CBE 30357

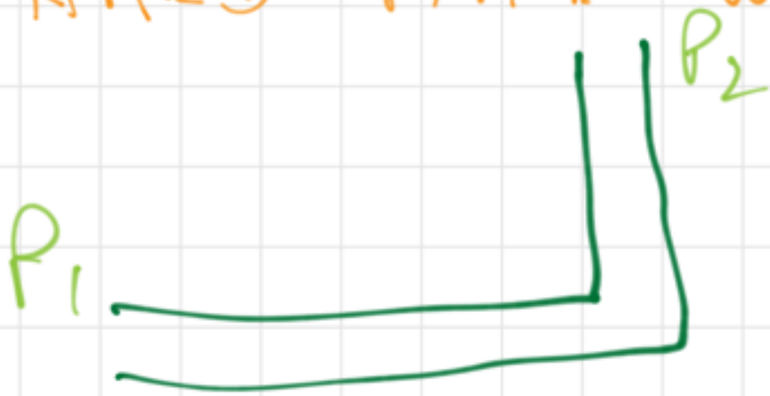
11/14/17

AS MUCH AS WE ENJOY USING THEM, AND FOR AS MANY PROBLEMS THAT CAN BE SOLVED...

THERE ARE SOME PROBLEMS THAT WE HAVE TROUBLE SOLVING

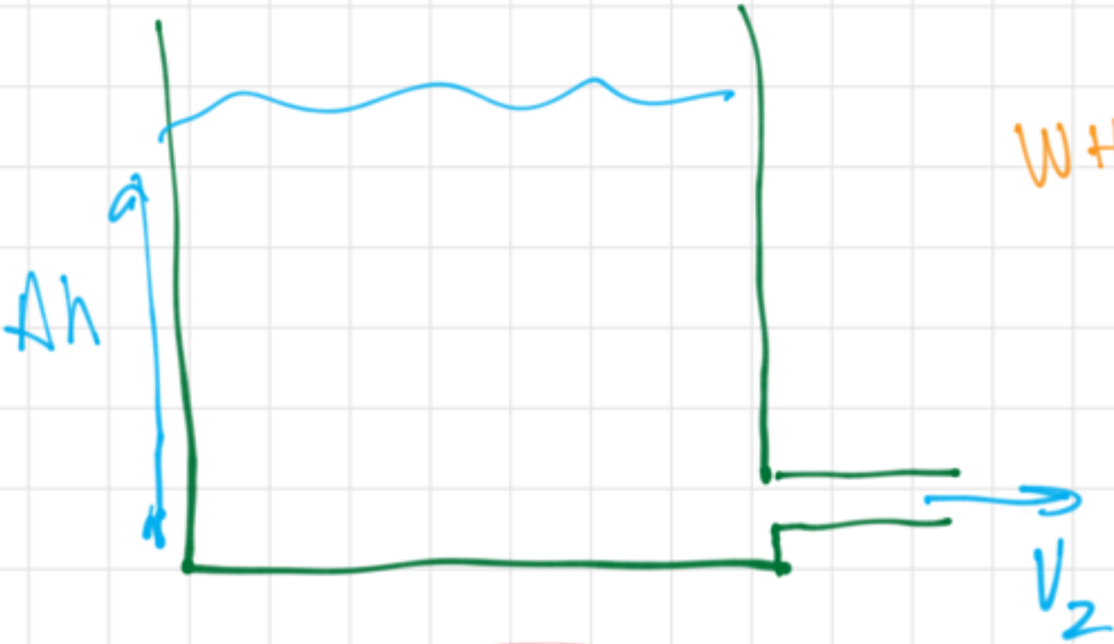
HOW BIG IS FLOW RATE FROM LEAK?

I WOULD HAVE A REALLY HARD TIME WITH THIS!



WHAT IS P_2?

HOW DO WE QUANTIFY DIFFERENCE?

