CBE 30357

10/31/17

TOPICS

0) MOVIE CLIP THAT HAS (OBLIQUE) COURSE RELEVANCE

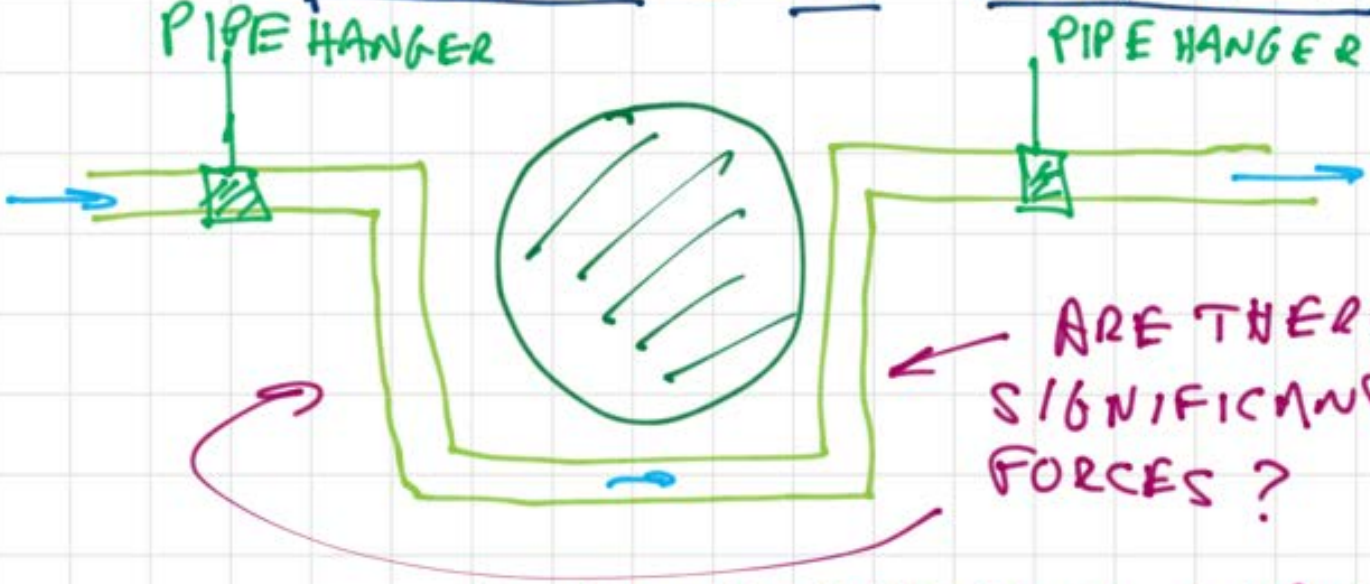
1) INTEGRATED FORM OF MASS BALANCE

$$\frac{dm}{dt} = \sum_{IN} \rho_i v_i A_i - \sum_{OUT} \rho_j v_j A_j$$

2) INTEGRATED MOMENTUM BALANCE

3) USE THESE TO ANALYZE BASIC FLOW SITUATIONS

PROBLEMS OF INTEREST



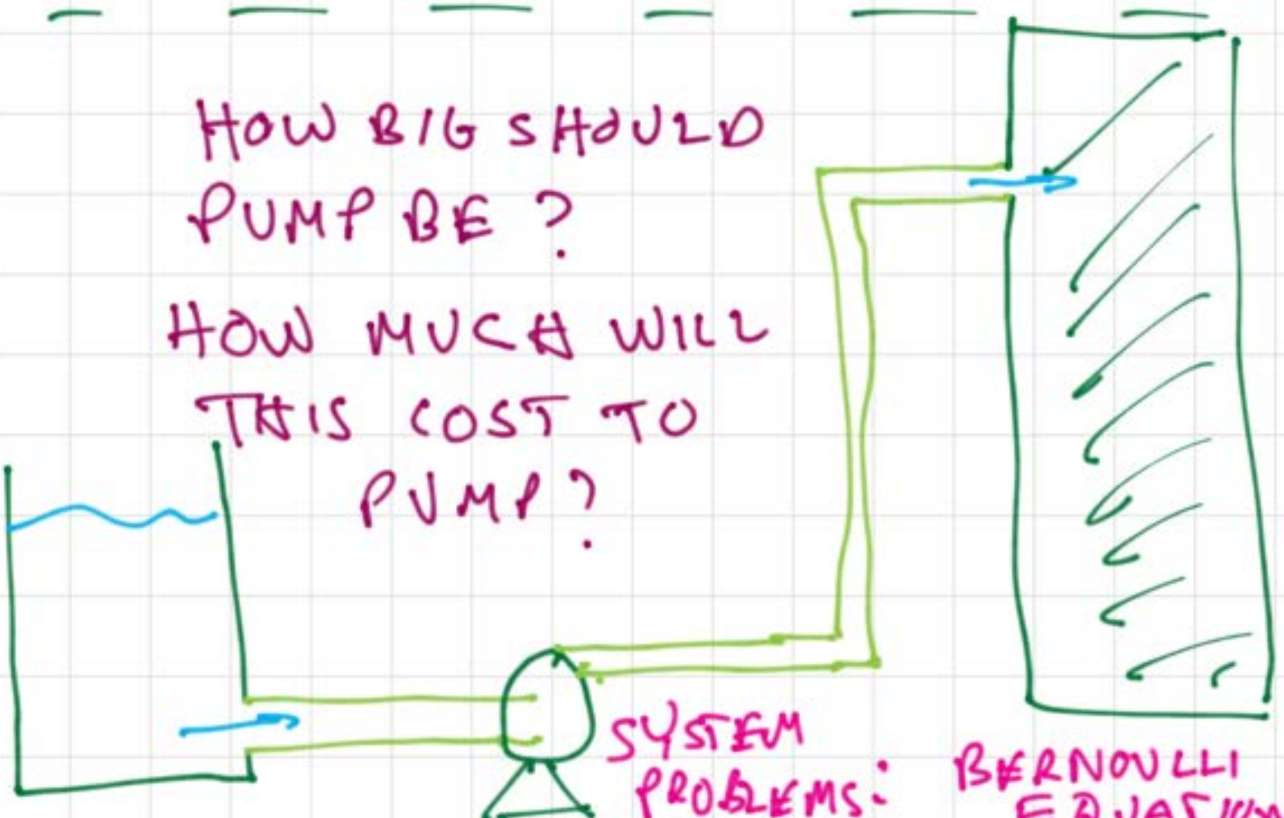
ARE THERE SIGNIFICANT FORCES?

