot \vec{n} \, dS$$

$$\rho V_x \; |V| \; A$$

$$\rho V \cos\theta \; V \; A$$

$$X \Rightarrow \rho V^2 A \cos\theta$$

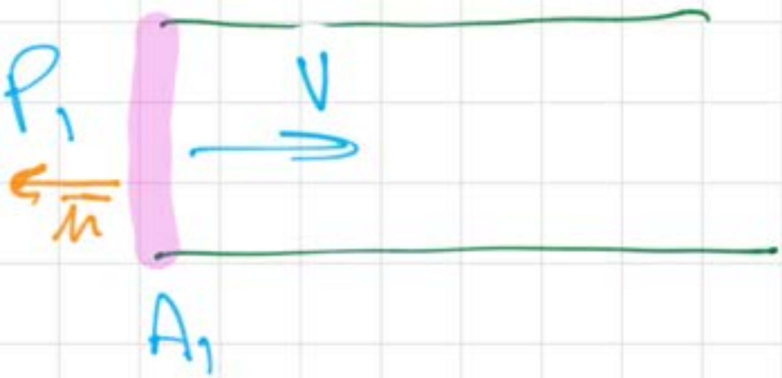
$$y \rightarrow \int \rho \vec{V} \; \vec{V} \cdot \vec{n} \, dS$$

$$\rho V_y \; |V| \; A$$

$$\rho V \sin\theta \; V \; A$$

$$y \Rightarrow \rho V^2 A \sin\theta$$

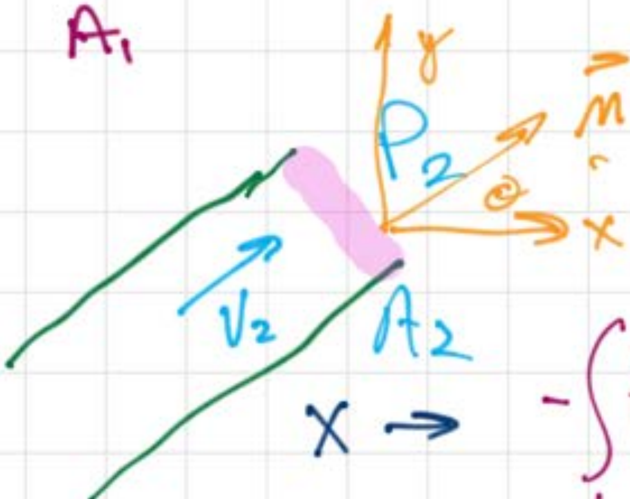
PRESSURE TERMS



FROM $\bar{n}$ — NO -

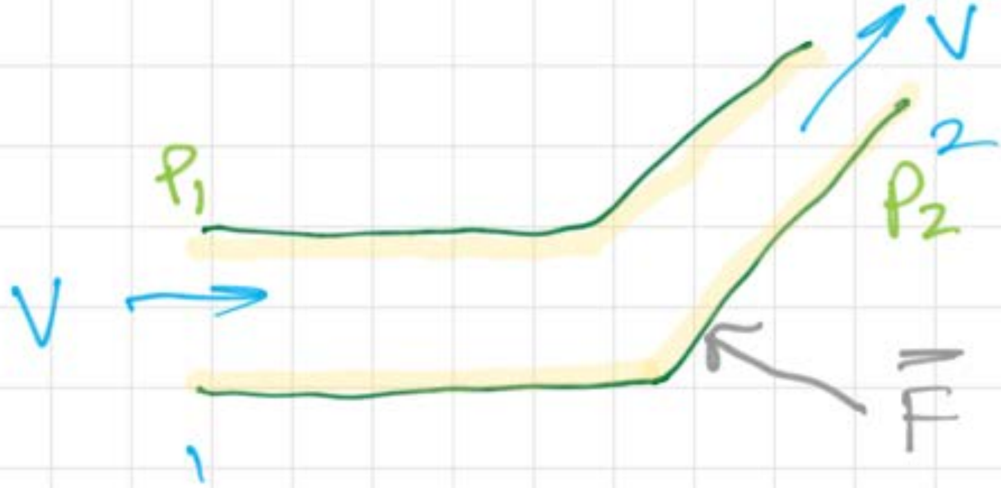
$$x - \quad -\int_{A_1} p\,\bar{n}\,dS = P_1 A_1$$