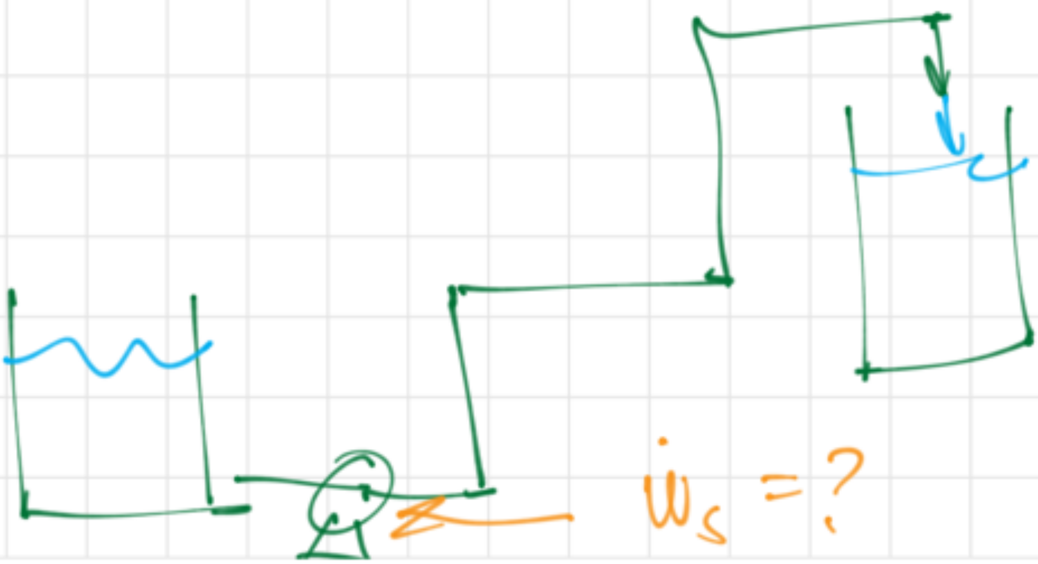


WHAT IS V_2?

HOW WOULD WE GET THIS IN PHYSICS?

$$\Delta \tfrac{1}{2} m v^2 \stackrel{?}{=} \Delta\, mgh$$

$$\frac{dv}{dt} = g \quad ?$$



$\dot{W}_s = ?$

PRESUMABLY, THE RELATION BETWEEN PRESSURE + VELOCITY IS CONTAINED IN MOMENTUM EQ.S ...

CAN WE FIND IT?

$$\rho(\vec{V}\cdot\vec{\nabla}\vec{V}) = -\vec{\nabla}P + \mu\nabla^2\vec{V} + \rho\vec{g}$$

IF $\vec{V}=0$ $\quad \vec{\nabla}P = \rho g$

$$\boxed{P = P_0 + \rho g h}$$

FOR $\mu=0$, $g=0$

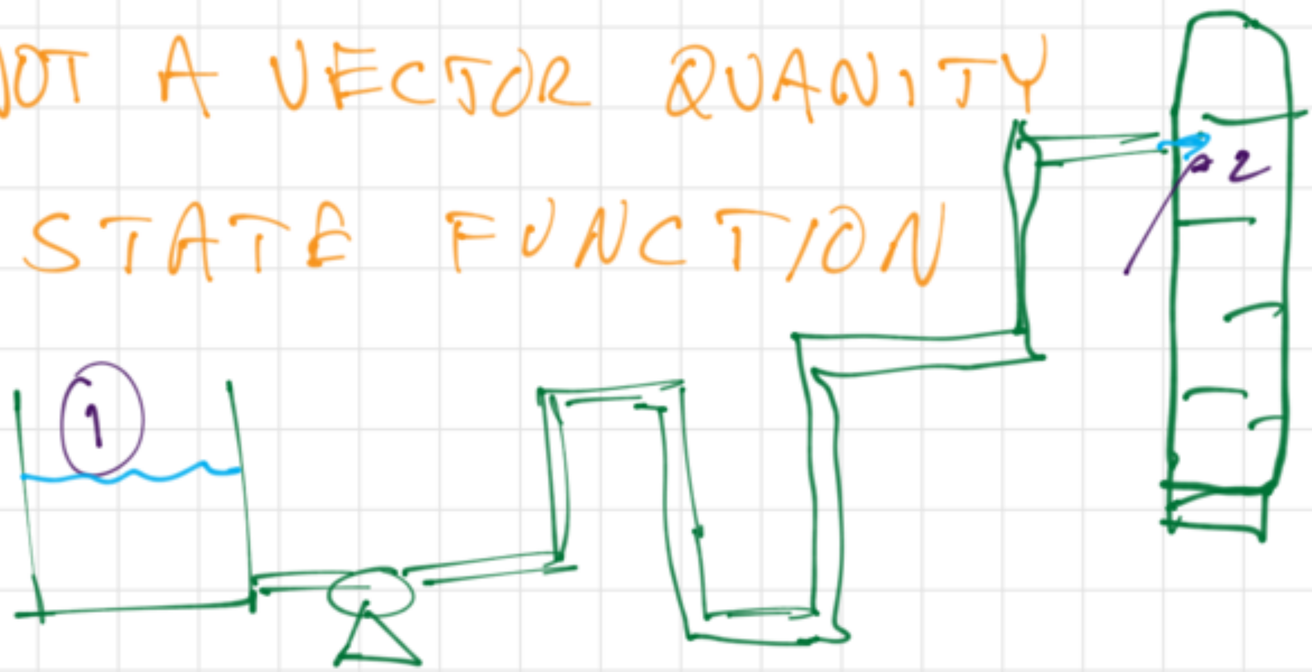
$$\rho(\vec{V}\cdot\vec{\nabla}\vec{V}) = -\vec{\nabla}P$$