DO YOU NEED ADDITIONAL SUPPORTS?

IF QUESION INVOLVES "FORCE" YOU WILL NEED MOMENTUM EQUATION! 🍕

HOW BIG SHOULD PUMP BE?

HOW MUCH WILL THIS COST TO PUMP?



SYSTEM PROBLEMS: BERNOULLI EQUATION

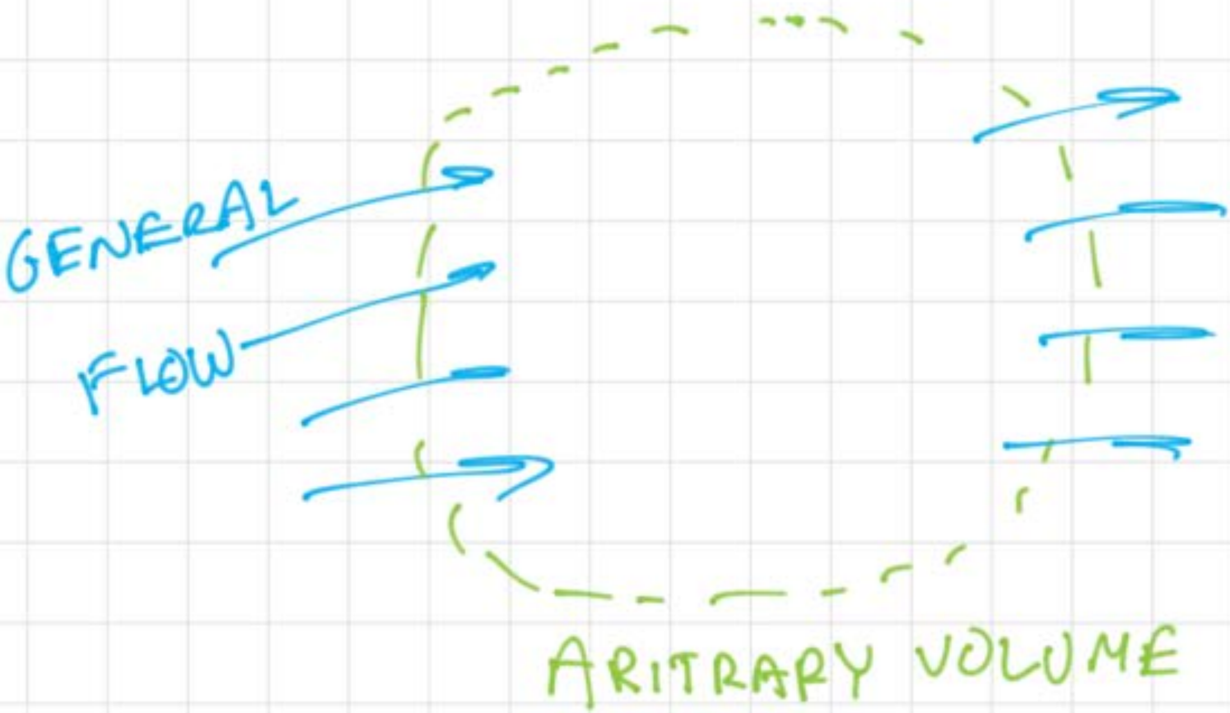
The scale of the problems does not lend itself to exact solutions:

** flow is turbulent
** geometry is complex
** may be happy with ~+/- 20%.

TO DO THIS, WE WILL USE
INTEGRATED FORMS OF
DIFFERENTIAL BALANCES

PROCEEDURE:

- PICK A GENERAL MACRO-SIZED CONTROL VOLUME
- CONSIDER AN ARBITRARY FLOW
- EMPLOY DIFFERENTIAL CONSERVATION EQUATION
- INTEGRATE TO GET GENERAL CONSERVATION EQUATION.



CONSERVATION OF MASS:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \rho \vec{v} = 0$$

THIS IS "TRUE" AT AN POINT SO IT MUST DESCRIBE ENTIRE VOLUME

INTEGRATE CONTINITY EQ. OVER VOLUME TO GET INTEGRAL VERSION.

