CBE 30357
10/5/17

From the work to date, you have "checked off" the first box regarding learning the subject of transport phenomena. You can identify the correct governing equation to start with, you can find the important terms and you can get a solution that matches the boundary conditions---for the simplest problems.

So far, we have solved unidirectional flows, that vary in only 1 direction, caused by gravity, pressure change or a moving surface.

This is a significant accomplishment!

TOPICS FOR TODAY

1) GENERALIZED PLOTS OF DATA:
ARE THEY USEFUL TO ENGINEERS?
CAN WE MAKE SENSE OF THESE?

2) DIMENSIONAL REASONING TO INTERPRET

3) OPTIMIZATION

"FAMOUS PLOT"

WE CAN PREDICT # OF "CARS" IN THE WORLD.

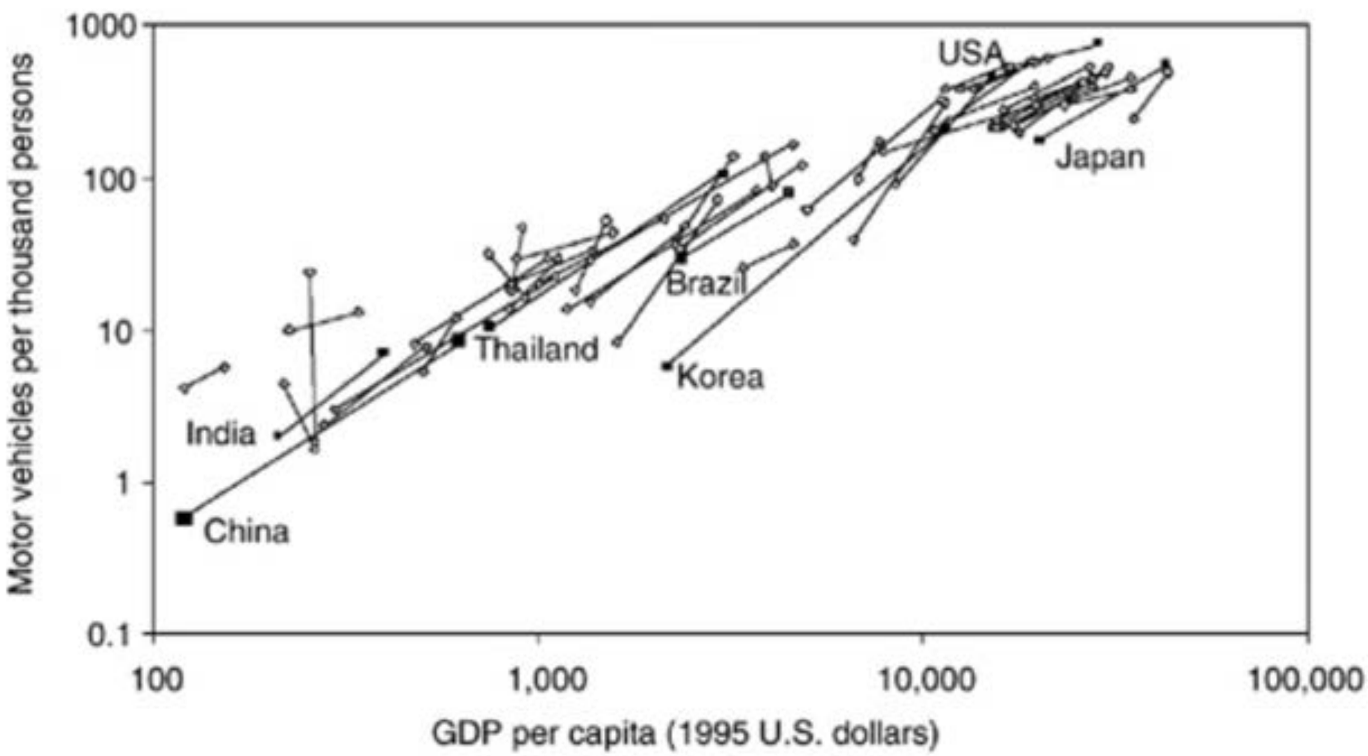
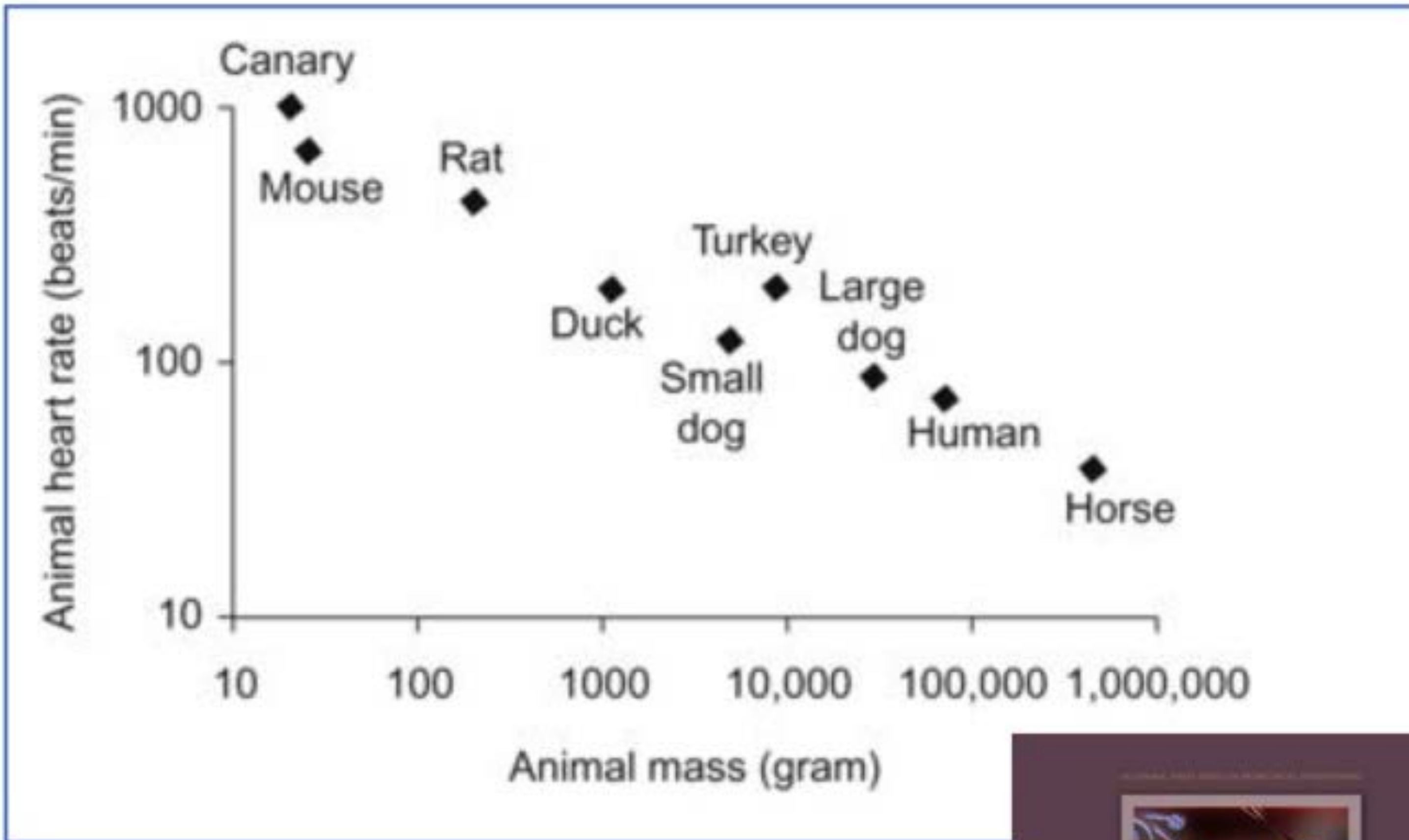