($\vec{n}$ POINTS IN - DIRECTION)



$$x \rightarrow \quad -\int_{A} P\,\bar{n}\,ds = -P_2 \cos\theta\, A_2$$

$$y \Rightarrow \quad -\int_{A} P\,\bar{n}\,ds = -P_2 \sin\theta\, A_2$$



$$\int (\dot{\bar{m}} \cdot \bar{c})\,ds = F_x \hat{i} + F_y \hat{j}$$

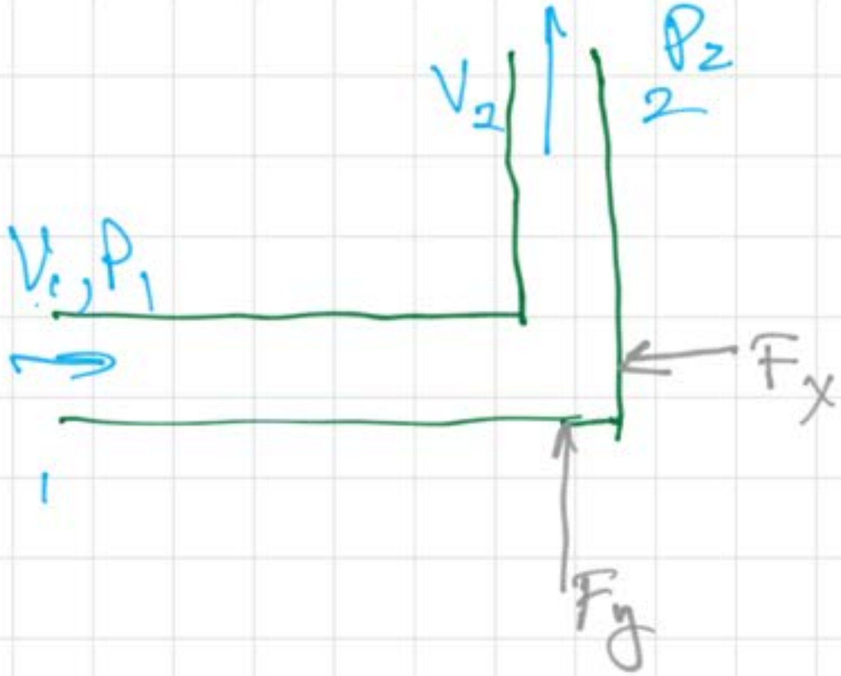
COMBINATION OF PRESSURE AND SHEAR STRESS
USUALLY CAN'T EVALUATE INTEGRALS EXACTLY

SO: COMPLETE BALANCE

x ~ $-\rho V_1^2 A_1 + \rho V_2^2 \cos\theta A_2 = P_1 A_1 - P_2 A_2 \cos\theta + F_x$

y ~ $0 + \rho V_2^2 \sin\theta A_2 = -P_2 A_2 \sin\theta + F_y$

A USUAL QUESTION IS "FIND $\bar{F}$"



90°
BEND

$$x \quad -\rho V_1^2 A_1 = P_1 A_1 + F_x$$

$$F_x = -P_1 A_1 - \rho V_1^2 A_1$$

FORCE MATCHES PRESSURE AND DEFLECTS ALL OF THE FLOW

$$y \quad +\rho V_2^2 A_2 = -P_2 A_2 + F_y$$

$$F_y = P_2 A_2 + \rho V_2^2 A_2$$

SIMPLIFIED FORM

Momentum equations for single flow inlet in + x direction.

