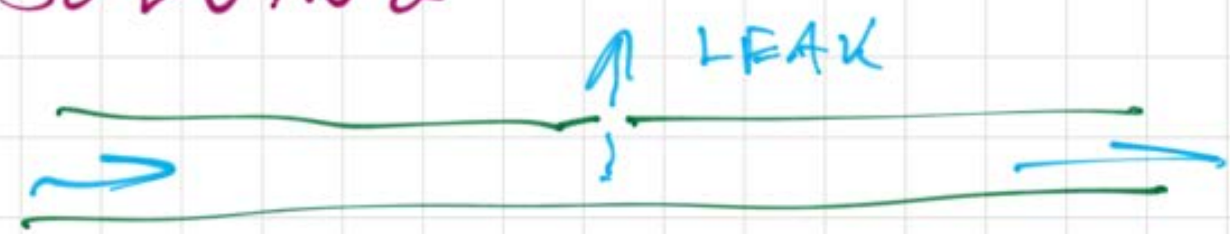
CBE 30357

11/15/17

AS MUCH AS WE ENJOY USING THEM, AND FOR AS MANY PROBLEMS THAT CAN BE SOLVED...

THERE ARE SOME PROBLEMS THAT WE HAVE TROUBLE SOLVING



HOW BIG IS FLOW RATE FROM LEAK?

REST OF SEMESTER

NEED TO RESCHEDULE

12	13	14	15	16 BERNOULLI EQ	17	18
19	20	21 BLOOD FLOW CARDIOLOGY FLUID MECH	22	23 Thanksgiving Day	24	25
26	27	28 BOUNDARY LAYER FLOWS	29	30 BOUNDARY LAYER FLOWS	Dec 1	2
3	4	5 LUBRICATION FLOWS	6	7 LUBRICATION MORE GENERAL ASPECTS OF TRANSPORT PHENOMENA	8 REVIEW LECT?	9
10	11	12	13 FINAL-Y 10:30 155 DEBLO	14	15	16

CBE CURRICULUM

Junior	MATH 30650, Differential Eq	3	CHEM 30324, Pchem	3
	CHEM 30333/31333 Achem & Lab	4	CBE 30338, Chem Proc Control	3
	CBE 30355 Transport 1		CBE 30356, Transport 2	3
	or CBE 30357 Biotransport	3	CBE 31358, Chem Eng Lab 1	3
	CBE 30367, Thermo 2	3	A&L 6	3
	CBE 30361, Materials	3		15
		16		
Senior	CBE 40443, Separations	3	CBE 40448, Process Design	3
	CBE 40445, Reaction Engineering	3	CBE Elective	3
	CBE 41459, Chem Eng Lab 2	3	Tech Elective	3
	CBE Elective	3	Advance Science Elective	3
	A&L 7	3	A&L8	3
		15		15
Total		128 credits	*Strongly recommended	

SO FAR:

- EXACT SOLUTIONS TO DIFFERENTIAL EQUATIONS OF FLUID MECHANICS
- SCALING, DIMENSIONAL ANALYSIS LIMITING PARAMETER RANGES
- MACROSCOPIC MOMENTUM EQ USED TO GET FORCES AND UNDERSTAND COMPLEX FLOWS

FOR SOME COMPLETENESS OF INTELLECTUAL PATH

$$\begin{cases} R_e\left(\frac{\partial \vec{V}^*}{\partial t^*}+\vec{V}^*\cdot\vec{\nabla}^*\vec{V}^*\right)=-\vec{\nabla}^* p^* + \nabla^{*2}\vec{V}^* \\ \vec{\nabla}^*\cdot\vec{V}^*=0 \qquad p^*=\frac{p}{(U_0 \mu/L)} \end{cases}$$

LIMIT AS $R_e \to 0$ VISCOUS PRESSURE

$$\vec{\nabla}^* p^* = \nabla^{*2}\vec{V} \qquad \vec{\nabla}\cdot\vec{V}=0$$

THE NEXT OBVIOUS LIMIT IS THAT $R_e \to \infty$!!

- MUST KEEP PRESSURE OR NO RESULT MAKES SENSE ...

$$p^* = \frac{p'}{\rho U_0^2} \qquad p' \equiv p - \rho g h$$

$$\left(\frac{\partial V^*}{\partial t^*}+\vec{V}^*\cdot\vec{\nabla}^*\vec{V}^*+\vec{\nabla}^* p^*\right)=\frac{1}{R_e}\nabla^{*2}\vec{V}^*$$

$$R_e \to \infty \qquad \frac{\partial}{\partial t}=0$$

$$\rho(\vec{V}\cdot\vec{\nabla}\vec{V})=-\vec{\nabla} p$$

$$\rho \vec{V}\cdot\vec{\nabla}\vec{V} = -\vec{\nabla} p$$

CHAIN RULE

$$\rho \vec{\nabla}\left(\frac{V^2}{2}\right) = -\vec{\nabla} p$$

INTEGRATE ALONG PATH

$$\rho\frac{V^2}{2}\Big|_2 - \rho\frac{V^2}{2}\Big|_1 = p_1 - p_2$$

$$\boxed{V\uparrow\ p\downarrow, \text{ or } V\downarrow\ p\uparrow}$$

IF FLUID SPEEDS UP, PRESSURE DROPS ... (AND VICE-VERSA)

IMPORTANT PHYSICAL OBSERVATION...