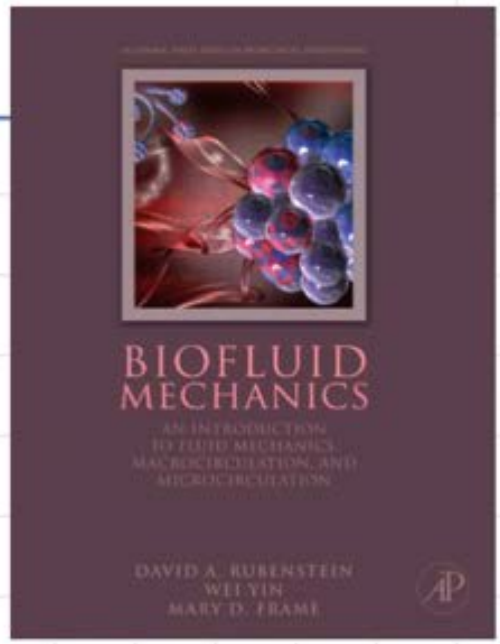
FIGURE 2-1 Motor vehicle fleets in relation to income, selected countries, 1970 and 1996. NOTE: Per capita gross domestic product (GDP) is transformed to dollars using market exchange rates (see footnote 2). SOURCES: Motorization data: International Road Federation (2001 and earlier); other data: World Bank (2001 and earlier).

SOME INTERESTING PHYSIOLOGICAL DATA.

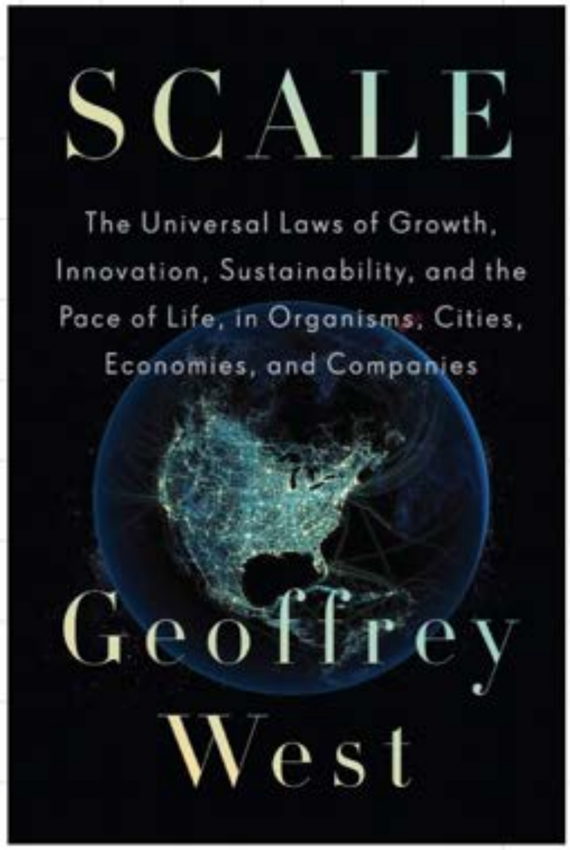
CAN WE INTERPRET?