$$\frac{1}{2}\frac{\partial V_x^2}{\partial x} \sim \frac{-\partial P}{\partial x}$$

$\therefore$ $\boxed{\Delta P = \rho \Delta V^2/2}$

SHOULD WE CONSIDER CONSERVATION OF ENERGY THAT WE HAVE SAVED ?
(AS IF FOR A RAINY DAY!)

- NOT A VECTOR QUANITY
- STATE FUNCTION



COULD CONSIDER CHANGE FROM STATE 1 → 2

ENERGY CONSERVATION

$$\frac{d}{dt}\left(U + m\left(\frac{V^2}{2} + \psi\right)\right) = \sum_{i=1}^{K} \dot{m}_i\left(\hat{H}_i + \frac{V_i^2}{2} + \psi_i\right)$$

POTENTIAL ENERGY / MASS

$$+ \dot{Q} + \dot{W}$$

RECALL

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$\Delta P = 0$

$$\dot{Q} = \dot{m}\left(\hat{H}_2 - \hat{H}_1\right)$$ HEAT X-CHANGER

$$\dot{W}_S = \dot{m}\left(\hat{H}_2 - \hat{H}_1\right)$$ PUMP TURBINE

$\Delta S = 0$

LET'S PICK:

$$\psi = gh$$

RECALL: $\hat{H} = \hat{U} + P\hat{V}$ $\qquad \hat{V} = \frac{1}{\rho}$

ALSO:

$$\Delta \hat{U} = C_V \Delta T + \left[T \underset{\substack{\parallel \\ 0 \text{ IF } S = \text{CONST}}}{\frac{\partial P}{\partial T}} - P \right] dV$$

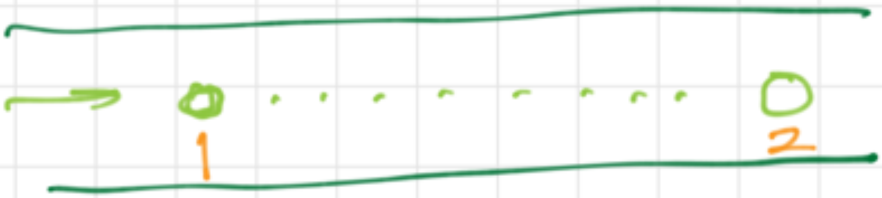
IF $\Delta T = 0$

$$\Delta \hat{H} = \Delta P \hat{V}$$

LET'S BREAK OUT ENTHALPY FROM FLOW TERMS.

$$\frac{d}{dt}\left(U + \left(\frac{1}{2}v^2 + gh\right) M \right) = \sum_i \dot{M}_i \left(\hat{U} + \frac{v^2}{2} + gh \right) + \dot{Q} + \dot{W}_s + \sum_i \dot{M}_i (P\hat{V})_i$$

NOW CONSIDER A VERY SIMPLE FLOW ...