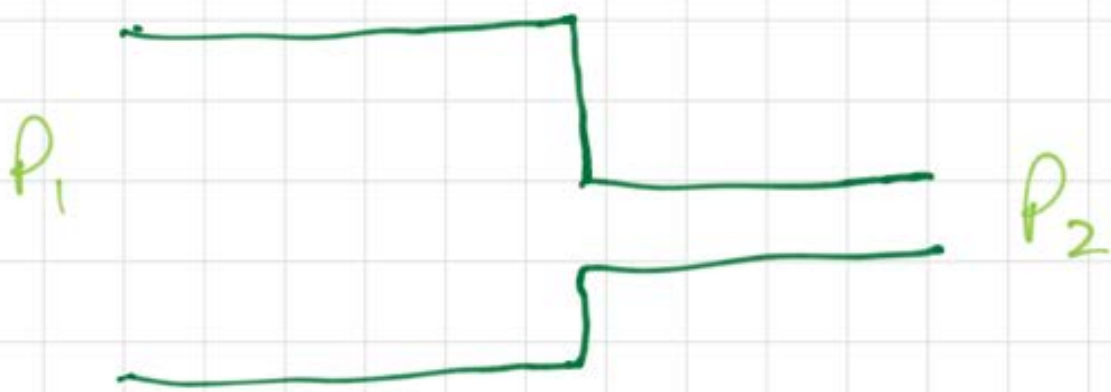
... ALMOST USELESS FOR A PRACTICAL PROBLEM

- HIGH Re LIMIT IS NOT THE SAME AS LOW Re
  - CAN'T COMPLETELY IGNORE VISCOSITY
- USUAL FLOWS ARE NOT JUST "STRAIGHT," CLEAN STREAMLINES

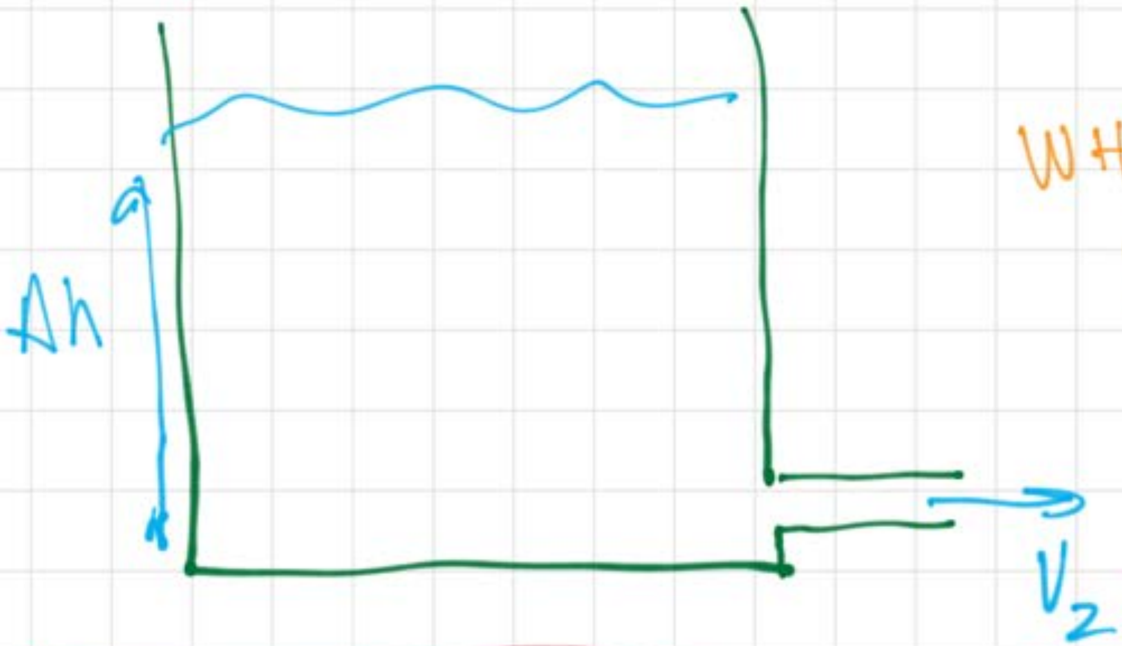
TEASE ... YES BUT HAVE YOU FORGOTTEN ABOUT — - -

I WOULD HAVE A REALLY HARD TIME WITH THIS!

WHAT IS p_2?



HOW DO WE QUANTIFY DIFFERENCE?

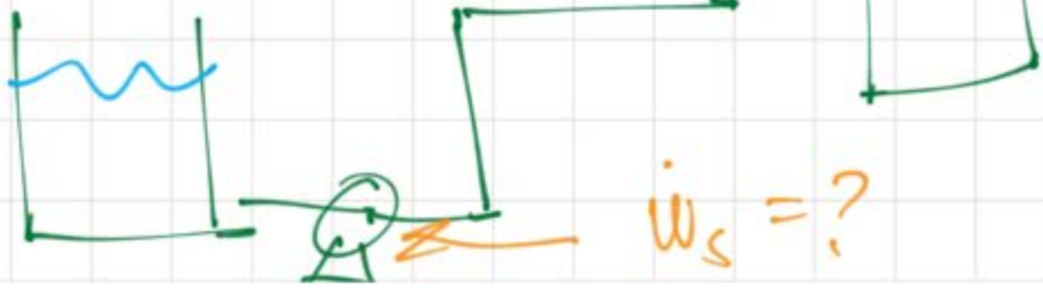


WHAT IS V_2?

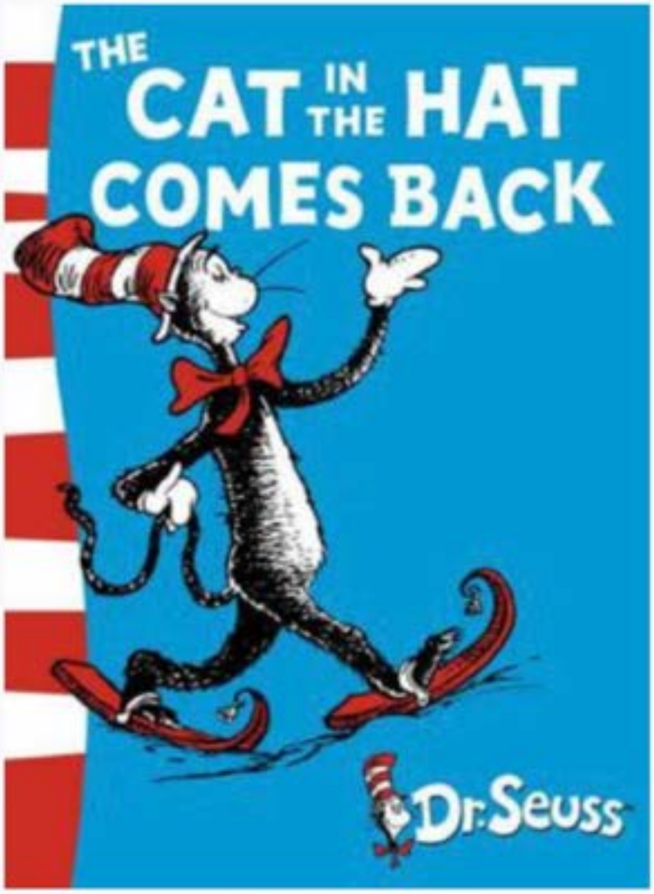
HOW WOULD WE GET THIS IN PHYSICS?