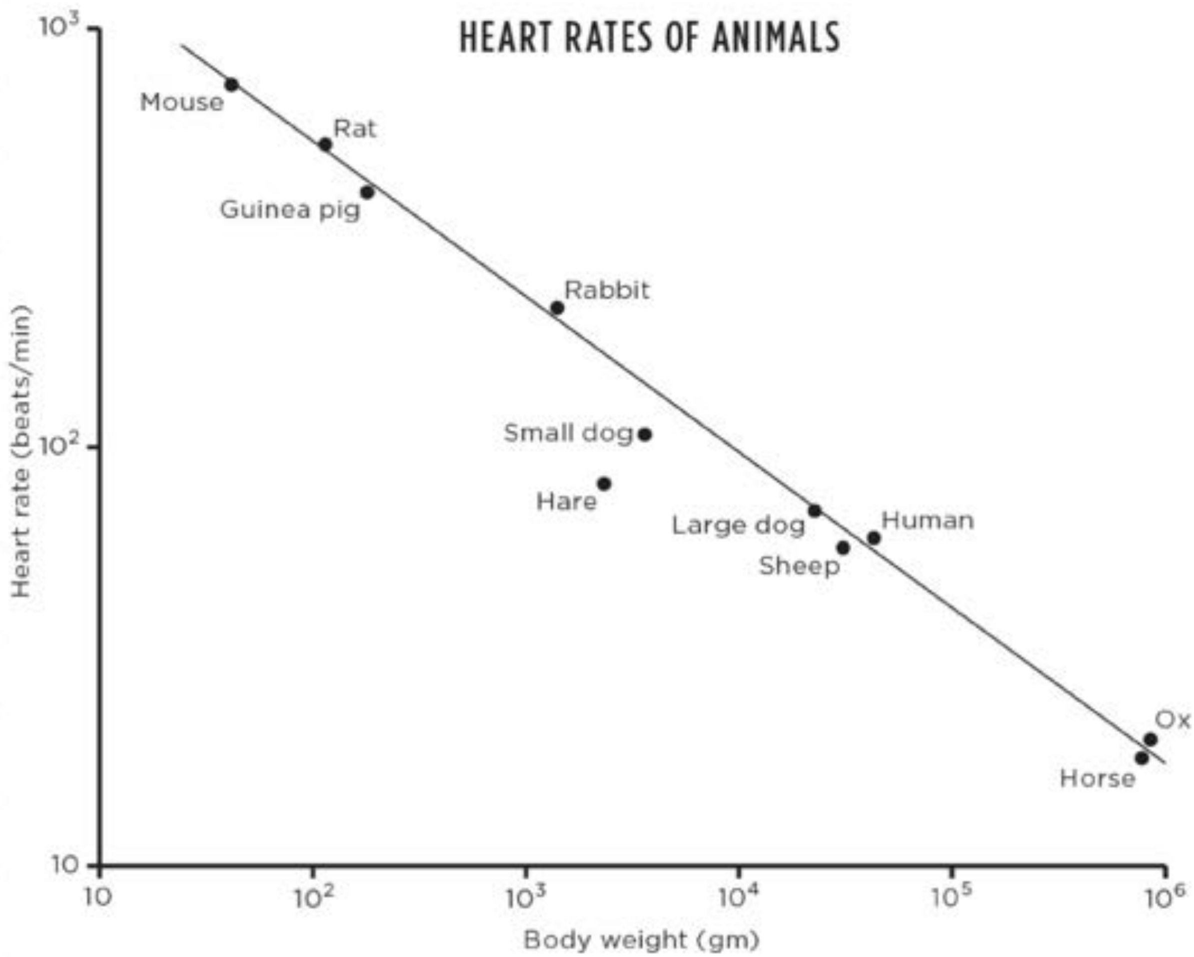


"ALLOMETRIC" DATA



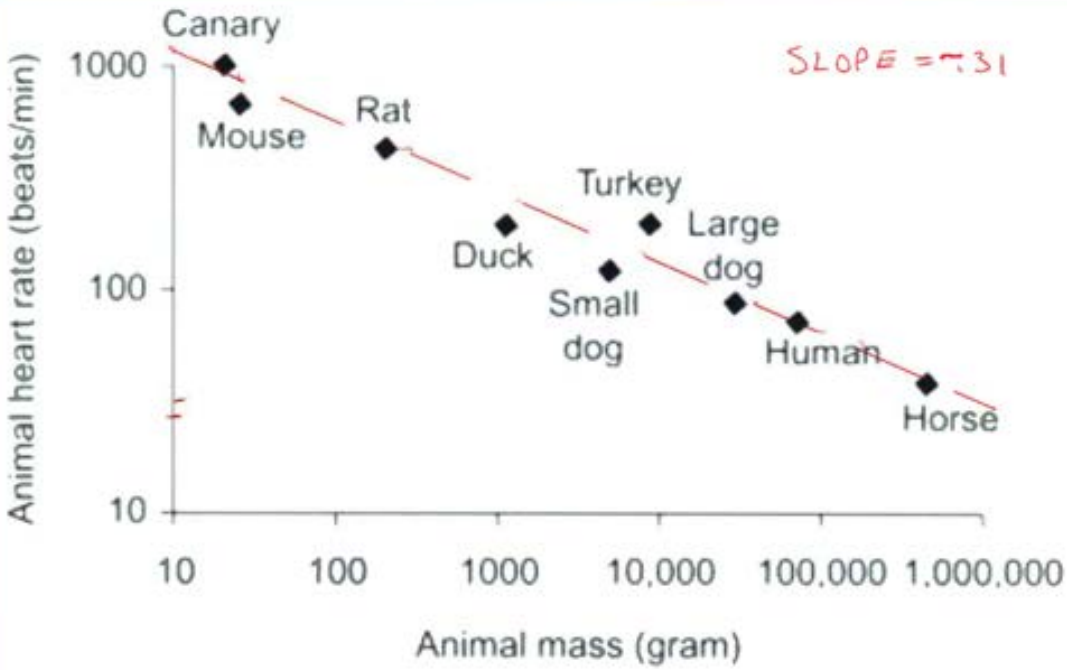
~ SAME DATA...

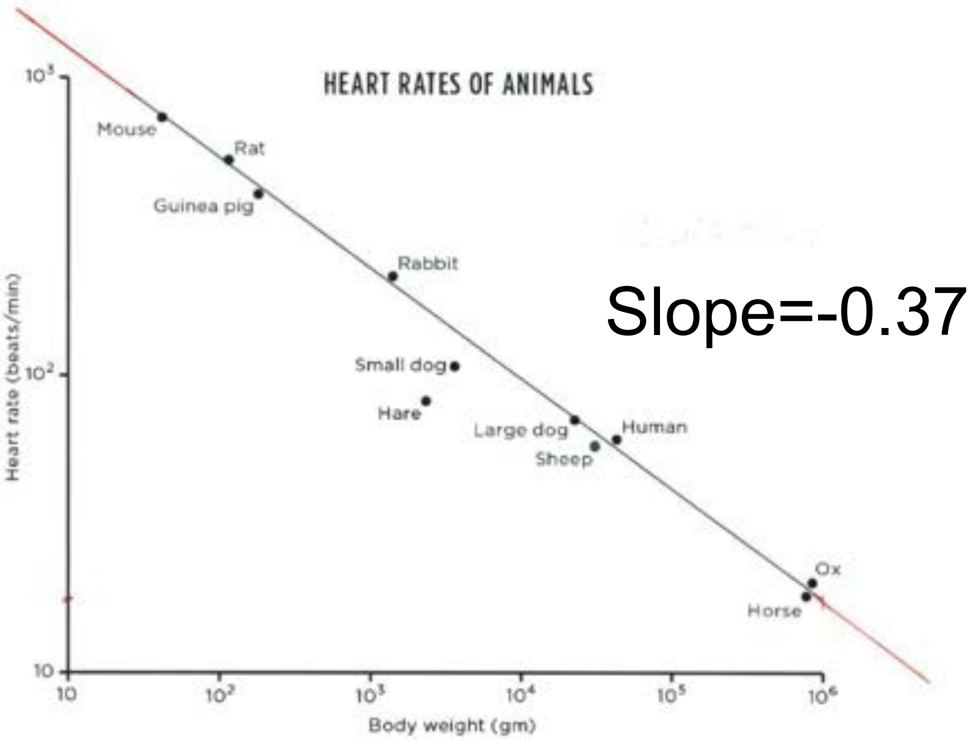

SCALE
The Universal Laws of Growth, Innovation, Sustainability, and the Pace of Life, in Organisms, Cities, Economies, and Companies
Geoffrey West


HEART RATES OF ANIMALS
Heart rate (beats/min)
Body weight (gm)
Mouse
Rat
Guinea pig
Rabbit
Small dog
Hare
Large dog
Human
Sheep
Ox
Horse
10
10^2
10^3
10^4
10^5
10^6

... DRAW A LINE ...

LAST YEAR I GOT -.35





ORIGIN OF THIS BEHAVIOR?