CONTINUITY:

VOLUME INTEGRALS

$$\int_V \frac{\partial \rho}{\partial t} dV = \int_V -\bar{\nabla} \cdot \rho \bar{v} \, dV$$

TOTAL MASS IN V

LEFT SIDE

$$\int_V \frac{\partial \rho}{\partial t} dV = \frac{\partial}{\partial t} \int \rho \, dV = \frac{dm}{dt}$$

$$\int_V \underline{\bar{\nabla} \cdot \rho \bar{v}} \, dV = \int_S \rho (\bar{v} \cdot \bar{n}) \, dS$$

Local variation of mass

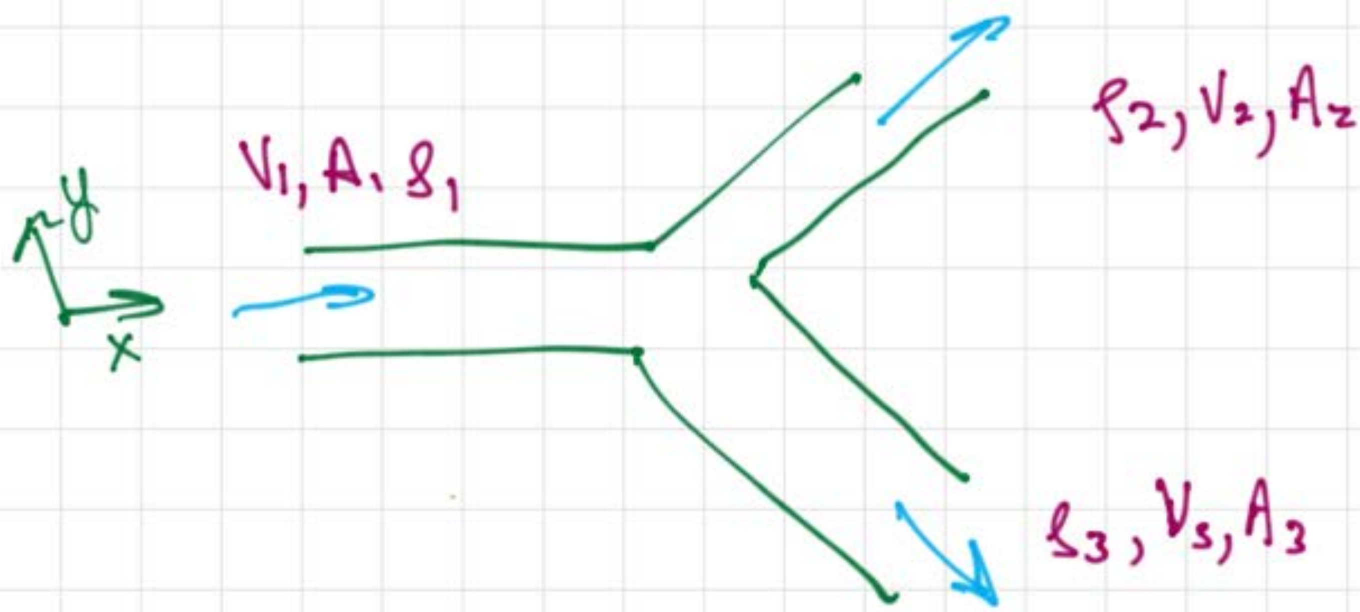
Integrate to get total variation.... however

We don’t know the details inside so we usually can’t do this integral: Use Divergence Theorem to convert volume integral to surface integral.

We now keep track of any changes inside by watching what goes in and out!

$$\therefore \quad \frac{dm}{dt} = -\int \rho(\bar{v}\cdot\bar{n})\,ds$$

LOOK AT A SIMPLE CASE
FOR WHICH WE KNOW ANSWER



PICK S.S. $\frac{dm}{dt} = 0$

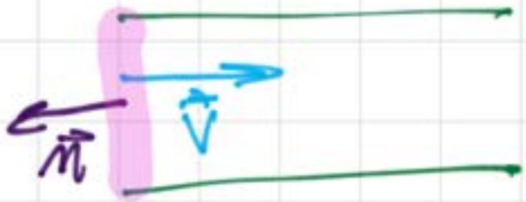
- INTEGRAL IS NON-ZERO ONLY
- AT OPENINGS WHERE $\vec{v}$ IS
- NON-ZERO

HOW TO EVALUATE THE INTEGRAL

INFLOW AT "1"...

$$-\int \rho(\bar{V}\cdot\bar{n})\,dS$$

AREA 1



A_1

OPPOSITE DIRECTION FROM $\bar{V}$

$$\bar{V}\cdot\bar{n} = -V_x \quad (-)$$

TAKE ρ = CONST

$$-\int_{A_1} \rho(\bar{V}\cdot\bar{n})\,dS = -\rho\int_{A_1} -V_x\,dS$$

$$= \rho\,\langle V\rangle_1 A_1$$

AVERAGE V_1

RATE OF MASS FLOW IN $= \rho\, Q_1$

DENSITY

VOLUMETRIC FLOW RATE

@ 2 $\vec{V} \cdot \bar{n}$ IS NOT JUST V_x OR V_y, BUT IT IS **ALL** OF THE 'V'

$$-\int_{A_2} \rho \underbrace{\vec{V} \cdot \vec{n}}_{\text{SAME DIRECTION}} ds = -\int_{A_2} \rho V_2 \, ds$$

NO "−" SIGN

$$= -\rho \langle V \rangle_2 A_2$$

$$= -\rho Q_2$$

@ 3 $$-\int_{A_3} \rho (\vec{V} \cdot \bar{n}) ds = \rho \langle V \rangle_3 A_3$$

$$= Q_3 A_3$$

$\therefore$ THE ANSWER MATCHES WHAT WE EXPECT

$$0 = \rho \langle V \rangle_1 A_1 - \rho \langle V \rangle_2 A_2 - \rho \langle V \rangle_3 A_3$$

$$\frac{dm}{dt} = \rho \sum_{\substack{i \\ \text{IN}}} \langle V \rangle_i A_i - \rho \sum_{\substack{j \\ \text{OUT}}} \langle V \rangle_j A_j$$

USEFUL IN THIS FORM.

FOR TURBULENT FLOWS IN PIPES & DUCTS, VELOCITY PROFILE IS USUALLY "FLAT" ENOUGH TO JUST WRITE "V_1" AS A SINGLE VALUE.

MOMENTUM CONSERVATION

MORE CARE AND EFFORT IS NEEDED !!