$$\Delta \tfrac{1}{2} m v^2 \stackrel{?}{=} \Delta m g h$$

$$\frac{dv}{dt} = g \quad ?$$

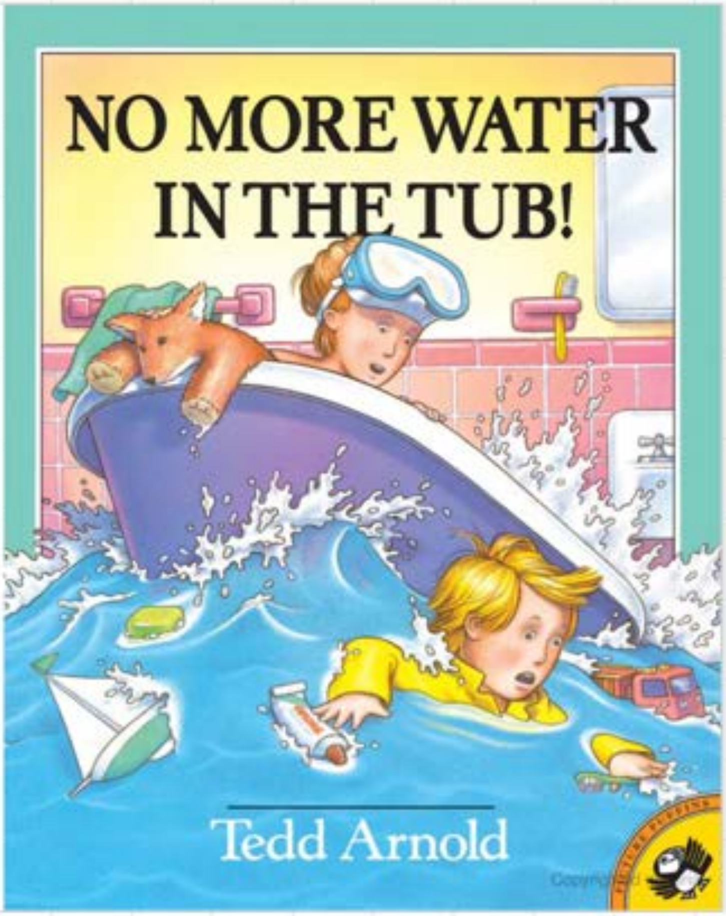


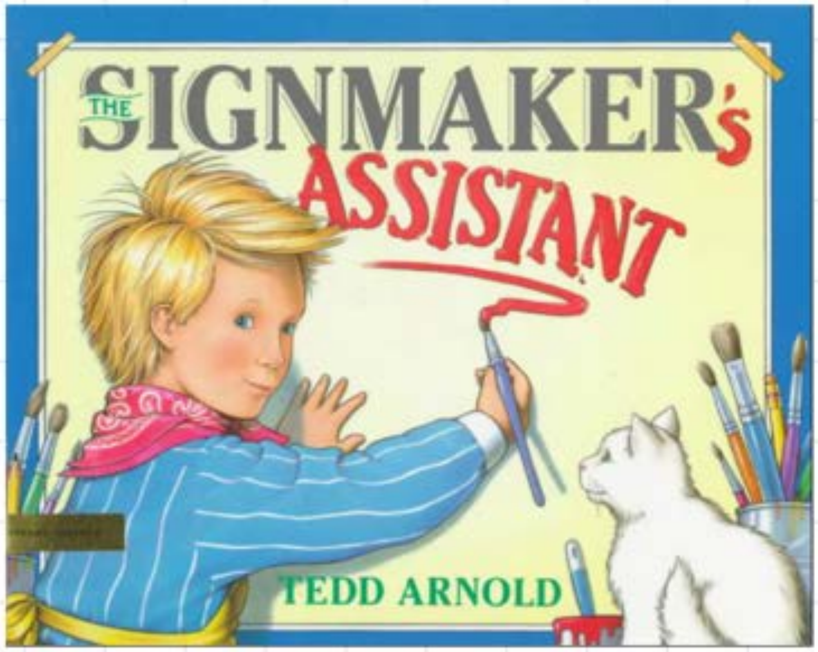
$\dot{W}_s = ?$

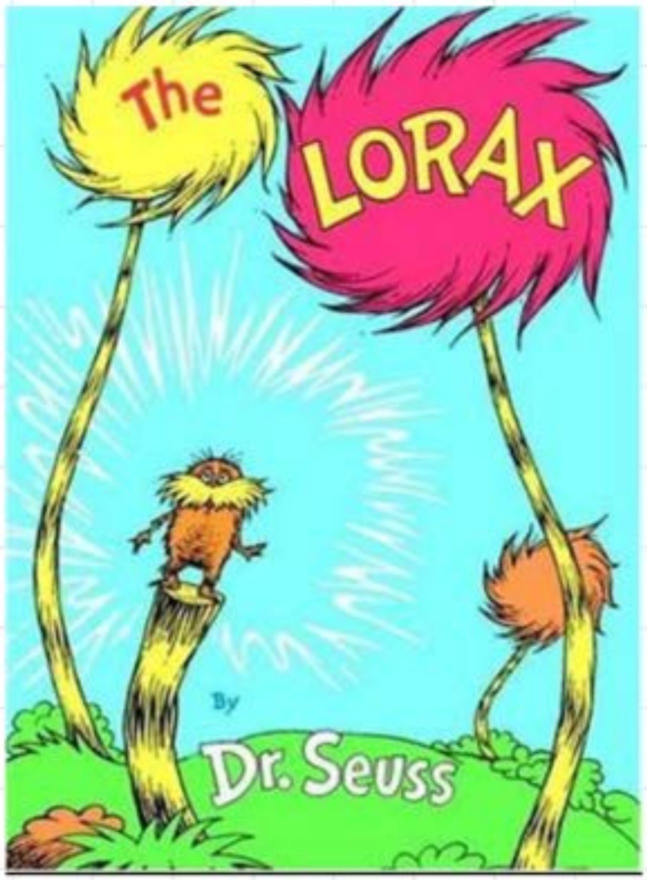


Author Dr. Seuss

WE NEED
CAT "Z" !!


NO MORE WATER
IN THE TUB!
Tedd Arnold


THE SIGNMAKER's
ASSISTANT
TEDD ARNOLD


The
LORAX
By
Dr. Seuss

COULD G. I. TAYLOR HAVE BEEN WRONG?

Motion of Axisymmetric Bodies in Viscous Fluids

G. I. Taylor,
Cambridge University

the y-axis will satisfy the conditions round the body moving transversely provided

$$U_P = \tfrac{1}{2} U_L.^1 \quad (3.8)$$

The resistance to slow motion of a long axisymmetric body moving in a viscous fluid is twice as great when the direction of motion is perpendicular to the axis as when moving along it.
