CBE 30357

10/26/17

TOPICS

1) REVIEW OF FLOW PAST A SPHERE SOLUTION AND CONSEQUENCES

G.I. TAYLOR VIDEO: 17:42

- PDE SOLVED BY SEPARATION OF VARIABLES USING A SOLUTION FORM SUGGESTED BY FLOW AT BOUNDARY
- $f(r)$ FROM "EULER EQ"
- FLOW DISTURBANCE DECAYS SLOWLY $\sim \frac{1}{r}$

2) SOME ADDITIONAL IMPLICATIONS...

TERMINAL SETTLING VELOCITY

VISCOSITY OF A SUSPENSION OF PARTICLES

DIFFUSIVITY OF SMALL PARTICLES AND MOLECULES

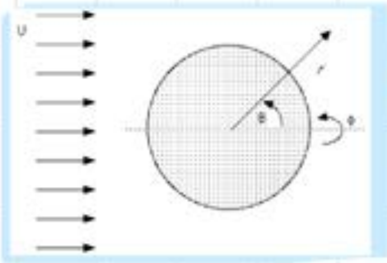
3) LIMIT IF $Re >> 0$

VISCOSITY WOULD DROP OUT OF EQUATIONS...

LESS "CORRECT"

NOT AS USEFUL

4) WHAT TO DO WHEN WE CAN'T SOLVE FLOWS IN EXACT DETAIL



$$\left\{ \begin{array}{l} \vec{\nabla}\cdot\vec{v}=0 \\ \vec{\nabla}p=\mu\nabla^2\vec{v} \end{array} \right\}$$

USE SPHERICAL COORDINATES
- CONTINUITY EQ.
- r-DIRECTION N.S. EQ } $Re\to 0$
- θ-DIRECTION N.S. EQ

SEQUENTIALLY CONSIDER 3 PDE'S USING AN ASSUMED FORM OF SOLUTION

$$\left.\begin{array}{l} v_r \sim f(r)\cos\theta \\ v_\theta \sim g(r)\sin\theta \end{array}\right\} \text{FROM B.C.'S}$$

THESE FORMS FOR V_r & V_θ WILL CAUSE TRIG FUNCTIONS TO CANCEL

EULER EQ:

NOTE: SAME SAME SAME SAME

$$r^4 f^{IV}(r) + 8r^3 f'''(r) + 8r^2 f''(r) - 8r f'(r) = 0$$

$$\therefore f(r) \sim r^\alpha$$

SUBS AND GET A POLYNOMIAL

$$f(r) = \frac{C_1}{3r^2} + \frac{C_2}{r} + \frac{1}{2}r^2 C_3 + C_4$$

THIS GIVES:

$$v_r = U\cos\theta\left(1 - \frac{3}{2}\frac{R}{r} + 2\frac{R^3}{r^3}\right)$$

$$v_\theta = -U\sin\theta\left(1 - \frac{3}{4}\frac{R}{r} + \frac{R^3}{4r^3}\right)$$

FASTEST $\bar{v}$ AS $r \to \infty$

NO ACCELERATION OF FLUID CAUSED BY DEFLECTION PAST SPHERE

INTEGRATE:

"SKIN DRAG" SHEAR STRESS, $\tau_{r\theta}\big|_R$

"FORM DRAG" PRESSURE $P\big|_R$

AROUND SPHERE TO GET DRAG

DRAG $= 6\pi\mu R U$

WE GET: SETTLING VELOCITY

$\Sigma F =$ GRAVITY + BUOYANCY + DRAG

$$\Sigma F = \rho g V \;-\; \rho_f g V \;-\; 6\pi\mu U R$$

$$= \rho g \tfrac{4}{3}\pi R^3 - \rho_f g \tfrac{4}{3}\pi R^3 - 6\pi\mu U R$$

$$6\pi\mu U R = \tfrac{4}{3}(\rho-\rho_f)\, g\, \pi R^3$$

$$U = \frac{2}{9}\frac{(\rho-\rho_f)\, g\, R^2}{\mu}$$

MATCHES EXPERIMENTS UP TO $Re \sim .8$.