VECTOR EQUATIONS ...

$$\rho\left(\frac{\partial \bar{v}}{\partial t} + \bar{v}\cdot\bar{\nabla}\bar{v}\right) = -\bar{\nabla}p + \bar{\nabla}\cdot\bar{\bar{\tau}} + \rho\bar{g}$$

START THIS WITH THE SAME VOLUME INTEGRAL

$$\int_V \rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v}\right)dV =$$

$$\int_V \left(\vec{\nabla}p + \vec{\nabla}\cdot\vec{\vec{\tau}} + \rho\vec{g}\right)dV$$

SAME AS $\frac{d\vec{P}}{dt} = \sum \vec{F}$

We selectively choose either leaving a term as a volume integral or converting to a surface integral depending on convenience.

$$\int_V \rho \frac{\partial \bar{v}}{\partial t} dV = \frac{d}{dt} \int_V \rho \bar{v} \, dV$$

RATE OF CHANGE OF ALL MOMENTUM IN V

$$\int_V \rho (\bar{v} \cdot \bar{\nabla} \bar{v}) dV = \int_S \bar{v} \rho (\bar{n} \cdot \bar{v}) dS$$

VECTOR VERSION OF DIVERGENCE THEORM

ALLOWS TRACKING OF MOMENTUM IN AND OUT

$$\int_V \vec{\nabla} \cdot \bar{\bar{\tau}} \, dV = \int_S \underbrace{\vec{n} \cdot \bar{\bar{\tau}} \, dS}$$

This integral contains all of the viscous stress acting on the walls and the entrances and exits. We don't usually evaluate it. It is the net "force"

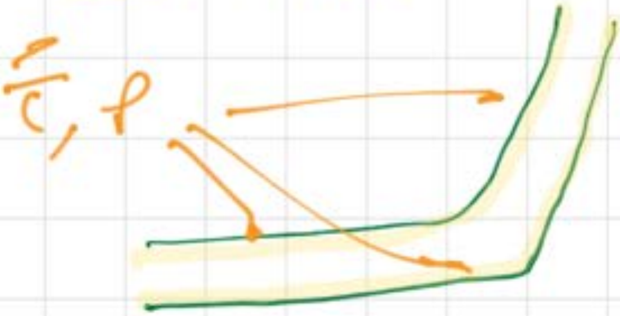
$$\int_V -\vec{\nabla} p \, dV = -\int_S p \, \vec{n} \, dS$$

This integral contains the pressure acting on all walls and flow openings.

WE WILL EVALUATE IT ONLY ON OPEN SURFACES!!

HOW TO INTERPRET p + $\bar{\tau}$ INTEGRALS

THEY REPRESENT FORCES ON SURFACE OF CONTROL VOLUME?



$$\int_{\text{SOLID SURFACES}} (\bar{n} \cdot \bar{\tau}) ds$$

$$- \int_{\text{SOLID SUFACE}} p \bar{n} ds$$

THIS DEFINES $\bar{F}$

$$\bar{F} \equiv - \int_{\text{SOLID SURFACE}} p \bar{n} ds + \int_{\text{SOLID SURFACE}} \bar{n} \cdot \bar{\tau} ds$$

$\bar{F}$ Is the total vector action of the pressure and shear on the solid surfaces of our pipe, device or control volume. We usually get this from the other terms. (Or we could do an experiment to measure it and use this to get some other term in the equation.)

GRAVITY INTEGRAL

$$\int_V \rho \bar{g}\, dV = m\bar{g}$$

THUS:

$$\frac{d}{dt}\left(\int \rho \bar{v}\, dV \right) + \int_S \bar{V}_S (\bar{n}\cdot\bar{v})\, dS =$$

$$-\int_{\text{OPEN SURFACE}} p\,\bar{n}\, dS + \int_{\text{OPEN SURFACE}} (\bar{n}\cdot\bar{\tau})\, dS + m\bar{g} + \bar{F}$$

ONLY RARELY WOULD THIS BE NEEDED

We need to consider how to best use these equations.
Need some body of experience.
Need simpler forms!

COMMENTS ABOUT FORCE TERMS

PRESSURE $\quad -\int_S p\,\bar{n}\,dS$

USUALLY WE KNOW PRESSURE ONLY AT OPEN SURFACES

SO I DON'T EVEN TRY!!

$$\int_S \bar{n}\cdot\bar{\tau}\,dS$$ = TOTAL FORCE ALONG WALLS

$= \bar{F}$

FORCE NEEDED TO HOLD FITTING IN PLACE

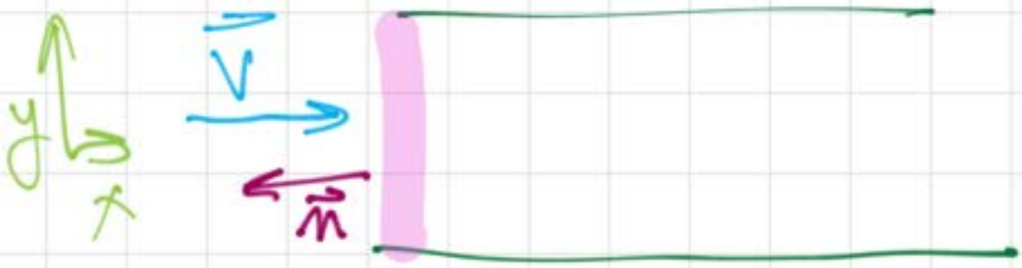
$$\int \rho \vec{g}\, dV = m\vec{g}$$

GRAVITY FORCE ON ENTIRE CONTROL VOLUME

WE WILL CONSIDER SOME EXAMPLE SITUATIONS

$$\int_S \underbrace{\rho \vec{v}}_{\text{MOMENTUM / VOLUME}} \underbrace{(\vec{v}\cdot\vec{n})}_{\text{ACTUAL INFLOW OR OUTFLOW VELOCITY (SCALAR)}} dS$$