This is known when the body is a prolate spheroid, but is true more generally.

Creeping Flows – Three-Dimensional Problems

For the limiting case of a sphere, $e \to 0$, and

$$C_{F1} = C_{F2} = 1,$$

as expected. On the other hand, for very slender spheroids, with $b/a = (1-e^2)^{1/2} \ll 1$, the force coefficients take the limiting form

$$\boxed{C_{F1} \approx \frac{2}{3}\frac{1}{\log(2a/b) - 1/2}, \quad C_{F2} \approx \frac{4}{3}\frac{1}{\log(2a/b) + 1/2},} \quad (8\text{–}179)$$

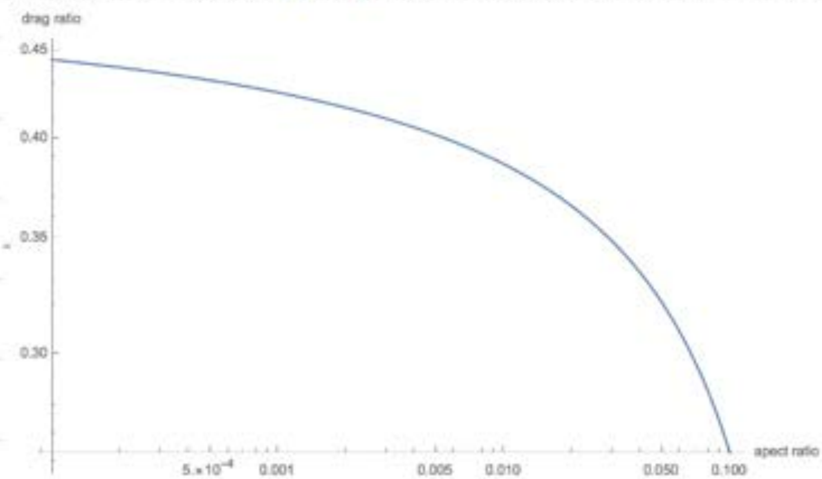
so that

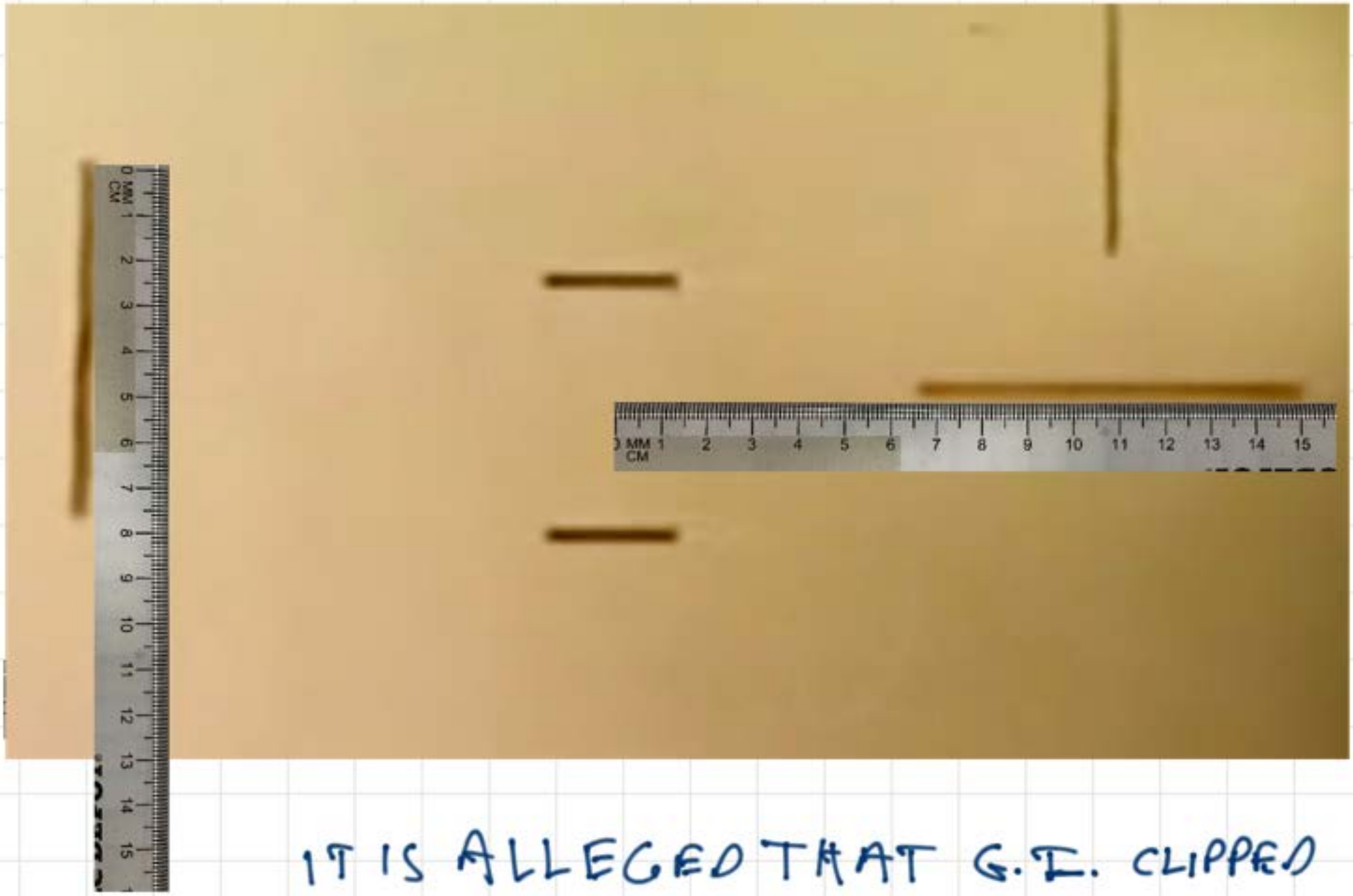
$$\frac{C_{F1}}{C_{F2}} \to \frac{1}{2} \text{ as } \frac{b}{a} \to 0. \quad (8\text{–}180)$$