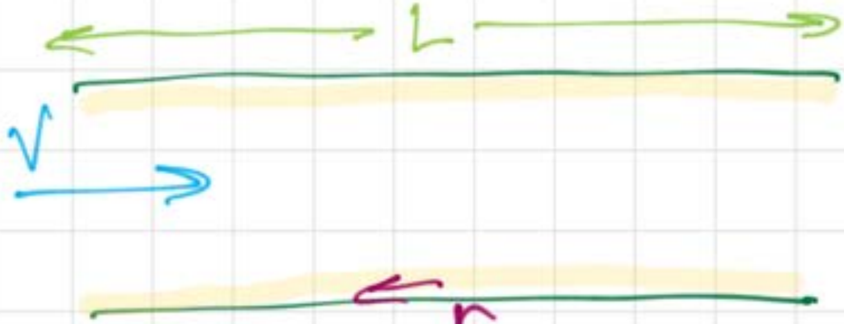
$$-\rho\langle V_x V_x\rangle A_1 + \rho\langle V_2 V_2\rangle A_2\cos(\theta) = P_1 A_1 - P_2 A_2\cos(\theta) + F_x$$
$$+\rho\langle V_2 V_2\rangle A_2\sin(\theta) = -P_2 A_2\sin(\theta) + F_y$$

PIPE FLOW: FORCE ON PIPE = ?

$$\underbrace{\cancel{-\rho V_1 V_1 A_1 + \rho V_1 V_1 A_1}}_{0} = P_1 A_1 - P_2 A_2 + F_x$$

$$F_x = -(P_1 - P_2)A$$ FORCE ON PIPE TO HOLD IN PLACE

ALSO FORCE ON FLUID



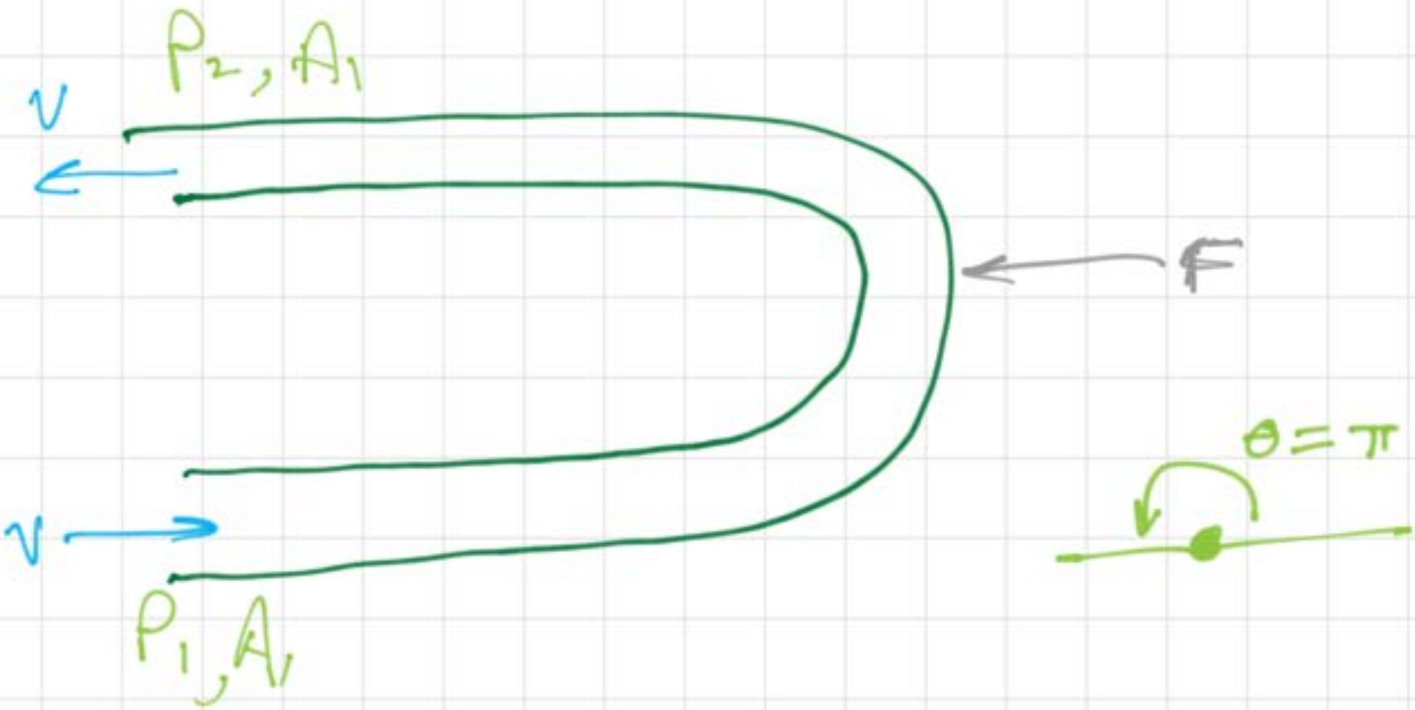
WE ALREADY KNOW:

$$\tau_w 2\pi R L = \int \vec{n}\cdot\vec{\tau}\, ds$$

$$0 = P_1 A_1 - P_2 A_1 + F_x$$
$$= (P_1 - P_2) A + \tau_w \pi R L$$

$$\boxed{\tau_w = -\frac{\Delta P}{L}\frac{R}{2}}$$

DEPENDS ON ΔP, NOT ANY SPECIFIC DETAIL OF FLOW



Momentum equations for single flow inlet in + x direction.

$$-\rho \langle V_x V_x \rangle A_1 + \rho \langle V_2 V_2 \rangle A_2 \cos(\theta) = P_1 A_1 - P_2 A_2 \cos(\theta) + F_x$$
$$+\rho \langle V_2 V_2 \rangle A_2 \sin(\theta) = -P_2 A_2 \sin(\theta) + F_y$$

$$-\rho V_1 V_1 A_1 + \rho V_1 V_1 A_1 \underbrace{\cos\pi}_{(-1)} = P_1 A_1 - P_2 A_1 \underbrace{\cos\pi}_{(-1)} + F_x$$

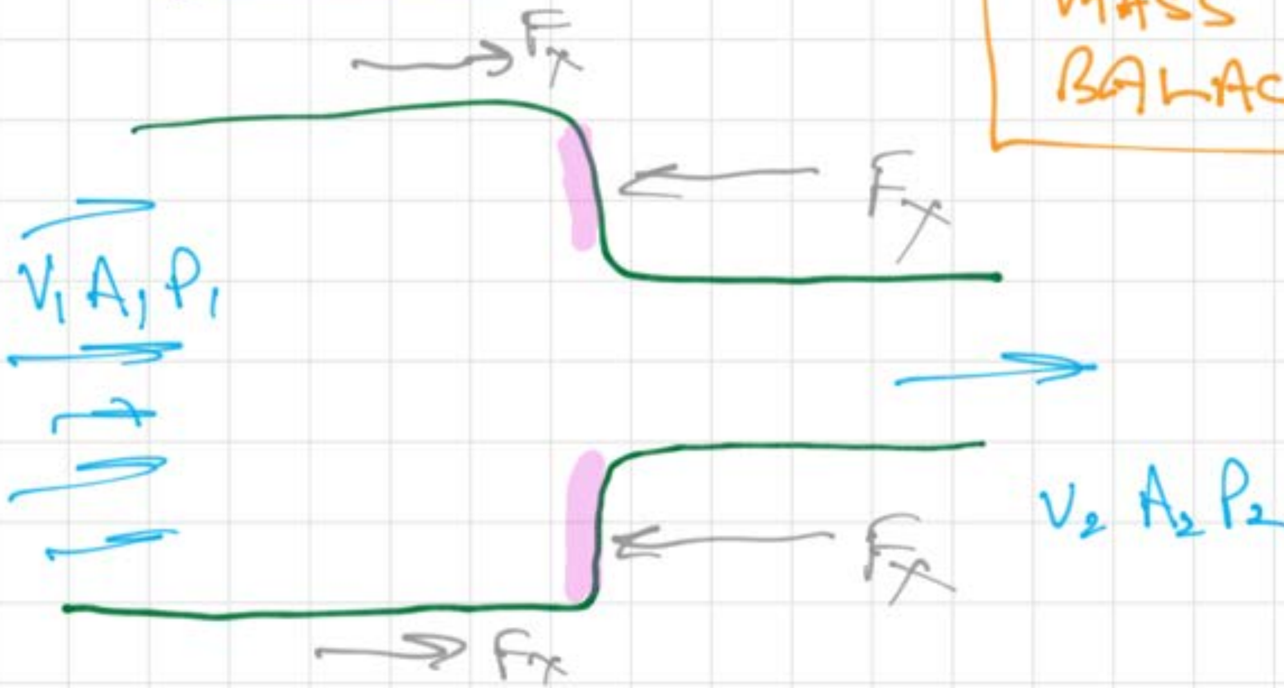
$$F_x = -2\rho V_1^2 A_1 - A_1 (P_1 + P_2)$$