$$= -\rho \langle v_x v_x \rangle_1 A_1 + \rho \langle v_x v_x \rangle_2 A_2$$

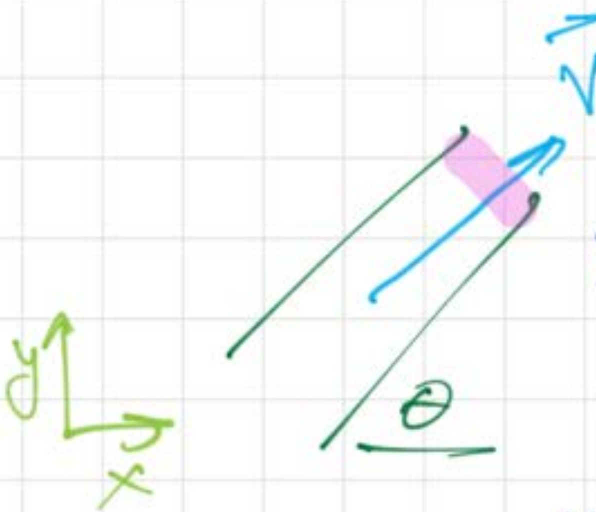


$$\int \rho\vec{V}\,(\vec{V}\cdot\vec{n})\,dS = -\int \rho\vec{V}\;V_x\,dS$$

$\vec{V}$ IS ALSO ONLY V_x $\therefore$

$$-\int \rho\;V_xV_x\,dS = -\rho\langle V_xV_x\rangle_1 A_1$$

OUTFLOW AT AN ANGLE IS DIFFERENT



$$x \Rightarrow \int \rho\,|\vec{V}|\cos\theta\,(\vec{V}\cdot\vec{n})\,dS$$

$$= \rho V_x Q = \rho|V|\cos\theta\;|V|\,A$$

$$y \Rightarrow \int \rho|\vec{V}|\sin\theta\,(\vec{V}\cdot\vec{n})\,dS$$

$$= \rho V_y Q = \rho|V|\sin\theta\;|V|A$$

EXAMPLE FLOW:

$\rho V_1 A_1 = \rho V_2 A_2$

$$x \quad 0 = -\rho \langle V_x V_x \rangle_1 A_1 + \rho \underbrace{V_x}\ \underbrace{|V|_2 A_2}_{,,Q}$$

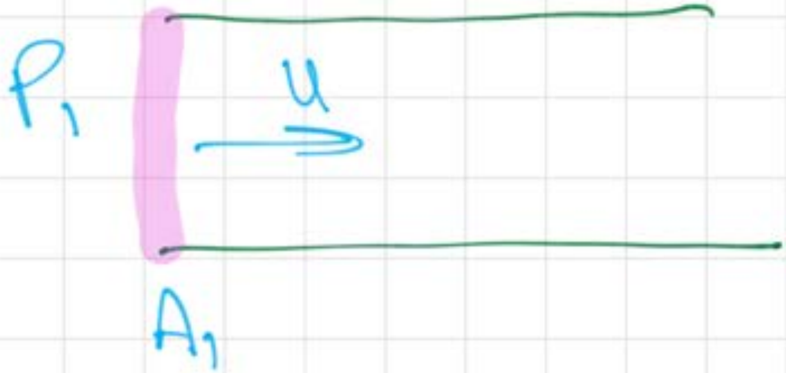
$$= -\rho V_1^2 A_1 + \rho \underbrace{V_2 \cos\theta}\ \underbrace{V_2 A_2}$$

$$y \quad 0 = 0 + \rho \underbrace{V_y}\ \underbrace{|V|_2 A}_{=Q}$$

$$= 0 + \rho \underbrace{V_2 \sin\theta}\ \underbrace{V_2 A_2}$$

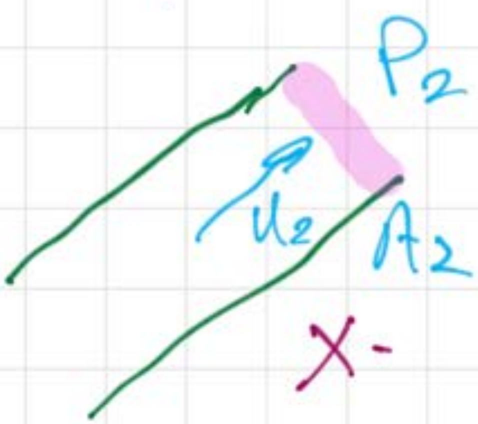
JUST THE FLOW TERMS ARE SHOWN!!

NOW GET PRESSURE TERMS...



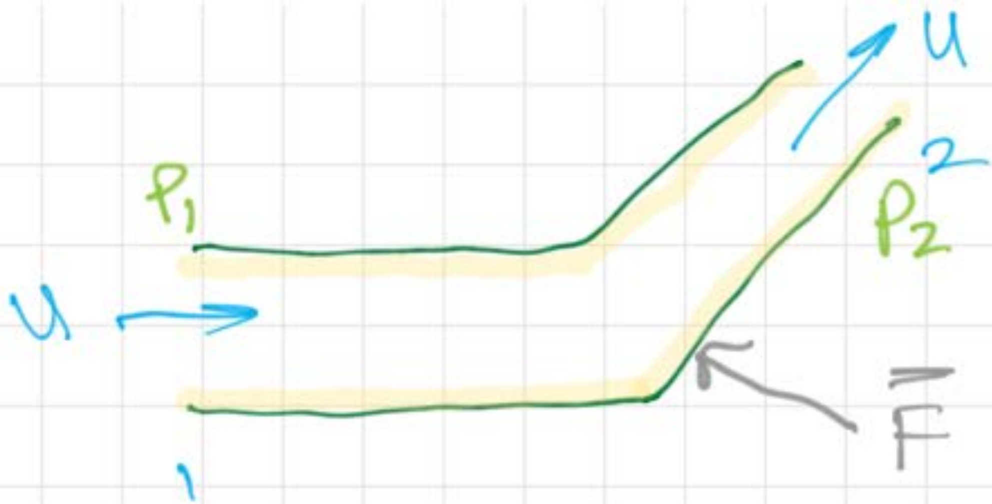
$$x - \quad -\int_{A_1} p\,\bar{n}\,ds = P_1 A_1$$