This latter result also can be obtained by means of the approximate *slender-body* procedure outlined in the next section. The fact that the ratio of the force coefficients C_{F1}/C_{F2} ...

A BETTER ANSWER, WHICH WAS KNOW LONG BEFORE G.I.'S MOVIE ...

```
LogLogPlot[ (cf1/cf2) /. b → 1, {a, .0001, .1}, AxesLabel → {"apect ratio", "drag ratio"}]
```

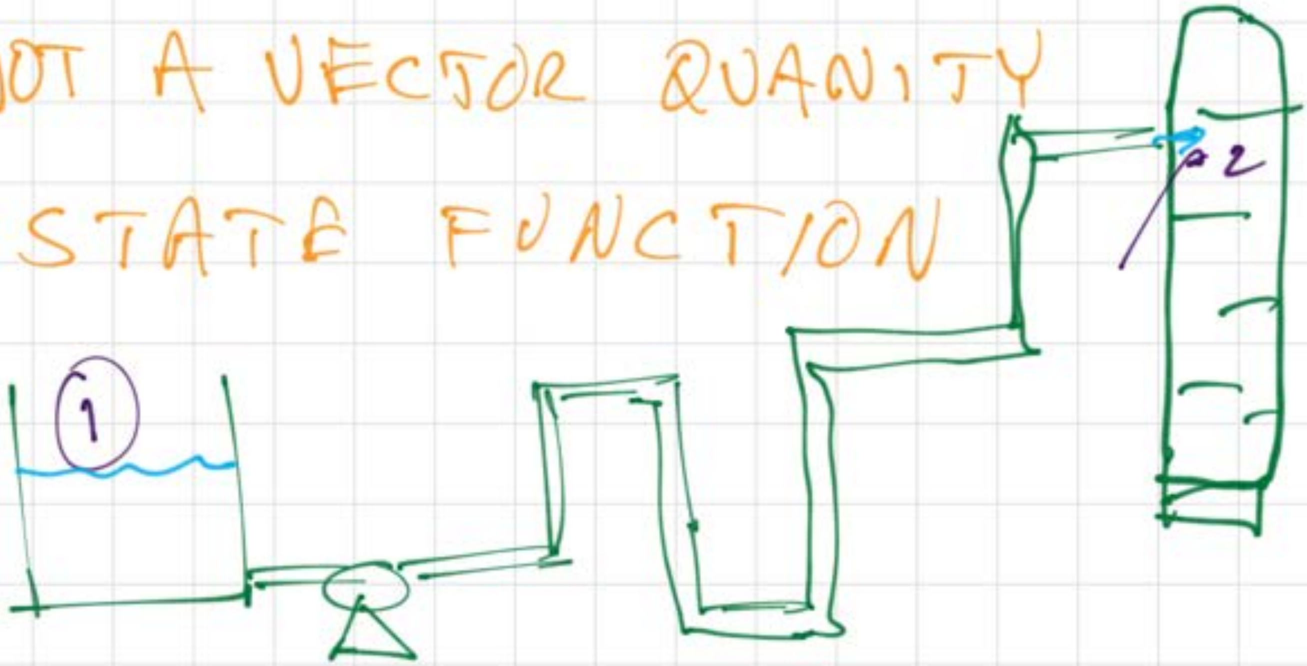




IT IS ALLEGED THAT G.I. CLIPPED THE ROD WITH WIRE CUTTERS !!

SHOULD WE CONSIDER
CONSERVATION OF ENERGY
THAT WE HAVE SAVED ?
(AS IF FOR A RAINY DAY!)