$f \sim M^{-.35}$

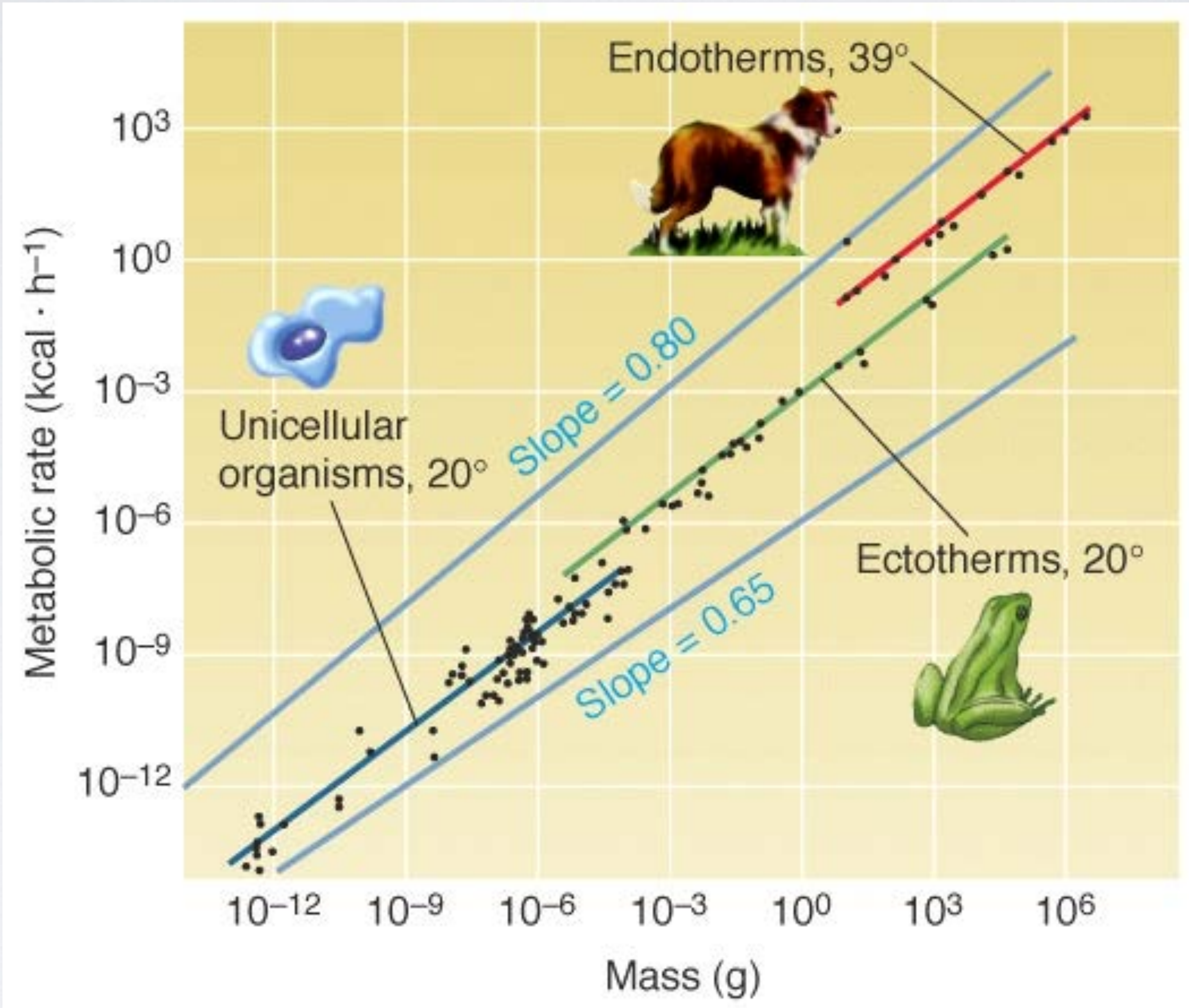
- The heart (attempts) to provide, in response to various stimuli, the flow rate of blood that is needed (at some instant) for all of a creatures needs
  - Flowrate to provide oxygen and other nutrients
  - To achieve this flowrate, "viscous losses" and gravity head must be overcome
- So the heart must simultaneously meet these two criteria

DIMENSIONAL REASONING

- flow rate times pressure gradient is "power"
- Flow rate will be a heart volume/time period
- Pressure gradient is caused by deceleration of "velocity squared"
- Heart power:
  - $(V_h{*}f)\ (\rho\ (V_h^{(1/3)}\ {*}f)^2 ==> \rho\ f^3 V_h^{5/3}$
- How does this power scale with animal size?

METABOLIC POWER(KLEIBER'S LAW)

$P \sim M^{.75}$



HEART RATE — MASS

- $\rho f^3 V_h^{5/3} \sim M^{.75}$
- Further $V_h \sim M$
  - http://www.biologyreference.com/Re-Se/Scaling.html
- Which gives…
  - $f \sim M^{-.31}$
- Interesting… I don't know how "correct" it is
- There are other allometric observations….

WHY DO BLOOD VESSELS BRANCH LIKE THIS?

TABLE II

VESSELS IN TABLE I GROUPED ACCORDING TO RANK

Vessel rank	Σr^2	Σr^3	Σr^4
	mm^2	mm^3	mm^4
0	2.2	3.4	5.1
1	3.8	1.9	0.94
2	4.0	1.2	0.36
3	8.6	0.54	0.043
4	20	0.51	0.013
5	140	1.5	0.021
6	200	1.8	0.019
7	650	2.4	0.0095
6′	380	5.5	0.10
5′	200	7.6	0.30
4′	120	6.3	0.37
3′	39	5.8	1.1
2′	25	19	14
1′	22	26	31
0′	9	27	81

The vessels of Table I have been grouped according to rank and the sums of r^2, r^3, and r^4 have been calculated for each rank.

The answer has to be that "nature" (millions of years of evolution) has performed optimization, to maximize fitness of organism, as constrained by physical laws.

Test of Murray's law

TABLE II

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0	2.2	3.4	5.1
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The vessels of Table I have been grouped according to rank and the sums of r^2, r^3, and r^4 have been calculated for each rank.

Murry revisited

On Connecting Large Vessels to Small

The Meaning of Murray's Law

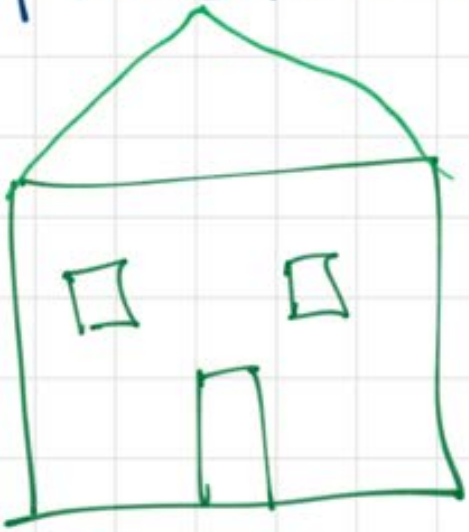
THOMAS F. SHERMAN

From the Department of Biology, Oberlin College, Oberlin, Ohio 44074

ABSTRACT A large part of the branching vasculature of the mammalian circulatory and respiratory systems obeys Murray's law, which states that the cube of the radius of a parent vessel equals the sum of the cubes of the radii of the daughters. Where this law is obeyed, a functional relationship exists between vessel radius and volumetric flow, average linear velocity of flow, velocity profile, vessel-wall shear stress, Reynolds number, and pressure gradient in individual vessels. In homogeneous, full-flow sets of vessels, a relation is also established between vessel radius and the conductance, resistance, and cross-sectional area of a full-flow set.

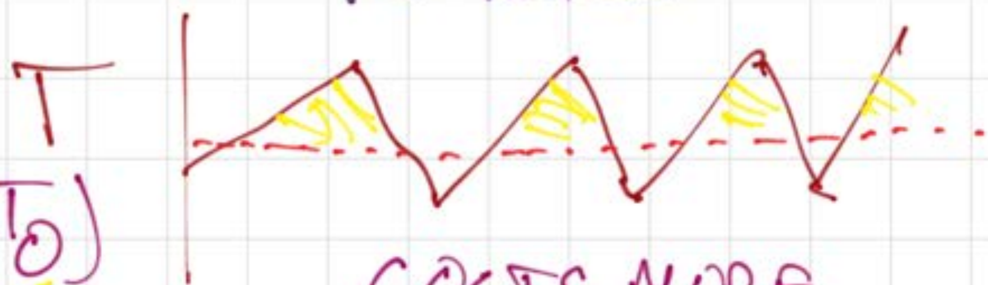
INTRODUCTION

HOW DO WE CHOOSE THE BEST OF MANY POSSIBLE DESIGNS?



MORE INSULATION
HIGHER CAPITAL COST
LOWER HEATING COST

MORE ADVANCED THERMOSTAT + FURNACE



$$\dot{Q} = hA(T - T_0)$$

EXTRA HEAT LOSS

COSTS MORE, LOWER ENERGY COST

CAR: MORE Al OR GRAPHITE COMPOSITE

CHEMICAL PROCESS: MORE TRAYS IN DISTILLATION COLUMN: LOWER REFLUX RATIO

A VERY GENERAL CHEMICAL ENGINEERING PRINCIPLE:

OPTIMIZATION:

- MAXIMIZE PROFITS
- MAXIMIZE CASH FLOW
- OFTEN, FOR A NEW PROCESS...