MATCHES

STOKES

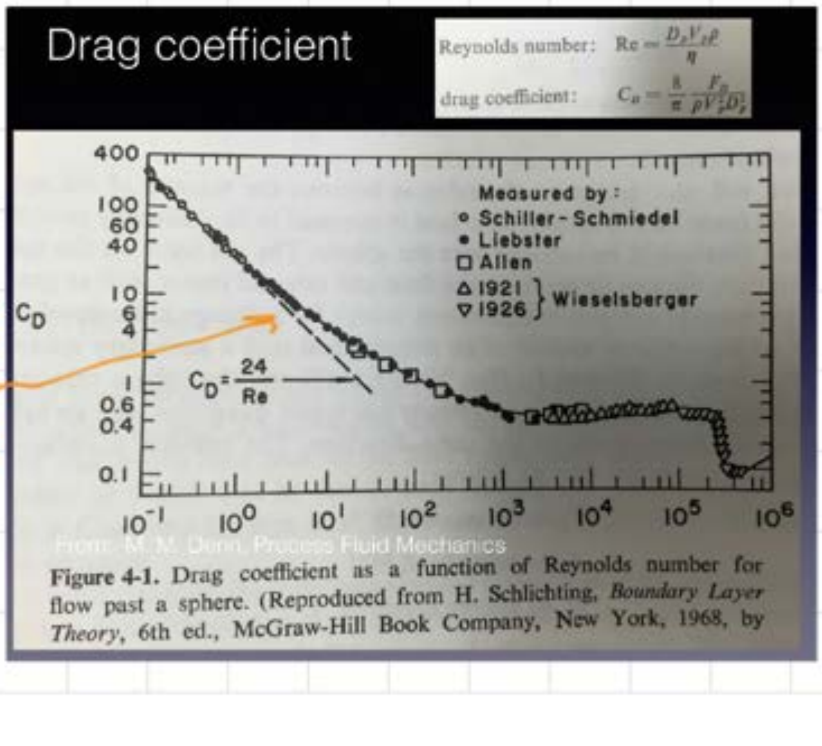


Figure 4-1. Drag coefficient as a function of Reynolds number for flow past a sphere. (Reproduced from H. Schlichting, *Boundary Layer Theory*, 6th ed., McGraw-Hill Book Company, New York, 1968, by

... A FEW MINUTES WITH G.I. TAYLOR...

VISCOSITY OF A DILUTE ($\phi < .1$) SUSPENSION OF SPHERES

SUSPENSION

$$\frac{\mu_s}{\mu_f} = 1 + \phi\left(\frac{\mu_f + \frac{5}{2}\mu_p}{\mu_f + \mu_p}\right)$$

FLUID

VOLUME FRACTION OF PARTICLES

VISCOSITY OF PARTICLES

$\mu_p \to \infty$ FOR SOLID

$$\frac{\mu_s}{\mu_f} = 1 + \frac{5}{2}\phi$$

(ALSO DUE TO EINSTEIN)

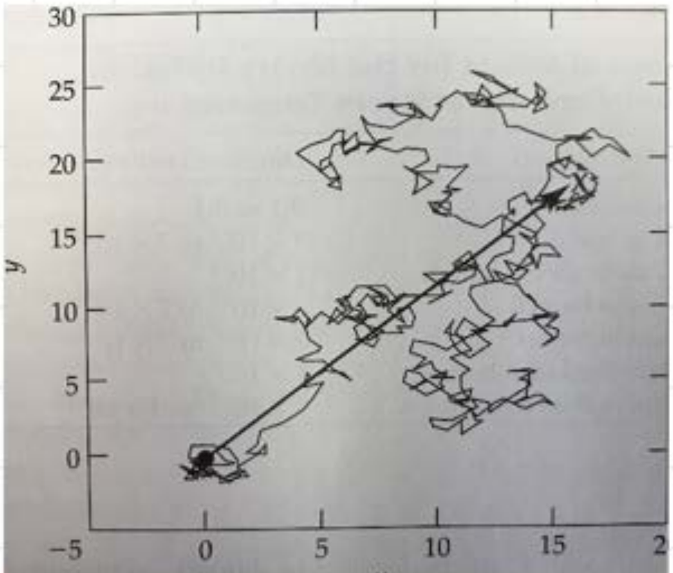
EITHER DROPS OR BUBBLES WILL SIGNIFICANTLY INCREASE VISCOSITY

DRAG ON A BUBBLE:

$$F_D = 4\pi \mu R U$$

ALMOST AS LARGE AS A SOLID PARTICLE !!