STEADY STATE

$$0 = \dot{m}\left(\hat{u} + \frac{v^2}{2} + gh\right)\Big|_1 - \dot{m}\left(\hat{u} + \frac{v^2}{2} + gh\right)\Big|_2$$

$$+ \dot{Q} + \dot{W}_s + \dot{m}\left(P_1\hat{v}_1 - P_2\hat{v}_2\right)$$

SHRINK TO A DIFFERENTIAL SLICE

$1/s$

$$d\left(\hat{u} + gh + \frac{v^2}{2} + P\hat{v}\right) = dQ + dW_s$$

WE KNOW:

$$d\hat{u} = Td\hat{s} - Pd\hat{v}$$

$$d\hat{u} = T d\hat{s} - dP\hat{v} + \hat{v} dP$$

$$d\hat{u} + d(P\hat{v}) = T d\hat{s} + \hat{v} dP$$

SUBS..

$$d\left(\hat{u} + gh + \frac{V^2}{2} + P\hat{v}\right) = dQ + dW_s$$

1/ρ

$$d\left(gh + \frac{V^2}{2} + \frac{1}{\rho} dP\right) = dQ + dW_s - T d\hat{s}$$

ALSO RECALL:

$$ds \geq \frac{dQ}{T}$$

$\therefore \quad dl_v \equiv T ds - dQ \geq 0$

SUB INTO RIGHT SIDE

INTEGRATE FROM 1 → 2

$\underline{\rho = \text{CONST}}$

$$\left(\frac{V_2^2}{2} + gh_2 + \frac{P_2}{\rho}\right) - \left(\frac{V_1^2}{2} + gh_1 + \frac{P_1}{\rho}\right)$$

$$= \delta w_s - l_v$$

GOOD FOR ANY CONTINUOUS PATH FOR $\rho = \text{CONST}$ AND FOR WHICH YOU CAN PRESCRIBE "l_v"

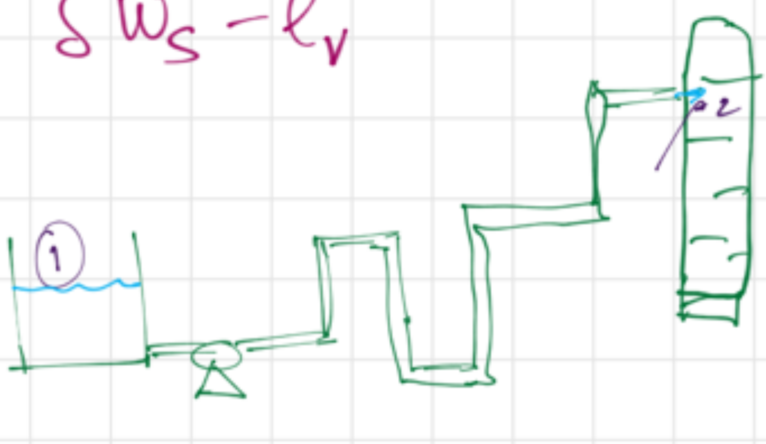
$l_v \Rightarrow$ "VISCOUS LOSSES"

ENGINEERING BERNOULLI EQUATION

- DERIVED FROM 1ST + 2ND LAWS OF THERMODYNAMICS

- CAN BE APPLIED ALONG ANY CONTINUOUS PATH FROM (STATE) "1" TO (STATE) "2"

$$\left(\frac{V_2^2}{2} + g h_2 + \frac{P_2}{\rho}\right) - \left(\frac{V_1^2}{2} + g h_1 + \frac{P_1}{\rho}\right) = \delta w_s - e_v$$



COULD CONSIDER CHANGE FROM STATE 1 → 2

FOR PIPE FLOW

$$l_v = \frac{2 L f v^2}{D}$$

MORE GENERALLY ...

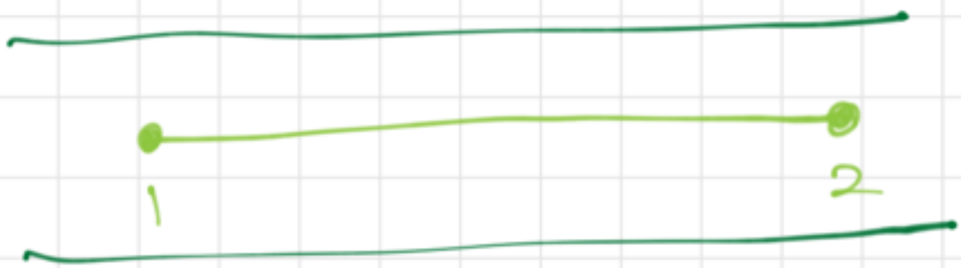
$$l_v = \frac{1}{2} K v^2$$

TABLES

FOR MULTIPLE FITTINGS AND PIPE SEGMENTS...

$$l_v = \sum_i \frac{1}{2} K_i v_i^2$$

SIMPLE EXAMPLES



A = CONST
V = CONST

$$\frac{P_2 - P_1}{\rho} = -l_v$$

$$= (dQ - T\,ds)$$

$$\doteq (dQ - c_v\,dT)$$

$$ds = \frac{c_v}{T}\,dT + \left.\frac{\partial P}{\partial T}\right|_v dv$$

$P_1 > P_2 \quad \therefore$

IF ADIABATIC $T_2 > T_1$
FLUID HEATS UP.