BALANCES 2X MOMENTUM

BALANCES ALL PRESSURE FORCE

AREA CHANGE

WILL NEED MASS BALACE!



$$0 = \rho_1 A_1 V_1 - \rho_2 A_2 V_2 \qquad \rho_1 = \rho_2$$

$$V_2 = V_1 \frac{A_1}{A_2}$$

SPEEDS UP.

$$-\rho V_1 V_1 A_1 + \rho V_2 V_2 A_2 = P_1 A_1 - P_2 A_2 + F_x$$

$$F_x = -\rho V_1^2 A_1 + \rho V_2^2 A_2 - P_1 A_1 + P_2 A_2$$

$$= -\rho V_1^2 A_1 \left(1 - \frac{A_1}{A_2}\right) - P_1 A_1 + P_2 A_2$$

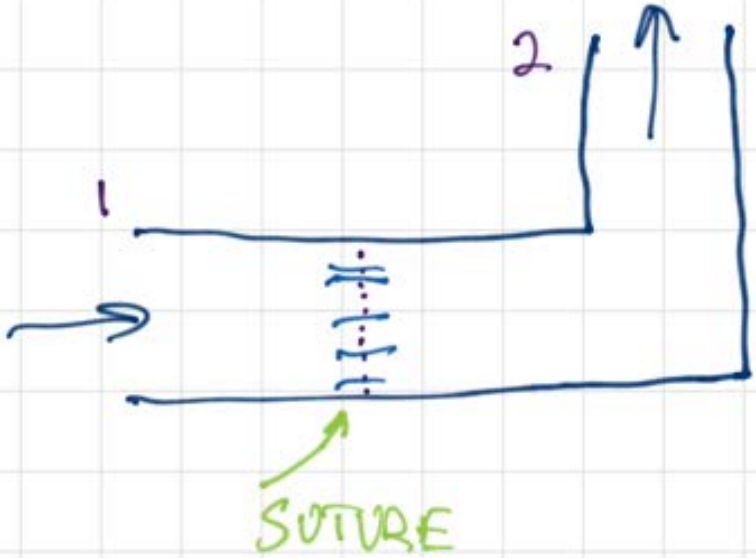
$$F_x = -\rho\, V_1^2 A_1 \Big(\underbrace{1 - \frac{A_1}{A_2}}_{<0}\Big) - P_1 A_1 + P_2 A_2$$