VISCOSITY OF LIQUID IN LIQUID SUSPENSION WILL MOST LIKELY BE HIGHER THAN EITHER COMPONENT



DIFFUSIVITY

EINSTEIN USED DRAG TO CALCULATE DIFFUSIVITY OF A PARTICLE

$$D = \frac{kT}{6\pi\mu R}$$

PARTICLE DIFFUSIVITY

BOLTZMANN CONSTANT

PARTICLE RADIUS

TABLE 1.1

Range of Values for the Binary Diffusion Coefficient, D_{ij}, at Room Temperature

Diffusing quantity	Diffusion coefficients ($cm^2\ s^{-1}$)
Gases in gases	0.1 to 0.5
Gases in liquids	1×10^{-7} to 7×10^{-5}
Small molecules in liquids	1×10^{-5}
Proteins in liquids	1×10^{-7} to 7×10^{-7}
Proteins in tissues	1×10^{-7} to 7×10^{-10}
Lipids in lipid membranes	1×10^{-9}
Proteins in lipid membranes	1×10^{-10} to 1×10^{-12}

DRAG ON SPHERE, TERMINAL VELOCITY IF Re > 1

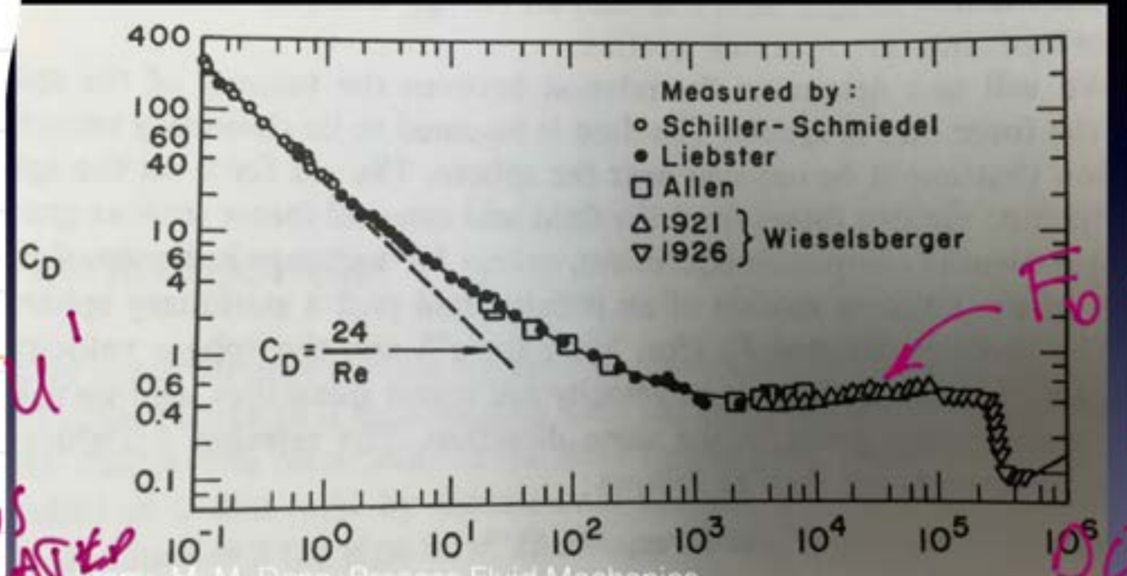
Drag coefficient

Reynolds number: $\mathrm{Re} = \frac{D_p V_p \rho}{\eta}$

drag coefficient: $C_D = \frac{8}{\pi} \frac{F_D}{\rho V_p^2 D_p^2}$

$F_D \sim u^1$

VISCOUS DOMINATED

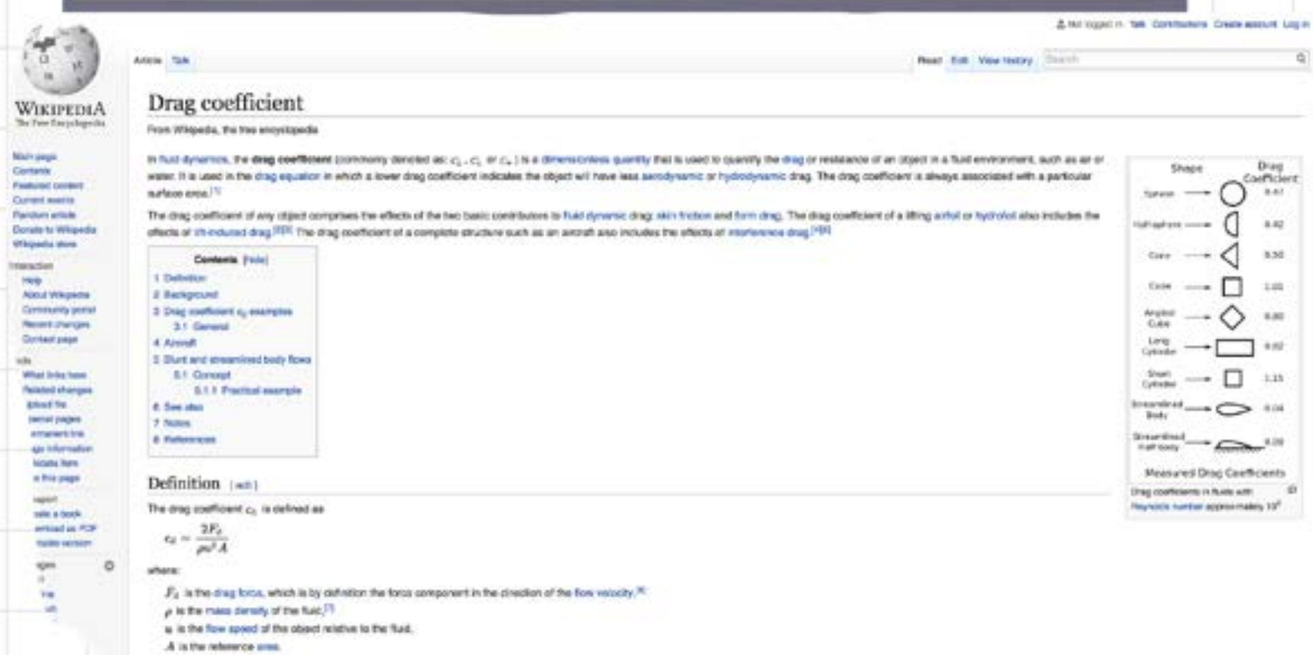


$F_D \sim u^2$

INERTIA DOMINATED

From: M. M. Denn, Process Fluid Mechanics

Figure 4-1. Drag coefficient as a function of Reynolds number for flow past a sphere. (Reproduced from H. Schlichting, *Boundary Layer Theory*, 6th ed., McGraw-Hill Book Company, New York, 1968, by



OUR LUNGS...