OR IF $T_2 = T_1$
$dQ > 0 \;\therefore$ EXOTHERMIC

BUT WE KNOW WHAT PRESSURE DROP SHOULD BE ...

$$\frac{P_2 - P_1}{\rho} = -l_v$$

FOR A PIPE:

$$f = \frac{(P_1 - P_2)\, D}{2 \rho V^2 L}$$

$$-\frac{P_2 - P_1}{\rho} = \frac{2 V^2 L f}{D}$$

FOR PIPE FLOW

$$l_v = \frac{2 L f V^2}{D}$$

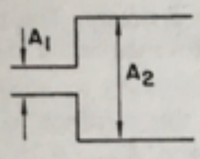
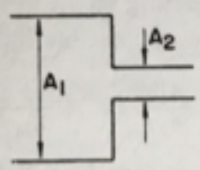
MORE GENERALLY ...

$$l_v = \tfrac{1}{2} K V^2$$

TABLES

TABLE 5-1

LOSSES IN FITTINGS AND VALVES FOR TURBULENT FLOW[a]

Fitting or valve	*Velocity heads lost, K_f*
90° elbow, standard	0.75
90° elbow, square	1.3
Coupling	0.04
Gate valve	
Open	0.17
Half-open	4.5
Globe valve, bevel seat	
Open	6.4
Half-open	9.5
Sudden expansion 	$\left(\frac{A_2}{A_1} - 1\right)^2$
Sudden contraction 	$\left(\frac{2}{m} - \frac{A_2}{A_1} - 1\right)^2$ m is the root of the quadratic $\frac{1 - m(A_2/A_1)}{1 - (A_2/A_1)^2} = \left(\frac{m}{1.2}\right)^2$
Rounded entrance	0.05

[a]The result for the sudden expansion is derived in Sec. 6.2. The result for the sudden contraction is from Martin, *Chem. Eng. Educ.*, Summer 1974, p. 138. Other values are from *Perry's Handbook*.

Example 5.7

A liquid is pumped through a 50-mm-diameter smooth pipe between two tanks at a rate of 3 kg/s in the section of the process stream shown in Fig. 5-5. The liquid has properties $\rho = 10^3$ kg/m³, $\eta = 10^{-3}$ Pa·s. The pressure above the

$$\Delta P = \tfrac{1}{2}\rho K V^2$$

DOWNSTREAM VELOCITY

$$\frac{P_2 - P_1}{\rho} = -l_v$$

$$= -\tfrac{1}{2} K V^2$$

$$K = 1.3$$

$$P_2 = P_1 - \tfrac{1}{2}(1.3)\rho V^2$$

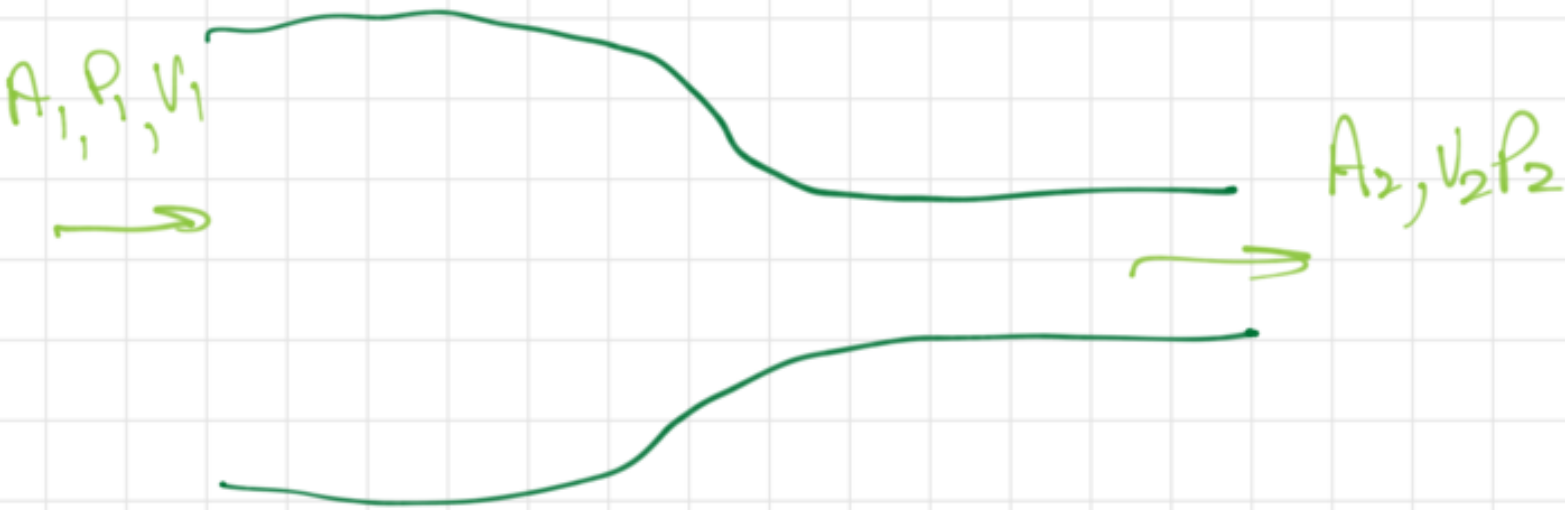
SOME EXAMPLES ....