- NOT A VECTOR QUANITY
- STATE FUNCTION



COULD CONSIDER CHANGE
FROM STATE 1 → 2

ENERGY CONSERVATION

$$\frac{d}{dt}\left(U + m\left(\frac{V^2}{2} + \Psi\right)\right) = \sum_{i=1}^{K} \dot{m}_i\left(\hat{H}_i + \frac{V_i^2}{2} + \Psi_i\right)$$

POTENTIAL ENERGY / MASS

$$+ \dot{Q} + \dot{W}$$

RECALL

KE, Ψ NOT IMPT

$\Delta P = 0$

$$\dot{Q} = \dot{m}\left(\hat{H}_2 - \hat{H}_1\right)$$ HEAT X-CHANGER

$$\dot{W}_S = \dot{m}\left(\hat{H}_2 - \hat{H}_1\right)$$ PUMP TURBINE

$\Delta S = 0$

LET'S PICK:

$$\Psi = gh$$

RECALL: $\hat{H} = \hat{U} + P\hat{V}$ $\quad \hat{V} = \frac{1}{\rho}$

ALSO:

$$\Delta \hat{U} = C_V \Delta T + \left[T\frac{\partial P}{\partial T} - P\right] dV$$

$$\underset{\text{IF } S = \text{CONST}}{\overset{\shortparallel}{0}}$$

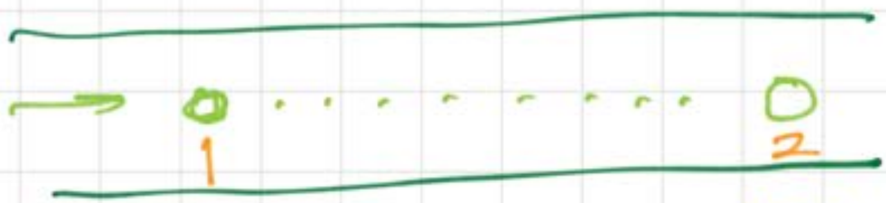
IF $\Delta T = 0$

$$\Delta \hat{H} = \Delta P \hat{V}$$

LET'S BREAK OUT ENTHALPY FROM FLOW TERMS.

$$\frac{d}{dt}\left(U + \left(\frac{1}{2}v^2 + gh\right)M\right) = \sum_i \dot{M}_i\left(\hat{U} + \frac{v^2}{2} + gh\right) + \dot{Q} + \dot{W}_s + \sum_i \dot{M}_i (P\hat{V})_i$$

NOW CONSIDER A VERY SIMPLE FLOW ...



STEADY STATE

$$0 = \dot{m}\left(\hat{u} + \frac{V^2}{2} + gh\right)\Big|_1 - \dot{m}\left(\hat{u} + \frac{V^2}{2} + gh\right)\Big|_2$$

$$+ \dot{Q} + \dot{W}_s + \dot{m}\left(P_1\hat{v}_1 - P_2\hat{v}_2\right)$$

SHRINK TO A DIFFERENTIAL SLICE

$$d\left(\hat{u} + gh + \frac{V^2}{2} + P\overbrace{\hat{v}}^{1/\rho}\right) = dQ + dW_s$$

WE KNOW:

$$d\hat{u} = T\,d\hat{s} - P\,d\hat{v}$$

$$d\hat{U} = T d\hat{S} - dP\hat{V} + \hat{V}dP$$

$$d\hat{U} + d(P\hat{V}) = T d\hat{S} + \hat{V} dP$$

SUBS..

$$d\left(\hat{U} + gh + \frac{V^2}{2} + P\hat{V}\right) = dQ + dW_s$$

1/S

$$d\left(gh + \frac{V^2}{2} + \frac{1}{S} dP\right) = dQ + dW_s - T d\hat{S}$$

ALSO RECALL:

$$dS \geq \frac{dQ}{T}$$

$$\therefore \quad dl_V \equiv T dS - dQ \geq 0$$

SUB INTO RIGHT SIDE

INTEGRATE FROM ① ⌒ ②

S = CONST

$$\left(\frac{V_2^2}{2} + g h_2 + \frac{P_2}{\rho}\right) - \left(\frac{V_1^2}{2} + g h_1 + \frac{P_1}{\rho}\right)$$

$$= \delta w_s - \ell_v$$

GOOD FOR ANY CONTINUOUS PATH FOR ρ = CONST AND FOR WHICH YOU CAN PRESCRIBE "ℓ_v"

$\ell_v \Rightarrow$ "VISCOUS LOSSES"

ENGINEERING BERNOULLI EQUATION

- DERIVED FROM 1ST & 2ND LAWS OF THERMODYNAMICS
- CAN BE APPLIED ALONG ANY CONTINUOUS PATH FROM (STATE) "1" TO (STATE) "2"

$$\left(\frac{V_2^2}{2} + g h_2 + \frac{P_2}{\rho}\right) - \left(\frac{V_1^2}{2} + g h_1 + \frac{P_1}{\rho}\right) = \delta w_s - l_v$$

COULD CONSIDER CHANGE FROM STATE 1 → 2

FOR PIPE FLOW

$$l_v = \frac{2 L f V^2}{D}$$

MORE GENERALLY ...

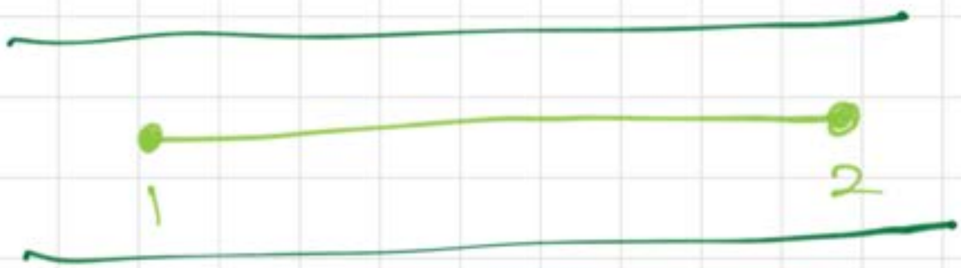
$$l_v = \frac{1}{2} K V^2$$

TABLES

FOR MULTIPLE FITTINGS AND PIPE SEGMENTS...

$$l_v = \sum_i \frac{1}{2} K_i V_i^2$$

SIMPLE EXAMPLES



$A = \text{CONST}$

$V = \text{CONST}$

$$\frac{P_2 - P_1}{\rho} = -l_v$$

$$= \left(dQ - T\,ds \right)$$

$$\doteq \left(dQ - c_v\,dT \right)$$

$$ds = \frac{c_v}{T}\,dT + \left.\frac{\partial P}{\partial T}\right|_v dv$$

$P_1 > P_2 \quad \therefore$

IF ADIABATIC $T_2 > T_1$

FLUID HEATS UP.