MINIMIZE TOTAL COSTS

IF SO:

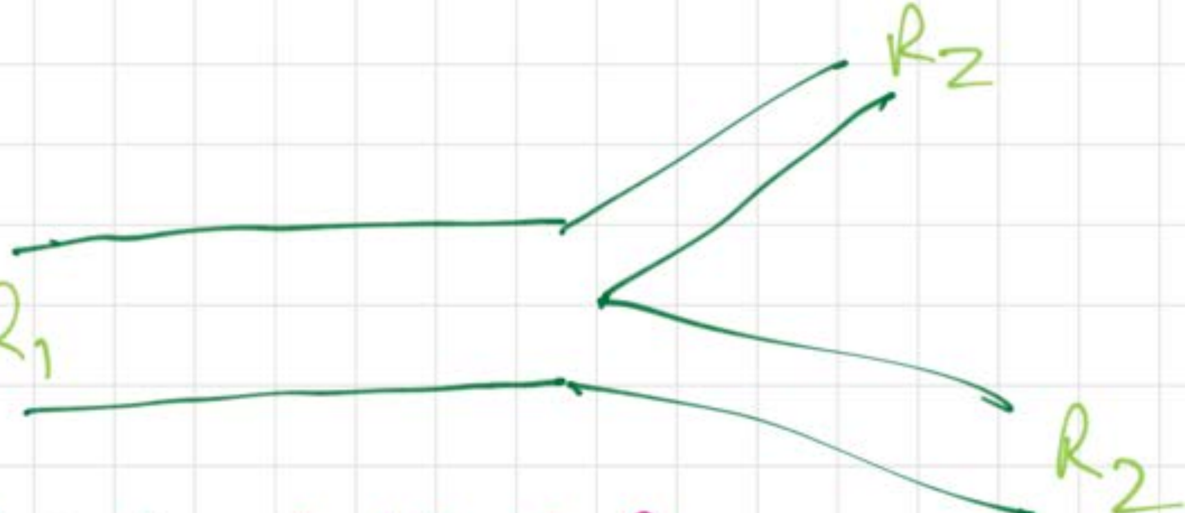
USUALLY A TRADEOFF BETWEEN OPERATING COSTS & CAPITAL COSTS

OR FOR A PHYSIOLOGICAL SITUATION

PUMPING COST

METABOLIC MAINTENANCE COST

SO LET'S SET UP
BLOOD FLOW OPTIMIZATION
WITH THE GOAL OF
DETERMINING HOW
BRANCHING OCCURS



WE NEED BEST R
FOR PRESCRIBED Q AT
EACH LEVEL.

BEST $\left[R_1(Q)\right]$ ALSO BEST $\left[R_2\left(\frac{Q}{2}\right)\right]$

"OBJECTIVE FUNCTION"

TOTAL COST = CAPITAL COST + OPERATING COST

"OPTIMUM" IS LOWEST TOTAL COST

FOR BLOOD FLOW WE COULD "PRESUME" THAT NATURE HAS APPROACHED "OPTIMUM" RATHER CLOSELY.

COST	=	COST OF PIPE	+	COST TO PUMP
TOTAL METABOLIC POWER REQUIRED FOR CIRCULATION SYSTEM	=	POWER TO MAINTAIN BLOOD + VESSELS	+	POWER NEEDED BY HEART TO PUMP

$$\text{TOTAL POWER} = \beta \pi R^2 L + \Delta P Q$$

"BEST" MINIMIZE:

$$TP = \Delta P\, Q + \beta \underbrace{\pi R^2 L}_{\text{VOLUME OF BLOOD VESSEL}}$$

LAMINAR FLOW:

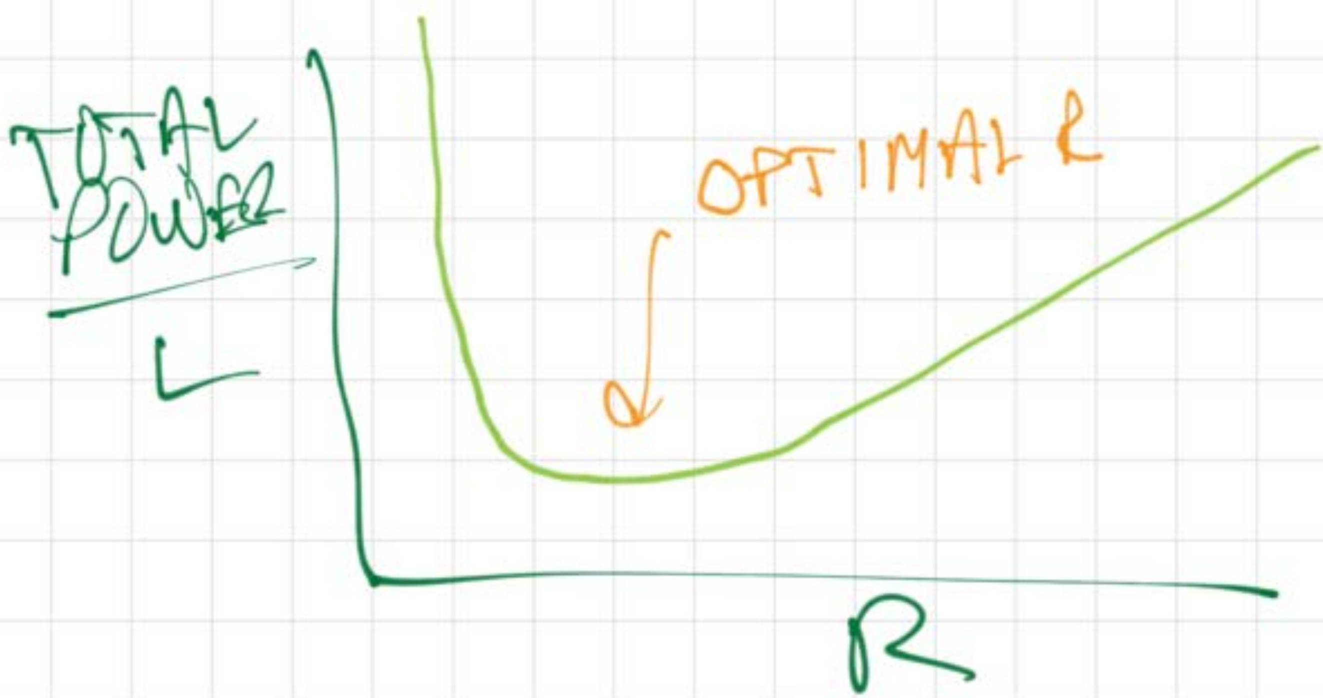
$$\Delta P = \frac{8 \mu Q L}{\pi R^4}$$

SUBSTITUTE...