Fitting Type		K
Pipe Entry Losses		
Square Inlet		0.50
Re-entrant Inlet		0.80
Slightly Rounded Inlet		0.25
Bellmouth Inlet		0.05

Fitting Type		K
Gradual Enlargements		
Ratio d/D q = 10° typical		
	0.9	0.02
	0.7	0.13
	0.5	0.29
	0.3	0.42
Gradual Contractions		
Ratio d/D q = 10° typical		
	0.9	0.03
	0.7	0.08
	0.5	0.12
	0.3	0.14
Valves		

Pipe Intermediate Losses		
Elbows R/D < 0.6	45°	0.35
	90°	1.10
Long Radius Bends (R/D > 2)	$11^1/_4$°	0.05
	$22^1/_2$°	0.10
	45°	0.20
	90°	0.50
Tees		
(a) Flow in line		0.35
(b) Line to branch flow		1.00
Sudden Enlargements		
Ratio	d/D	
	0.9	0.04
	0.8	0.13
	0.7	0.26
	0.6	0.41
	0.5	0.56
	0.4	0.71
	0.3	0.83
	0.2	0.92
	<0.2	1.00

Valves	
Gate Valve (fully open)	0.20
Reflux Valve	2.50
Globe Valve	10.00
Butterfly Valve (fully open)	0.20
Angle Valve	5.00
Foot Valve with strainer	15.00
Air Valves	zero