NET BALANCE ON PRESSURE

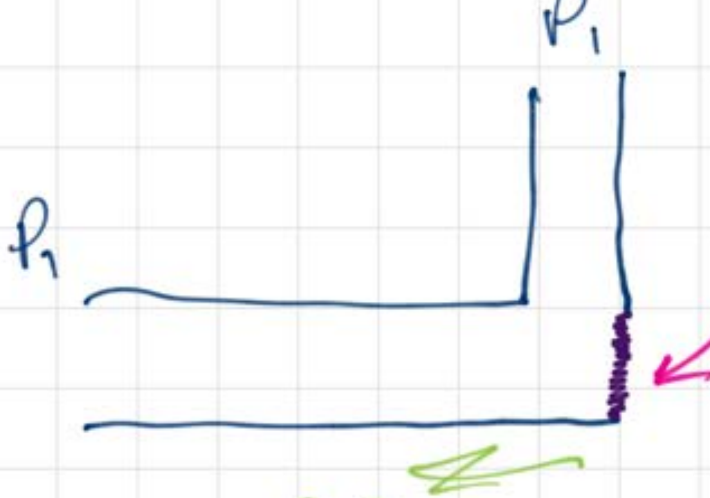
FOR HIGH R_e FLOWS,

NORMAL FORCES ARE MUCH LARGER THAN SHEAR FORCES

SO, JUST NEED TO CONSIDER PRESSURE



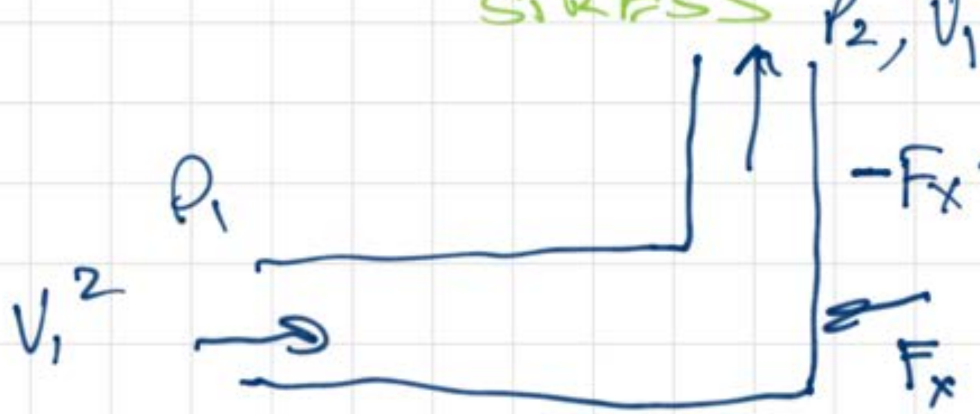
HOW MUCH STRESS?