$$\text{TOTAL POWER} = \frac{8\mu Q^2 L}{\pi R^4} + \beta \pi R^2 L$$

$$\frac{\text{TOTAL POWER}}{L} = \frac{8\mu Q^2}{\pi R^4} + \beta \pi R^2$$

$$= \frac{\alpha}{R^4} + \gamma R^2$$



$$0 = \frac{dT_P}{dR} = \frac{-32\mu Q^2}{R^5} + 2\beta\pi R$$

$$Q^2 = \frac{\beta R^6 \pi^2}{16\mu}$$

$$Q = \sqrt{\frac{\beta\pi^2}{16\mu}}\, R^3$$

FOR A SPECIFIED Q, THIS IS THE BEST R

IN TERMS OF VELOCITY

$$\pi R^2 V = \sqrt{\frac{\pi^2 B}{16\mu}} R^3$$

$$V = \sqrt{\frac{B}{16\mu}} R$$

NOW ANSWER THIS QUESTION AT A BRANCH

$$Q_1 = 2Q_2$$

BEST R_1

$$\sqrt{\frac{B\pi^2}{16\mu}}\,R_1^3 = 2\sqrt{\frac{B\pi^2}{16\mu}}\,R_2^3$$

$$R_1^3 = 2R_2^3$$

$$R_2 = \frac{R_1}{\sqrt[3]{2}}$$

TOTAL FLOW AREA INCREASES

MURRY'S LAW

WE WILL GET SAME SCALING RELATION IF WE KEEP WALL SHEAR STRESS CONSTANT ACROSS A BRANCH

$$R_1^3 = 2R_2^3$$

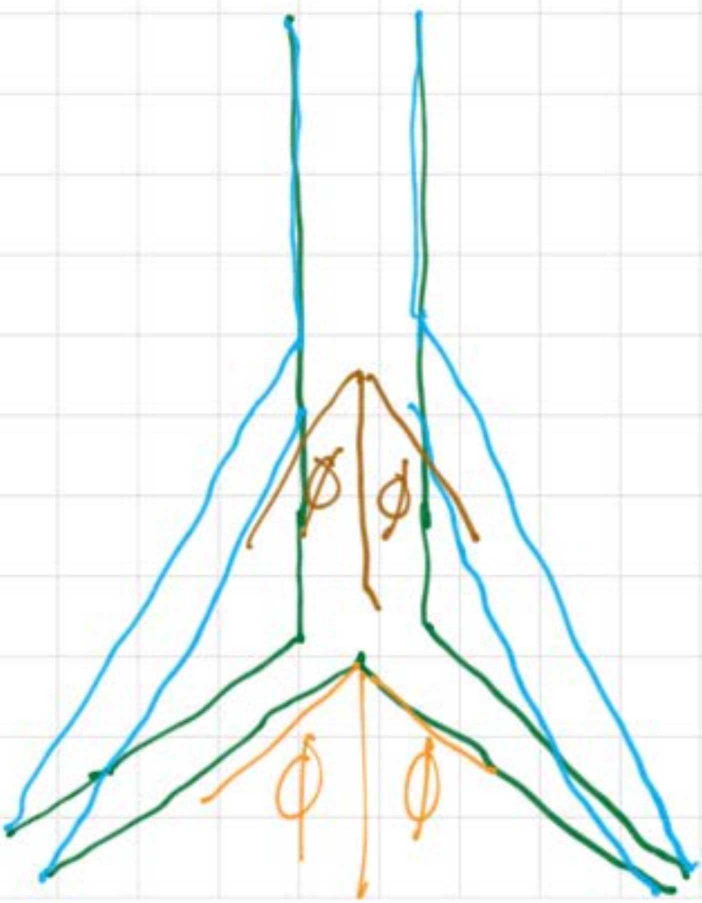
AS BRANCHES OCCUR, AREA FOR FLOW INCREASES, ... BLOOD SLOWS DOWN!

MURRY'S LAW

WOULD GET $R_1^3 = R_2^3 + R_3^3$

EVEN IF $R_2 \neq R_3$

CAN EXTEND OPTIMIZATION TO "WHERE" BRANCHING SHOULD OCCUR:



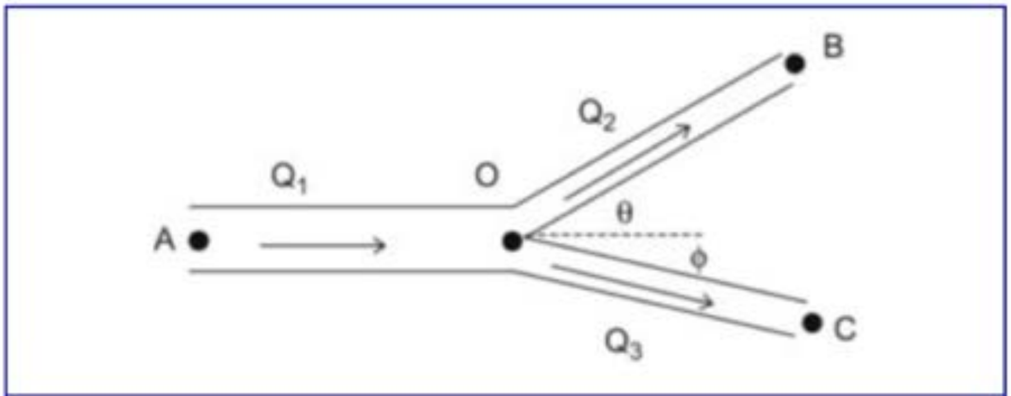
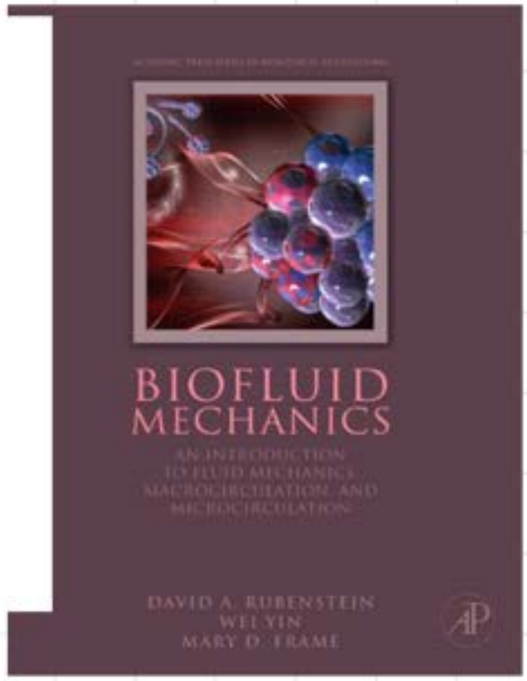


Figure 5.14 Flow rate at a bifurcation



$$r_1^3 = r_2^3 + r_3^3 \tag{5.26}$$

$$\cos\theta = \frac{r_1^4 + r_2^4 - r_3^4}{2r_1^2 r_2^2}$$

$$\cos\phi = \frac{r_1^4 - r_2^4 + r_3^4}{2r_1^2 r_3^2}$$

$$\cos(\theta + \phi) = \frac{r_1^4 - r_2^4 - r_3^4}{2r_2^2 r_3^2}$$

FOR AN EVEN SPLIT $\theta = \phi$

$\cos\theta = \frac{1}{\sqrt[3]{2}}$ $\quad \theta \approx \frac{\pi}{4.8}$

Murray:

THE PHYSIOLOGICAL PRINCIPLE OF MINIMUM WORK. I. THE VASCULAR SYSTEM AND THE COST OF BLOOD VOLUME

BY CECIL D. MURRAY

DEPARTMENT OF BIOLOGY, BRYN MAWR COLLEGE

Communicated January 26, 1926

Introduction.—Physiological organization, like gravitation, is a "stubborn fact," and it is one task of theoretical physiology to find quantitative laws which describe organization in its various aspects. Just as the laws of thermodynamics were known before the kinetic theory of gases was developed, so it is not impossible that some quantitative generalizations may be arrived at in physiology which are independent of the discrete mechanisms in living things, but which apply to organic systems considered statistically. One such generalization is the principle of the maintenance of steady states—a principle which furnishes definite equations (of the type indicating equality of intake and output of elementary substances) applicable to the hypothetical normal individual. The purpose of these studies is to discuss the possible application of a second principle, the principle of minimum work, to problems concerning the operation of physiological systems.