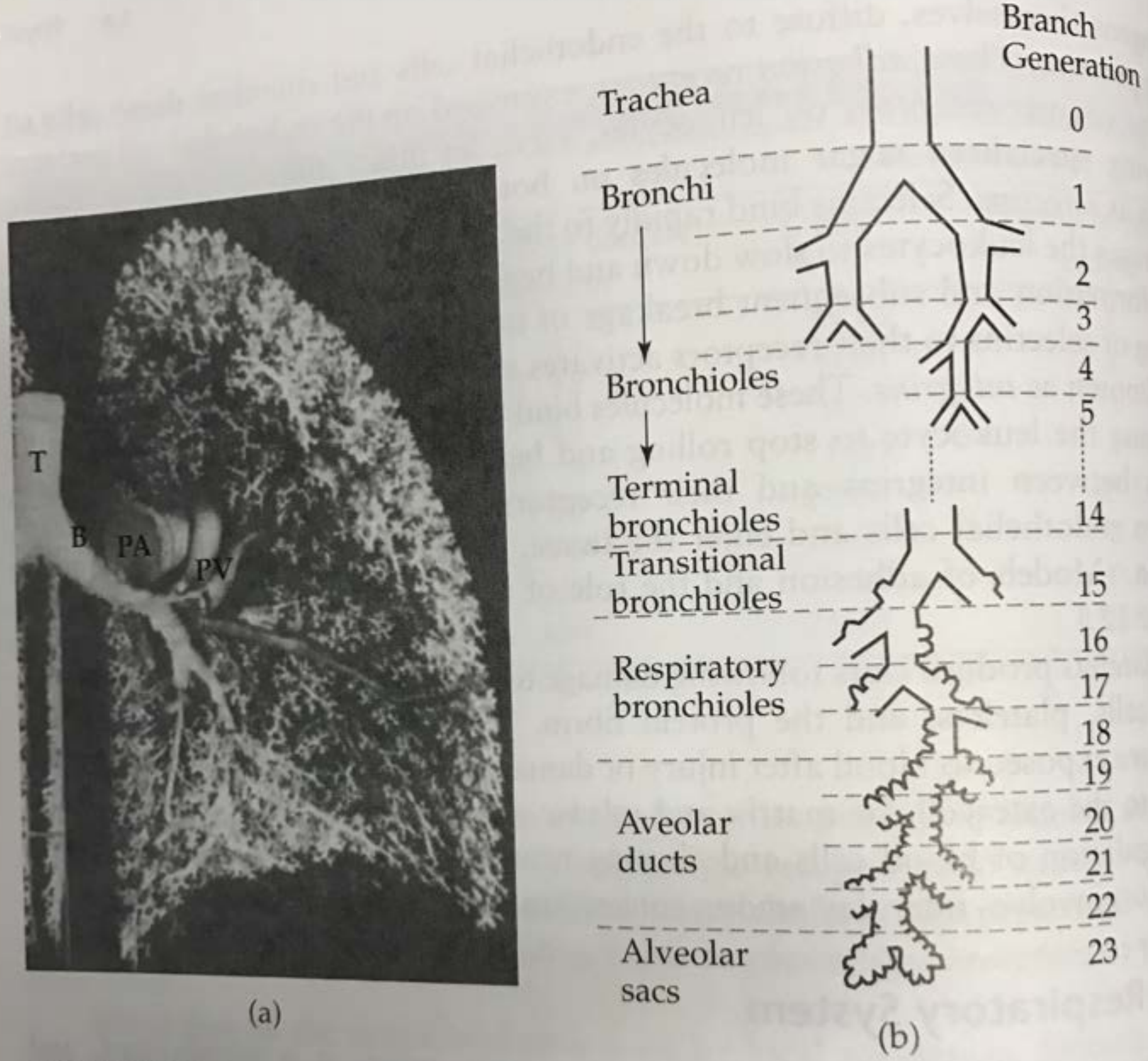