OR IF $T_2 = T_1$

$dQ > 0 \;\therefore$ EXOTHERMIC

BUT WE KNOW WHAT PRESSURE DROP SHOULD BE ...

$$\frac{P_2 - P_1}{\rho} = -l_v$$

FOR A PIPE:

$$f \equiv \frac{(P_1 - P_2)\, D}{2 \rho V^2 L}$$

$$-\frac{P_2 - P_1}{\rho} = \frac{2 V^2 L f}{D}$$

FOR PIPE FLOW

$$l_v = \frac{2 L f V^2}{D}$$

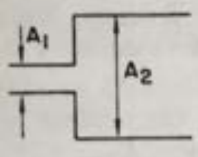
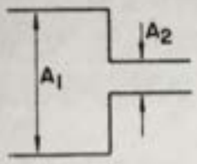
MORE GENERALLY ...

$$l_v = \tfrac{1}{2} K V^2$$

$\underline{\underline{\text{TABLES}}}$

TABLE 5-1

LOSSES IN FITTINGS AND VALVES FOR TURBULENT FLOW[a]

Fitting or valve	*Velocity heads lost, K_f*
90° elbow, standard	0.75
90° elbow, square	1.3
Coupling	0.04
Gate valve	
Open	0.17
Half-open	4.5
Globe valve, bevel seat	
Open	6.4
Half-open	9.5
Sudden expansion 	$\left(\frac{A_2}{A_1}-1\right)^2$
Sudden contraction 	$\left(\frac{2}{m}-\frac{A_2}{A_1}-1\right)^2$ m is the root of the quadratic $\frac{1-m(A_2/A_1)}{1-(A_2/A_1)^2}=\left(\frac{m}{1.2}\right)^2$
Rounded entrance	0.05

[a]The result for the sudden expansion is derived in Sec. 6.2. The result for the sudden contraction is from Martin, *Chem. Eng. Educ.*, Summer 1974, p. 138. Other values are from *Perry's Handbook*.

Example 5.7

A liquid is pumped through a 50-mm-diameter smooth pipe between two tanks at a rate of 3 kg/s in the section of the process stream shown in Fig. 5-5. The liquid has properties $\rho = 10^3$ kg/m^3, $\eta = 10^{-3}$ Pa·s. The pressure above the

$$\Delta P = \tfrac{1}{2}\rho K V^2$$

DOWNSTREAM VELOCITY

$$\frac{P_2 - P_1}{\rho} = -\ell_v$$

$$= -\tfrac{1}{2} K V^2$$

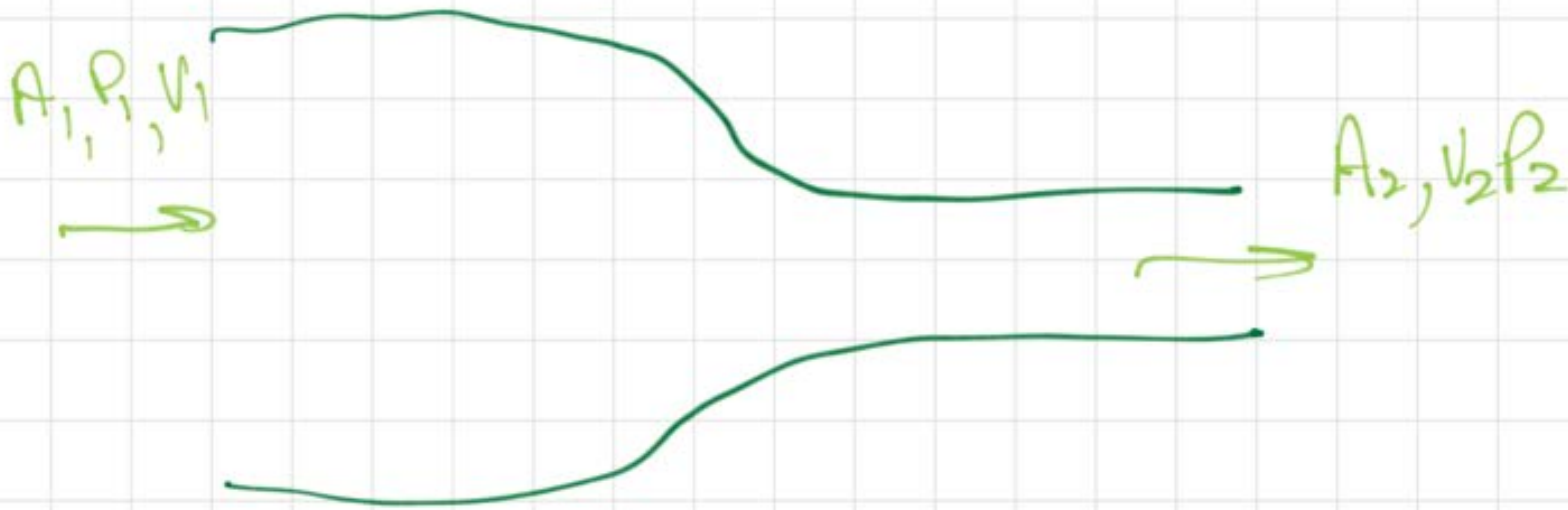
$$K = 1.3$$

$$P_2 = P_1 - \tfrac{1}{2}(1.3)\rho V^2$$

SOME EXAMPLES....