($\vec{n}$ POINTS IN − DIRECTION)



$$x - \quad -\int_{A} p\,\bar{n}\,ds = -P_2 \cos\theta\, A_2$$

$$y - \quad -\int_{A} p\,\bar{n}\,ds = -P_2 \sin\theta\, A_2$$



$$\int (\bar{n} \cdot \bar{\tau})\, ds = F_x \hat{i} + F_y \hat{j}$$

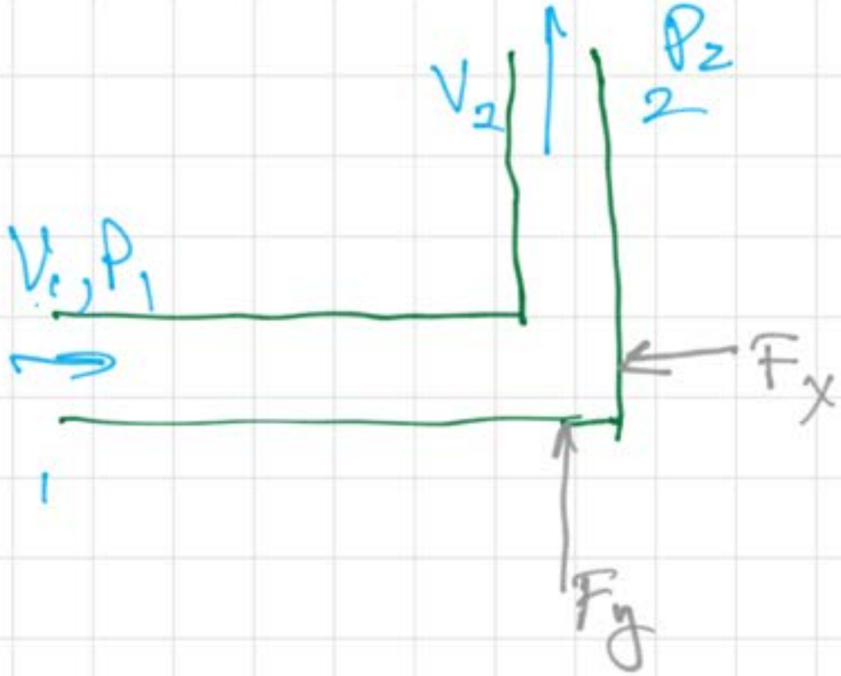
COMBINATION OF PRESSURE AND SHEAR STRESS USUALLY CAN'T EVALUATE INTEGRALS EXACTLY

SO: COMPLETE BALANCE

$$x \sim \quad -\rho V_1^2 A_1 + \rho V_2^2 \sin\theta A_2 = P_1 A_1 - P_2 A_2 \cos\theta + F_x$$

$$y \sim \quad 0 + \rho V_2^2 \sin\theta A_2 = -P_2 A_2 \sin\theta + F_y$$

A USUAL QUESTION IS "FIND $\bar{F}$"



90°
BEND

$$x \quad -\rho V_1^2 A_1 = P_1 A_1 + F_x$$

$$F_x = \underbrace{-P_1 A_1 - \rho V_1^2 A_1}$$

FORCE MATCHES PRESSURE
AND DEFLECTS ALL OF THE
FLOW

$$y \quad +\rho V_2^2 A_2 = -P_2 A_2 + F_y$$

$$F_y = P_2 A_2 + \rho V_2^2 A_2$$

SIMPLIFIED FORM

Momentum equations for single flow inlet in + x direction.

$$-\rho\langle V_x V_x\rangle A_1 + \rho\langle V_2 V_2\rangle A_2\cos(\theta) = P_1 A_1 - P_2 A_2\cos(\theta) + F_x$$

$$+\rho\langle V_2 V_2\rangle A_2\sin(\theta) = -P_2 A_2\sin(\theta) + F_y$$

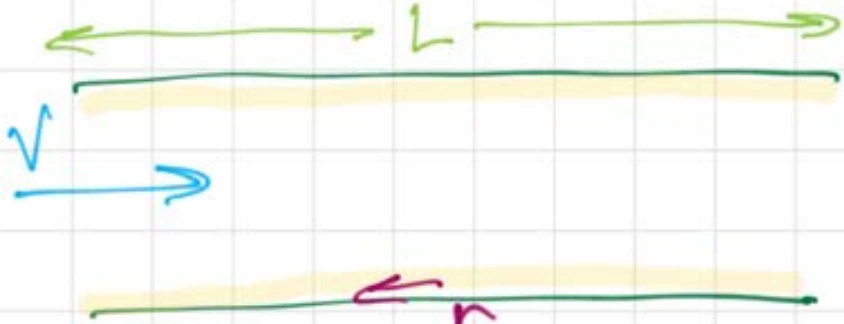
PIPE FLOW: FORCE ON PIPE = ?

$$\underbrace{-\rho V_1 V_1 A_1 + \rho V_1 V_1 A_1}_{0} = P_1 A_1 - P_2 A_2 + F_x$$

$$F_x = -(P_1 - P_2)A$$

FORCE ON PIPE TO HOLD IN PLACE

ALSO FORCE ON FLUID



WE ALREADY KNOW:

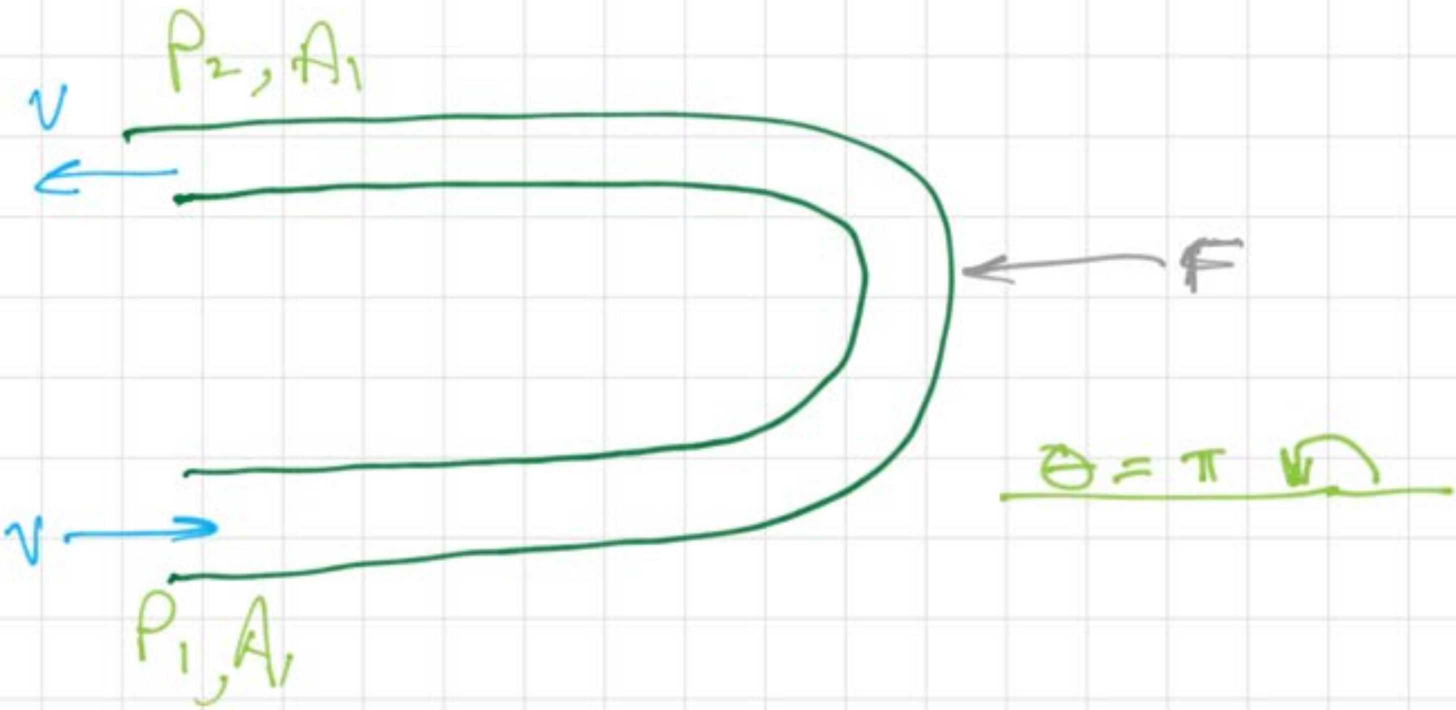
$\tau_W 2\pi R L = \int \vec{n} \cdot \vec{\tau} ds$

$$0 = P_1 A_1 - P_2 A_1 + F_x$$
$$= (P_1 - P_2) A + \tau_W 2\pi R L$$

$$\boxed{\tau_W = \frac{\Delta P}{L}\frac{R}{2}}$$

$\Delta P \equiv P_2 - P_1$

DEPENDS ON ΔP, NOT ANY SPECIFIC DETAIL OF FLOW



Momentum equations for single flow inlet in + x direction.

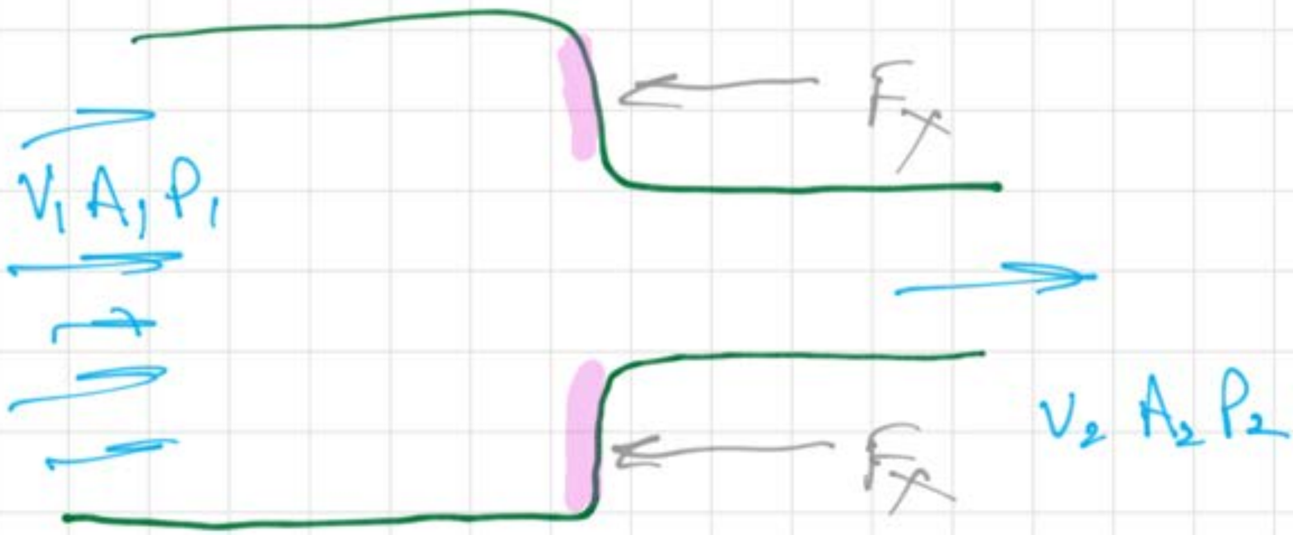
$$-\rho\left\langle V_x V_x \right\rangle A_1 + \rho\left\langle V_2 V_2 \right\rangle A_2 \cos(\theta) = P_1 A_1 - P_2 A_2 \cos(\theta) + F_x$$