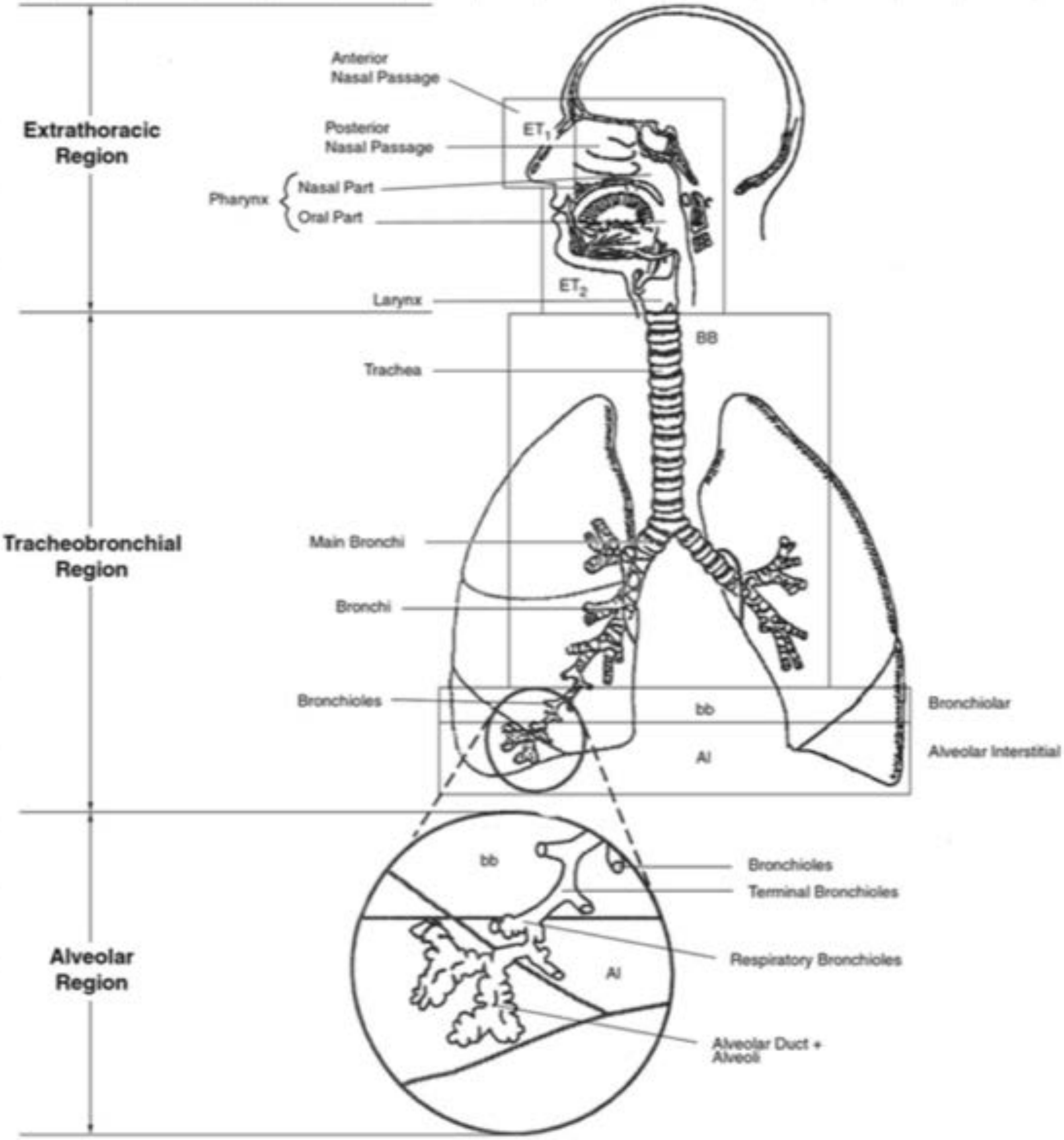