Classic Chemical Engineering: Pipe flow

- 1937

EQUIPMENT DESIGN

FLUID-FLOW DESIGN METHODS

R. P. GENEREAUX
E. I. du Pont de Nemours & Company, Inc.,
Wilmington, Del.

SINCE most chemical engineering plant designs require consideration of fluid transportation, a familiarity with the fundamental principles involved is of considerable value in obtaining suitable and economic results. In the following text certain fundamental principles are adopted for use in solving fluid-flow problems.

Calculation of Flow in Pipes

The most common problem is the determination of pipe size and pressure drop. Many of the publications cited in a bulletin of the National Research Council (*2*) describe

to 4000) in which flow changes from viscous to turbulent or vice versa, and above Re = about 4000 lies the turbulent region. Most plant flows are in the turbulent region, for which the theoretical relations are not so well known. Pipe wall roughness does affect the friction factor. The plot of f vs. Re data forms a relatively narrow band indicating a curve of negative slope, the slope decreasing as the Reynolds number increases. No such simple and accurate formula as that for viscous flow has been obtained. However, two methods are used which give adequate accuracy for design purposes.

Optimization

- Capital costs versus operating costs (1940)

ECONOMIC PIPE SIZE

IN THE TRANSPORTATION OF VISCOUS AND NONVISCOUS FLUIDS

B. R. SARCHET AND A. P. COLBURN
University of Delaware, Newark, Del.

The economic pipe size, for which the sum of pipe and pumping costs is a minimum, has been derived for both the turbulent and viscous regions of flow. The resulting equations are represented by convenient nomographs. By solving the optimum-diameter equations simultaneously with the critical Reynolds number, a convenient relation has been found to indicate whether any given flow will be turbulent or viscous in a pipe of optimum diameter. Although the optimum velocity of many liquids in turbulent flow runs from 3 to 4 feet per second, much lower optimum velocities are calculated for very viscous liquids.

exponential function of diameter. For example, for ordinary steel pipe in nominal sizes up to one-inch diameter the cost increases approximately as the first power of the diameter. For larger sizes the cost is closely proportional to the 1.5 power of diameter. The annual cost of a unit length of pipe may therefore be expressed generally as:

$$C_p = C_3 D^n \qquad (1)$$

The annual cost of pressure drop is evaluated by determining the cost of compressing gases or pumping liquids to overcome pressure drop. The cost is zero when not charged to the operation, such as in some cases when water is drawn from a pressure main. The product of flow rate and pressure drop is an exact expression of the work done in overcoming friction in the case of liquids but not in the case of gases. The percentage error in assuming this to be true in the case of gases is approximately half the percentage pressure drop[1], an error which is unimportant for the small pressure drops encountered

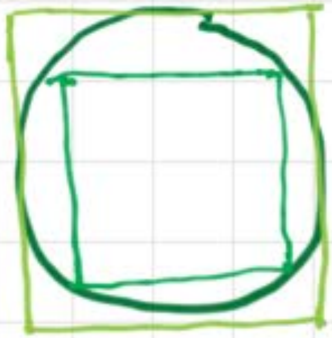
For a pressure driven flow, we could get an exact solution for an infinitely wide channel and a circular pipe.

This is nothing short of "great" --except if the channel is square!

So, what do we do?

IF $h/w < \sim .1$ WE ARE PROBABLY OK... ("HOW OK"?)

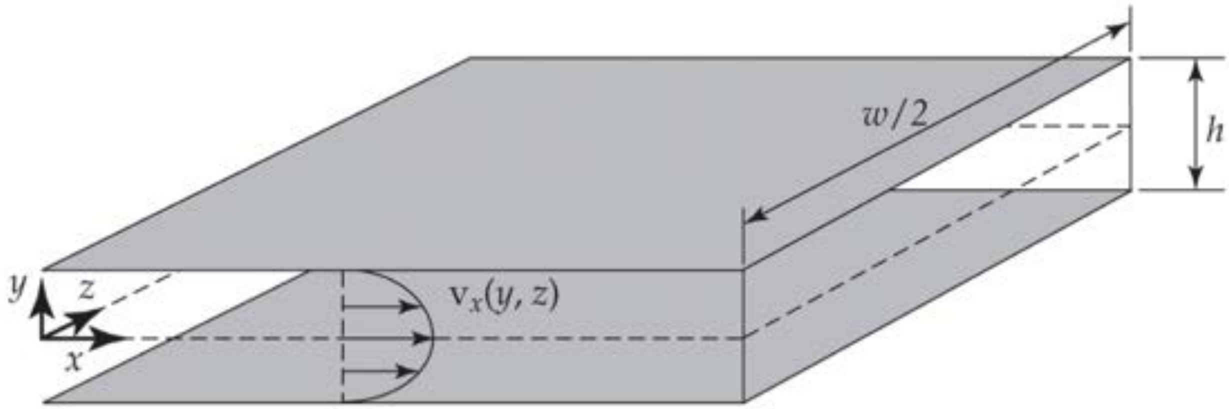
IF $h/w \approx 1$ WE EXPECT THAT INFINITELY WIDE CHANNEL IS NOT ACCURATE...



MAYBE WE CAN "BRACKET" THE ANSWER?

WE CAN DO BETTER THAN THIS!!

Figure 3.7 Flow in a rectangular channel of height h and width w. The section is through the plane $z = 0$.



FOR A STEADY, FULLY-DEVELOPED 2-D FLOW OF A NEWTONIAN FLUID . . . .

$$0 = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right)$$

POISSON'S EQ

SOLUTIONS ARE WELL KNOWN!!

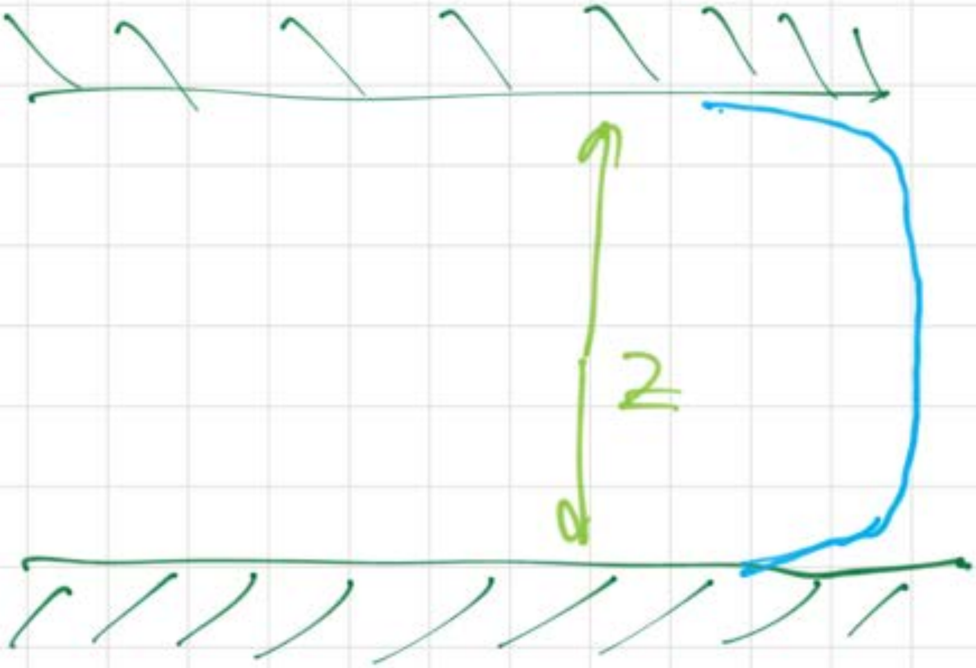
THIS EQUATION IS EASILY SOLVED BY SEPARATION OF VARIABLES