$$+\rho\left\langle V_2 V_2 \right\rangle A_2 \sin(\theta) = -P_2 A_2 \sin(\theta) + F_y$$

$$-\rho V_1 V_1 A_1 + \rho V_1 V_1 A_1 \cos\pi = P_1 A_1 - P_2 A_1 \cos\pi + F_x$$

$$F_x = -2\rho V_1^2 A_1 - A_1 (P_1 + P_2)$$

AREA CHANGE



$$0 = \rho_1 A_1 V_1 - \rho_2 A_2 V_2 \qquad \rho_1 = \rho_2$$

$$V_2 = V_1 \frac{A_1}{A_2}$$

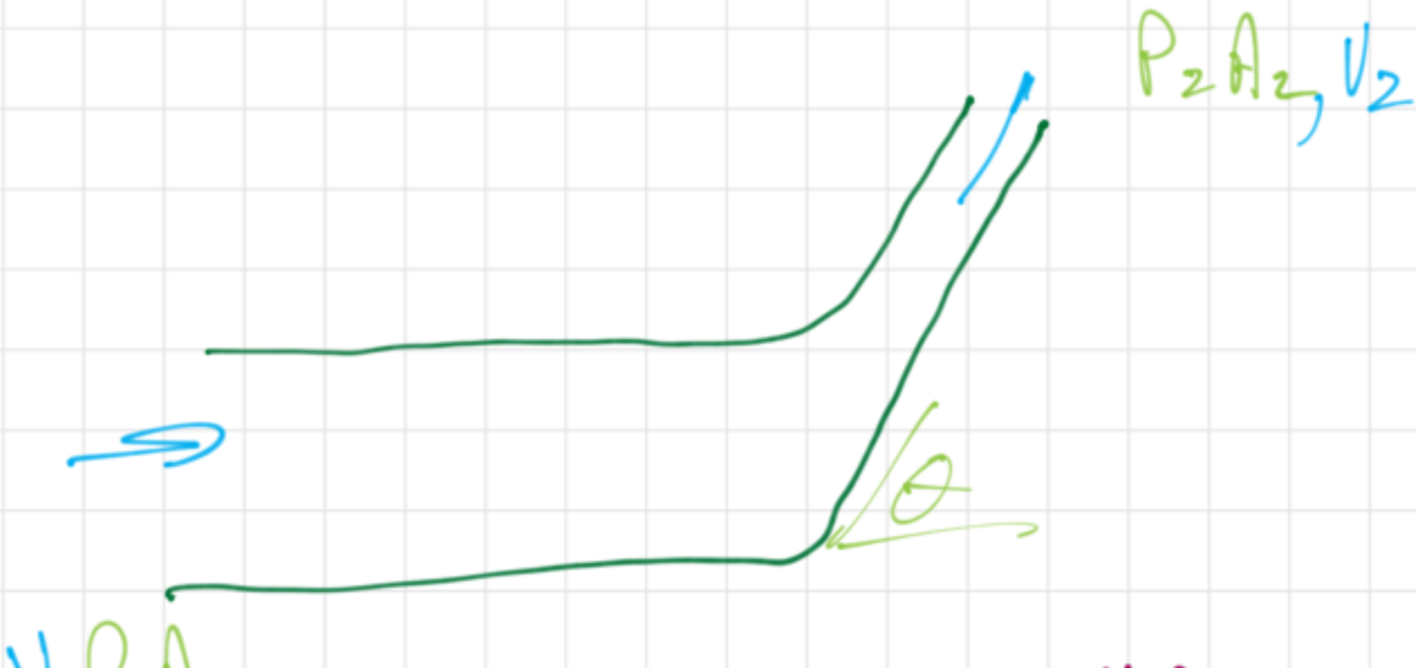
SPEEDS UP.

$$-\rho V_1 V_1 A_2 + \rho V_2 V_2 A_2 = -P_1 A_1 + P_2 A_2 + F_x$$

$$-F_x = \rho V_1^2 A_1 - \rho V_2^2 A_2 - P_1 A_1 + P_2 A_2$$

$$= \rho V_1^2 A_1 \left(1 - \frac{A_1}{A_2}\right) - P_1 A_1 + P_2 A_2$$

I MIGHT BE ABLE TO GET P_2 FROM A CORRELATION FOR LOSSES (...TO COME...)



MASS: $0 = \rho_1 V_1 A_1 - \rho_2 V_2 A_2$

$$V_2 = V_1 \frac{A_1}{A_2}$$

$$x \quad -\rho V_1 V_1 A_1 + \rho V_2 \cos\theta \, V_2 A_2 = P_1 A_1 - P_2 A_2 \cos\theta + F_x$$

$$y \quad 0 \quad + \rho V_2 \sin\theta \, V_2 A_2 = 0 - P_2 A_2 \sin\theta + F_y$$