FIGURE 1.13 (a) Cast of a human lung, showing the trachea (T), one bronchus (B), the pulmonary artery (PA), and the pulmonary vein (PV). (b) Schematic of the organization of the airways in the human lung. (From Ref. [13], used with permission.)

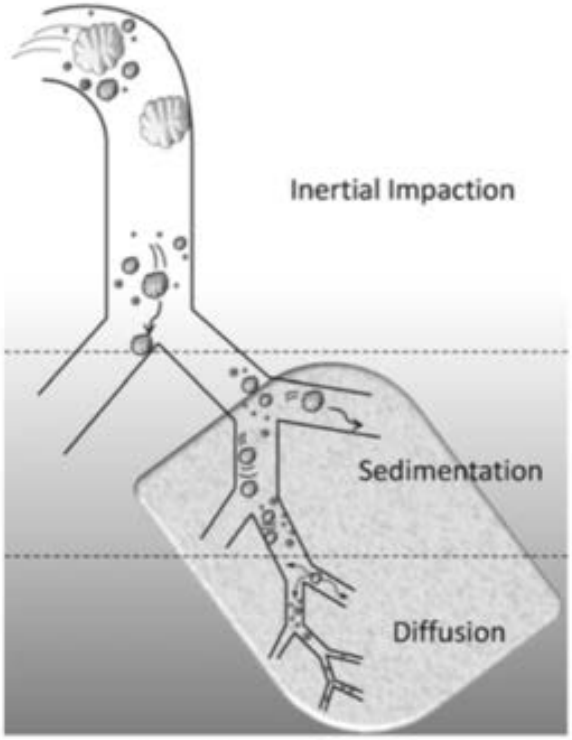


TORTUOUS PATH, PARTICLES STICK TO WALL: CLEARED BY CILIA.