EVEN IF NO FLOW

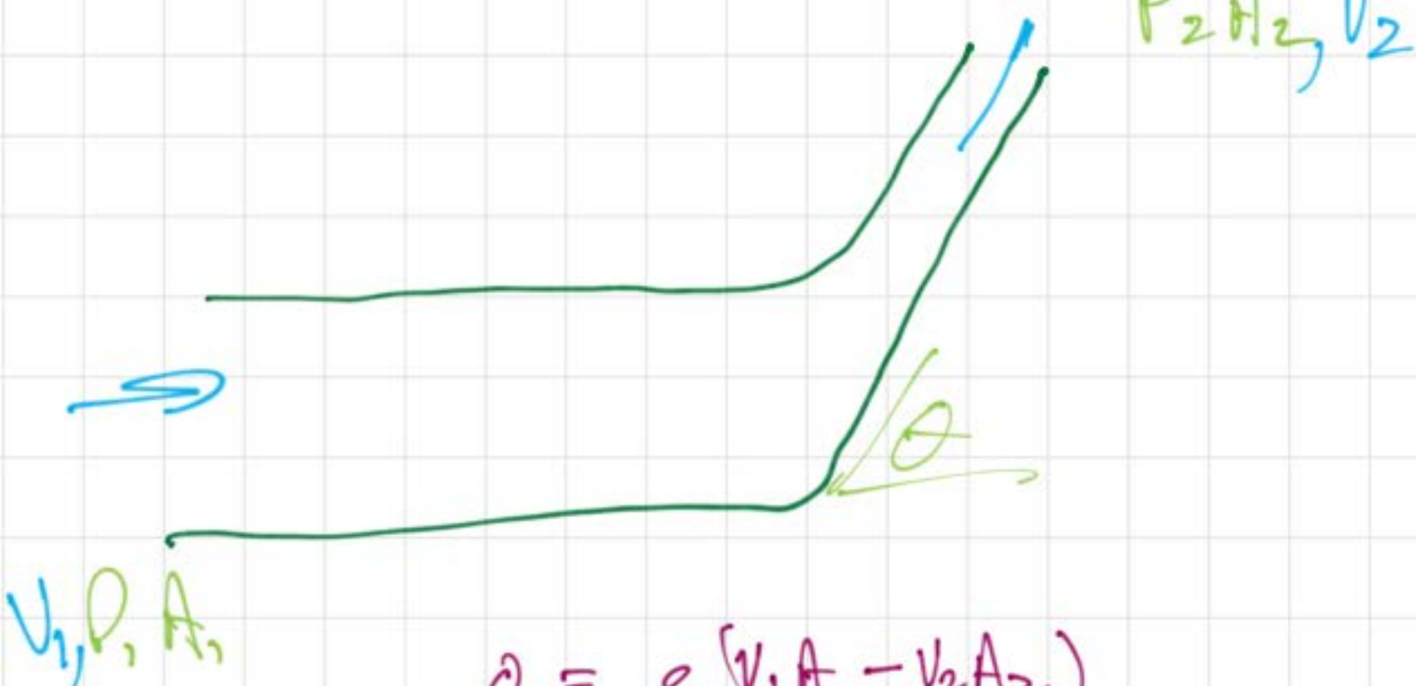
STILL HAVE TO MATCH PRESSURE WITH A FORCE...

PIPE IS UNDER TENSILE STRESS



$$-F_x = P_1 A_1 + \rho V_1^2 A_1$$

I MIGHT BE ABLE TO GET P_2 FROM A CORRELATION FOR LOSSES (...TO COME...)



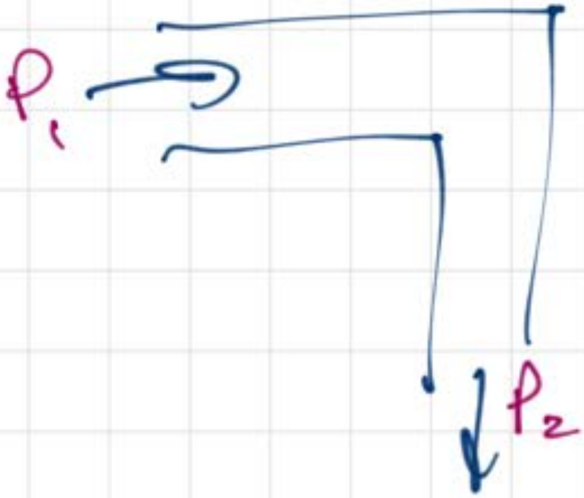
$$0 = \rho (V_1 A - V_2 A_2)$$

$$V_2 = V_1 \left(\frac{A_1}{A_2} \right)$$

$$x \quad -\rho V_1 V_1 A_1 + \rho V_2 \cos\theta \, V_2 A_2 = P_1 A_1 - P_2 A_2 \cos\theta + F_x$$

$$y \quad 0 \quad + \rho V_2 \sin\theta \, V_2 A_2 = 0 - P_2 A_2 \sin\theta + F_y$$