Fitting Type		K
Pipe Entry Losses		
Square Inlet		0.50
Re-entrant Inlet		0.80
Slightly Rounded Inlet		0.25
Bellmouth Inlet		0.05
Pipe Intermediate Losses		
Elbows R/D < 0.6	45°	0.35
	90°	1.10
Long Radius Bends (R/D > 2)	11¼°	0.05
	22½°	0.10
	45°	0.20
	90°	0.50
Tees		
(a) Flow in line		0.35
(b) Line to branch flow		1.00
Sudden Enlargements		
Ratio d/D		
0.9		0.04
0.8		0.13
0.7		0.26
0.6		0.41
0.5		0.56
0.4		0.71
0.3		0.83
0.2		0.92
<0.2		1.00

Fitting Type	K
Gradual Enlargements	
Ratio d/D q = 10° typical	
0.9	0.02
0.7	0.13
0.5	0.29
0.3	0.42
Gradual Contractions	
Ratio d/D q = 10° typical	
0.9	0.03
0.7	0.08
0.5	0.12
0.3	0.14
Valves	
Gate Valve (fully open)	0.20
Reflux Valve	2.50
Globe Valve	10.00
Butterfly Valve (fully open)	0.20
Angle Valve	5.00
Foot Valve with strainer	15.00
Air Valves	zero



$$\frac{P_2-P_1}{\rho}+\frac{V_2^2-V_1^2}{2} = -\ell_v$$

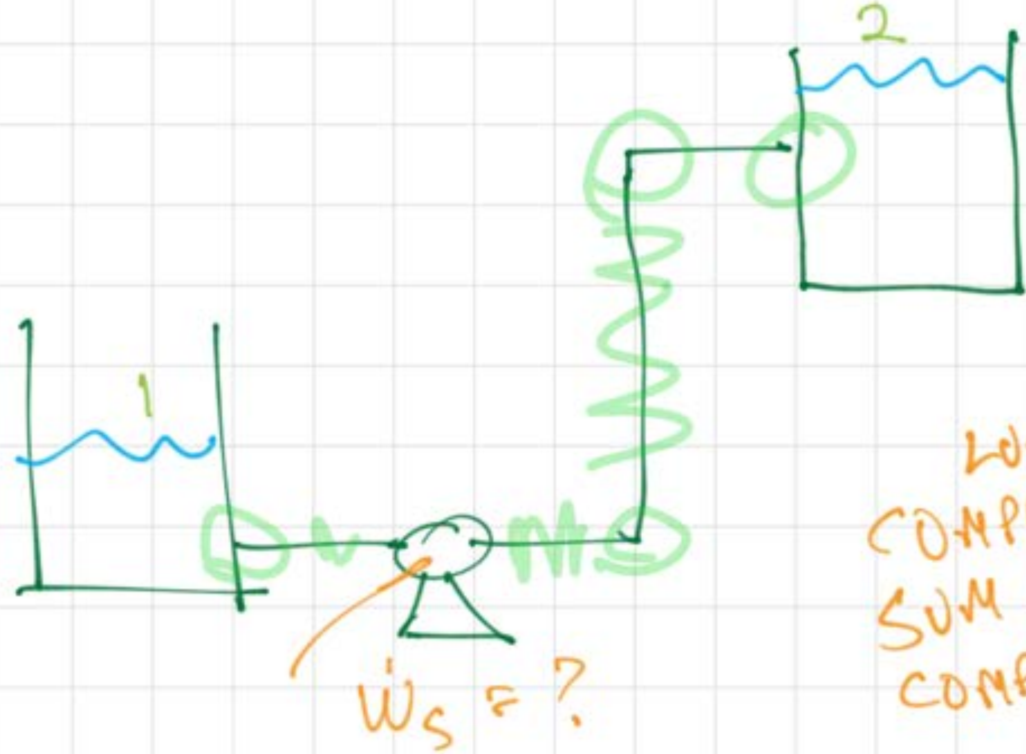
$$V_2 = \frac{V_1 A_1}{A_2}$$

IF $\ell_v = 0$ ...

$$P_2 - P_1 = \frac{\rho}{2}\left(V_1^2 - V^2\left(\frac{A_1}{A_2}\right)^2\right)$$

$$= \frac{\rho V_1^2}{2}\left(1 - \left(\frac{A_1}{A_2}\right)^2\right)$$

$P_2 < P_1$ $\qquad$ >0

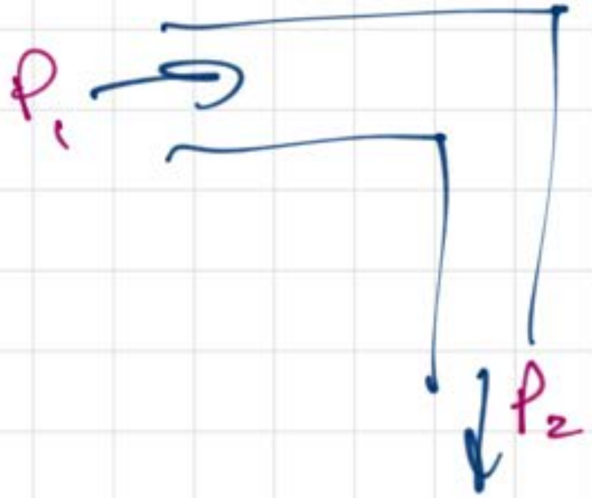


$$(gh_2 - gh_1) = \delta W_s - \underbrace{\sum_i l_{v_i}}$$

$$\dot{W}_s = \dot{m} g (h_2 - h_1) + \dot{m} \sum_i l_{v_i}$$

$$\sum_i l_{v_i} =$$ 2 ELBOWS + 1 EXIT + 1 ENTRANCE + 4 STRAIGHT PIECES OF PIPE ...

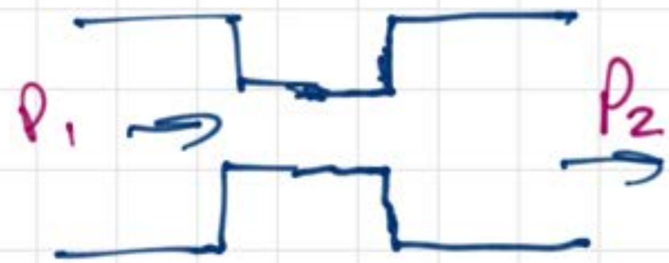
SOME OTHER INFO



WE EXPECT THAT EVEN IF THERE IS NO CHANGE OF AREA,

$P_1 > P_2$

FOR THESE FLOWS



EMPIRICAL FORMALISM

$$\Delta P = \tfrac{1}{2} \rho K V^2$$

K DEPENDS ON GEOMETRY