$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} = -l_v$$

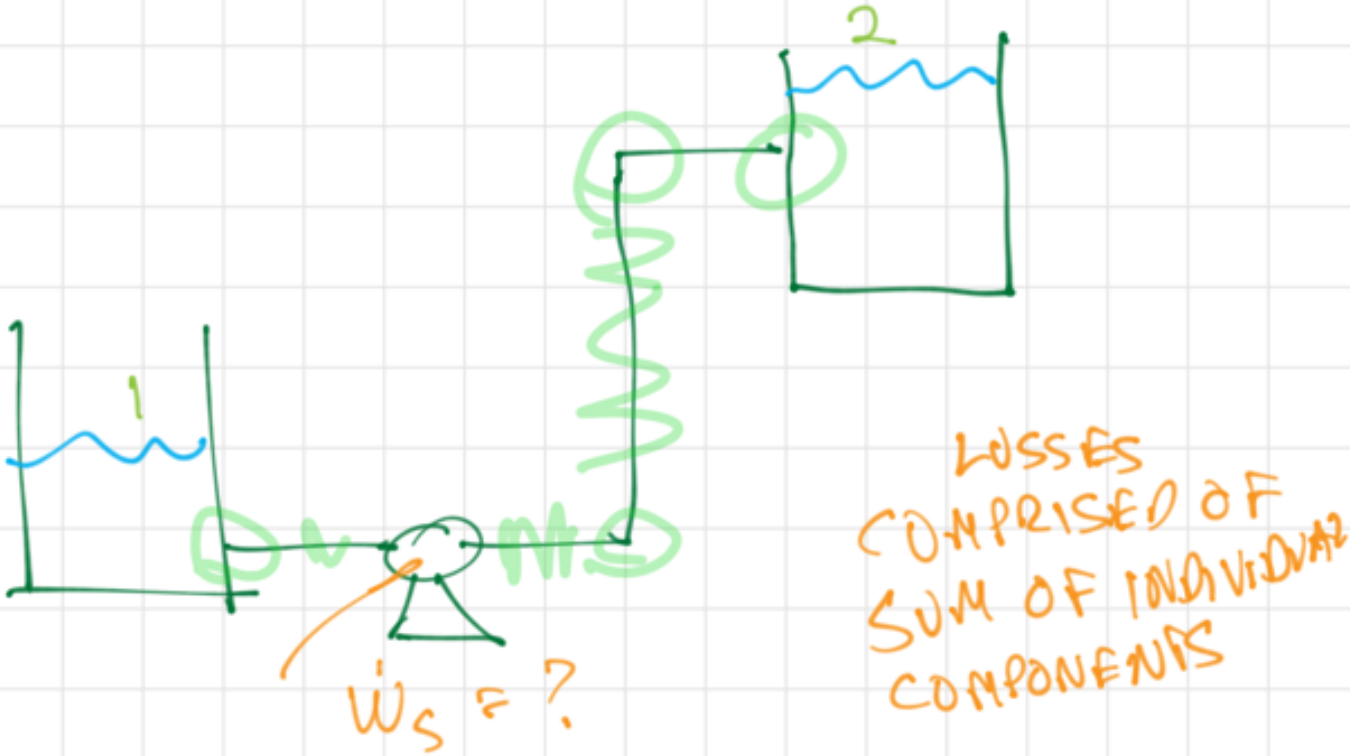
$$V_2 = \frac{V_1 A_1}{A_2}$$

IF $l_v = 0 \ldots$

$$P_2 - P_1 = \frac{\rho}{2}\left(V_1^2 - V^2\left(\frac{A_1}{A_2}\right)^2\right)$$

$$= \frac{\rho V_1^2}{2}\left(1 - \left(\frac{A_1}{A_2}\right)^2\right)$$

$P_2 < P_1$ $\qquad$ > 0

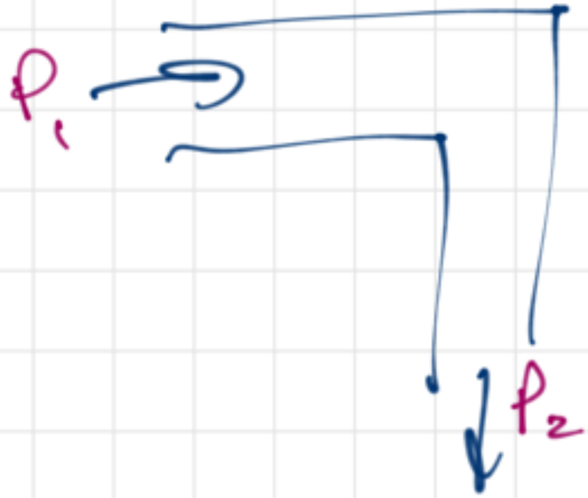


$$(gh_2 - gh_1) = \delta W_s - \underbrace{\sum_i l_{v_i}}$$

$$\dot{W}_s = \dot{m} g(h_2 - h_1) + \dot{m} \sum_i l_{v_i}$$

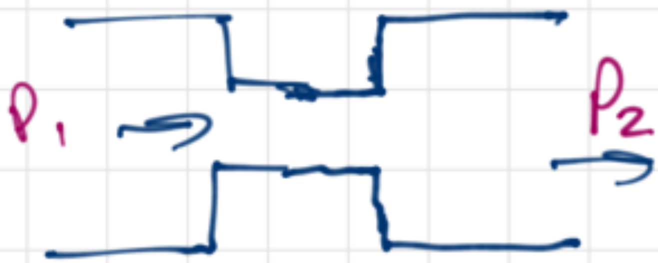
$\sum_i l_{v_i}$ = 2 ELBOWS + 1 EXIT + 1 ENTRANCE + 4 STRAIGHT PIECES OF PIPE ...

SOME OTHER INFO



WE EXPECT THAT EVEN IF THERE IS NO CHANGE OF AREA,

$P_1 > P_2$ FOR THESE FLOWS



EMPIRICAL FORMALISM

$$\Delta P = \tfrac{1}{2} \rho K V^2$$

K DEPENDS ON GEOMETRY