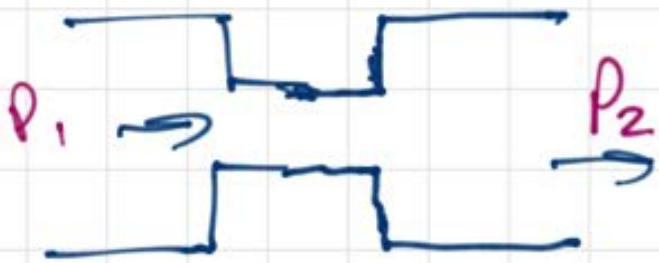
SOME OTHER INFO



WE EXPECT THAT EVEN IF THERE IS NO CHANGE OF AREA,

$P_1 > P_2$

FOR THESE FLOWS



EMPIRICAL FORMALISM

$$\Delta P = \tfrac{1}{2} \rho K V^2$$

K DEPENDS ON GEOMETRY

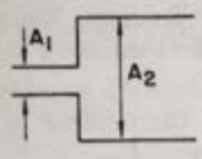
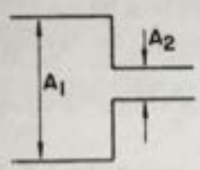
SOME EXAMPLES....

Fitting Type		K
Pipe Entry Losses		
Square Inlet		0.50
Re-entrant Inlet		0.80
Slightly Rounded Inlet		0.25
Bellmouth Inlet		0.05
Pipe Intermediate Losses		
Elbows R/D < 0.6	45°	0.35
	90°	1.10
Long Radius Bends (R/D > 2)	$11\frac{1}{4}$°	0.05
	$22\frac{1}{2}$°	0.10
	45°	0.20
	90°	0.50
Tees		
(a) Flow in line		0.35
(b) Line to branch flow		1.00
Sudden Enlargements		
Ratio	d/D	
	0.9	0.04
	0.8	0.13
	0.7	0.26
	0.6	0.41
	0.5	0.56
	0.4	0.71
	0.3	0.83
	0.2	0.92
	<0.2	1.00

Fitting Type		K
Gradual Enlargements		
Ratio d/D	q = 10° typical	
	0.9	0.02
	0.7	0.13
	0.5	0.29
	0.3	0.42
Gradual Contractions		
Ratio d/D	q = 10° typical	
	0.9	0.03
	0.7	0.08
	0.5	0.12
	0.3	0.14
Valves		
Gate Valve (fully open)		0.20
Reflux Valve		2.50
Globe Valve		10.00
Butterfly Valve (fully open)		0.20
Angle Valve		5.00
Foot Valve with strainer		15.00
Air Valves		zero

TABLE 5-1

LOSSES IN FITTINGS AND VALVES FOR TURBULENT FLOW[a]

Fitting or valve		*Velocity heads lost, K_f*
90° elbow, standard		0.75
90° elbow, square		1.3
Coupling		0.04
Gate valve		
Open		0.17
Half-open		4.5
Globe valve, bevel seat		
Open		6.4
Half-open		9.5
Sudden expansion		$\left(\frac{A_2}{A_1} - 1\right)^2$
Sudden contraction		$\left(\frac{2}{m} - \frac{A_2}{A_1} - 1\right)^2$ m is the root of the quadratic $\frac{1 - m(A_2/A_1)}{1 - (A_2/A_1)^2} = \left(\frac{m}{1.2}\right)^2$
Rounded entrance		0.05

[a]The result for the sudden expansion is derived in Sec. 6.2. The result for the sudden contraction is from Martin, *Chem. Eng. Educ.*, Summer 1974, p. 138. Other values are from *Perry's Handbook*.

Example 5.7

A liquid is pumped through a 50-mm-diameter smooth pipe between two tanks at a rate of 3 kg/s in the section of the process stream shown in Fig. 5-5. The liquid has properties $\rho = 10^3$ kg/m³, $\eta = 10^{-3}$ Pa·s. The pressure above the

$$\Delta P = \frac{1}{2}\rho K V^2$$

DOWNSTREAM VELOCITY