THE TEXT SHOWS THIS ON pp 138–139

EIGEN VALUE PROBLEM

and the solution for the velocity field is

$$v_x(y, z) = \frac{\Delta p h^2}{8\mu L}\left(1 - \frac{4y^2}{h^2}\right) - \frac{\Delta p h^2}{8\mu L}\sum_{n=0}^{\infty} \frac{32(-1)^n \cosh\big((2n+1)\pi z/h\big)\cos\big((2n+1)\pi y/h\big)}{(2n+1)^3\pi^3\cosh\big((2n+1)\pi w/2h\big)}.$$



SIMILAR SHAPE TO $V_x(y)$.

PRESSURE DROP OR

SHEAR STRESS ...

INTEGRATE

$$\tau_{yx} = \frac{\mu}{W}\int_{-W/2}^{W/2} \left.\frac{\partial v_x}{\partial y}\right|_{y=-h/2} dz$$

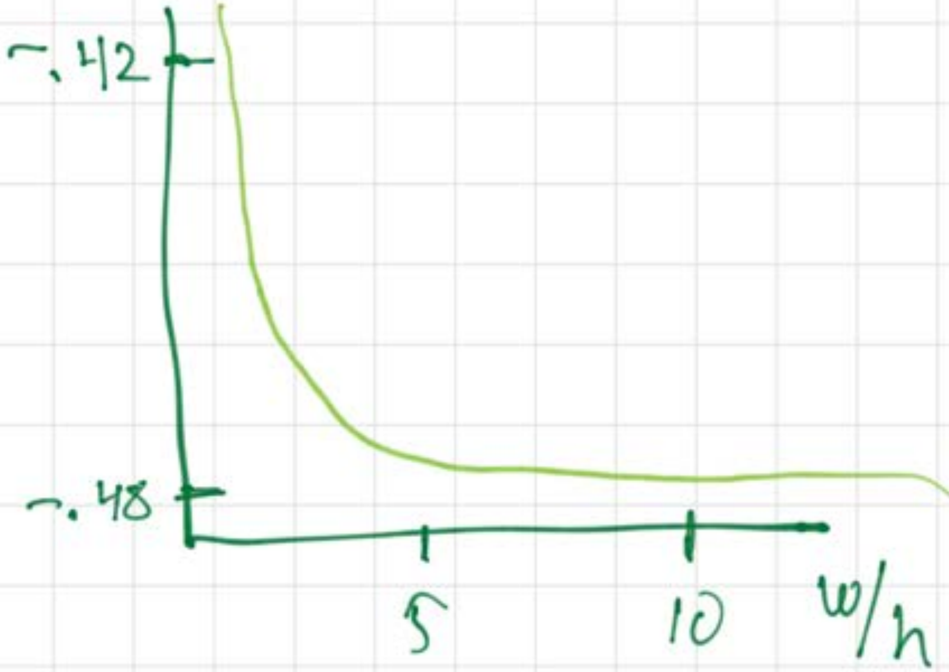
$$= \frac{\Delta P h}{2L}\left[1 - 16\frac{h}{W}\sum_{n=0}^{\infty}\frac{(-1)^n \tanh(2n+1)\pi\frac{W}{2h}}{(2n+1)^3\pi^3}\right]$$

ASPECT RATIO

$$\tau_{yx} \cong -\frac{dP}{dx}h\left(\frac{1}{2} - .25\,\varepsilon\right)$$

$$\varepsilon \equiv \frac{h}{W}$$

BOTTOM WALL, DOES NOT INCLUDE SIDES



SHOULD WE TAKE TIME TO LEARN HOW TO DO ANALYICAL SOLUTION???

- - - -

HOW ABOUT . . .

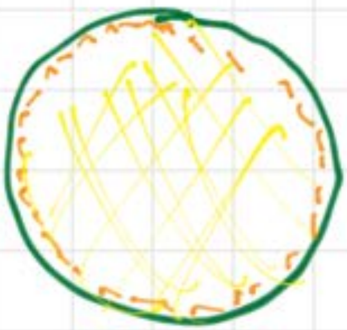
NOT THIS PDE.

A NUMERICAL SOLUTION IS JUST A FEW LINES OF CODE ... AND ANY SHAPE CAN BE DONE..

CAN WE DO SOMETHING ELSE ANALYTICALLY FOR NON-CIRCULAR CHANNELS, AND FOR PIPES THAT ARE NOT FULL AND ARE GRAVITY DRIVEN?

"HYDRALIC" DIAMETER

$$d_H \equiv \frac{4 \text{ CROSS SECTION AREA}}{\text{WETTED PERIMETER}}$$



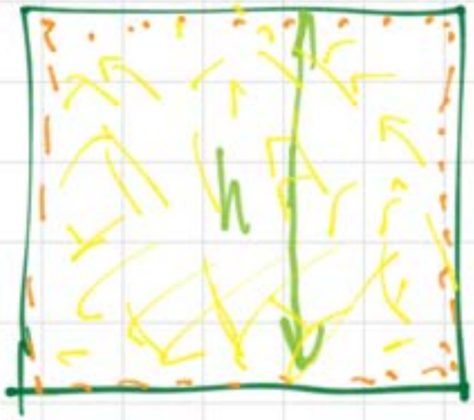
— $2\pi \cdot R$

πR^2

$$d_H = \frac{4 \pi R^2}{2\pi R} = 2R = D$$

SO THE "IDEA" WORKS FOR CIRCLE

SQUARE



$\cdots\ 4h$

$\cdots\ h^2$

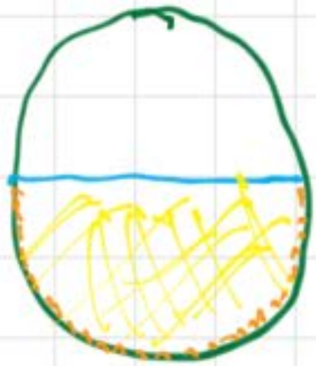
$$d_H = \frac{4h^2}{4h} = h$$

$$d_H = \frac{4wh}{2w + 2h}$$

THIS IS CURIOUS

As $\frac{h}{w} \to 0 \quad d_H \to 2h$

THE REASON THAT WE TALK ABOUT "WETED PERIMETER"



$$— = \frac{\pi D}{2}$$

$$— = \frac{\pi D^2}{8}$$

$$d_H = \frac{4\ \pi D^2/8}{\pi D/2} = D$$

THERE ARE OTHER CAVEATS FOR OPEN CHANNEL FLOWS...

WE CAN USE NUMERICAL SOLUTION TO CHECK UTILITY OF HYDRAULIC DIAMETER