LONG PATH: ONLY LAST ~5 BRANCHES ABSORB

OR, COULD BREATH BACK OUT.

PARTICLE CLEARING MECHANISMS



Inertial Impaction	Sedimentation	Diffusion
Directly related to d^2	Directly related to d^2	Inversely related to d
$Stk = \frac{\rho_p \cdot d^2 \cdot V}{18 \cdot \eta \cdot R}$	$V_{ts} = \frac{(\rho_p - \rho_a) \cdot d^2 \cdot g}{18 \cdot \eta}$	$Dif = \frac{k \cdot T}{3\pi \cdot \eta \cdot d}$

Aerodynamic Diameter: $d_{ae} = d \cdot \sqrt{\frac{\rho}{\rho_0}}$

Fig. 2. The influence of particle size on deposition. *d*: particle diameter; *Stk*: Stokes number; ρ_p: particle density; *V*: air velocity; η: air viscosity; *R*: airway radius; V_{ts}: terminal settling velocity; ρ_a: air density; *g*: gravitational acceleration; *Dif*: diffusion coefficient; *k*: Boltzmann's constant; *T*: absolute temperature; d_{ae}: aerodynamic diameter; ρ_0: unity density.

B, mass, *m*, and velocity, *v*, according to Eq. (1) (Gonda, 2004):

WHERE DO EQUATIONS COME FROM?

SOLUTION TO NAVIER-STOKES EQUATIONS FOR FLOW PAST A SPHERE: $Re \Rightarrow 0$

SEEMS LIKE IT IS SAFE TO SAY THAT YOU ARE OPTIMIZED FOR THE TRADEOFFS OF EFFICIENT BREATHING VERSUS PROTECTION FROM PARTICLES

The respiratory tract is especially designed, both anatomically and functionally, so that air can reach the most distal areas of the lungs in the cleanest possible condition. Nasal hairs, nasal turbinates, vocal chords, the cilia of the bronchial epithelium, the sneeze and cough reflexes, etc., all contribute to this filtering process. And, on most occasions it is properly done. But human beings are full of paradoxes: an efficient system, designed to avoid certain

particles from penetrating into the lungs, is at the same time used to intentionally deposit drugs in the airways and even for these to reach the alveoli in the best possible condition. It is thus necessary to get around the defense systems by evading reflex arcs, mucus layers, ciliary movements, etc., so that, with the inspiratory flow, the molecules that can improve diseases are deposited in the lungs. A system that evolved over time in order to filter and clean the air should be dodged in order to deposit other substances that we deliberately want to reach the inside of the organism. Without a

What if the Reynolds number is large, >>1 ?

RESCALE PRESSURE

$$p^* = \frac{p}{\rho U^2}$$

$$Re\left(\frac{\partial \vec{v}^*}{\partial t^*} + \vec{v}^* \cdot \vec{\nabla}^* \vec{v}^* + \nabla^* p^*\right) = \nabla^{*2} \vec{v}^* \simeq 0$$

TERMED "INVISCID" OR "IDEAL" FLOW

IF NO VISCOSITY:

- CAN'T IMPOSE NO SLIP

CAN CALCULATE LIFT BUT NOT DRAG ON A WING

We need to be able to solve problems when:

A. Details of flow are not known

B. Large scale of flow suggests that details should not matter.

TO DO THIS WE WILL USE INTEGRATED FORMS OF DIFFERENTIAL BALANCES

FOR MASS CONSERVATION

FOR ALL MASS INSIDE VOLUME INTEGRAL

CONTINUITY:

$$\int_V \frac{\partial \rho}{\partial t} dV = \int_V -\vec{\nabla} \cdot \rho \vec{v} \, dV$$

TOTAL MASS IN V

$$\int_V \frac{\partial \rho}{\partial t} dV = \frac{\partial}{\partial t} \int \rho \, dV = \frac{dm}{dt}$$

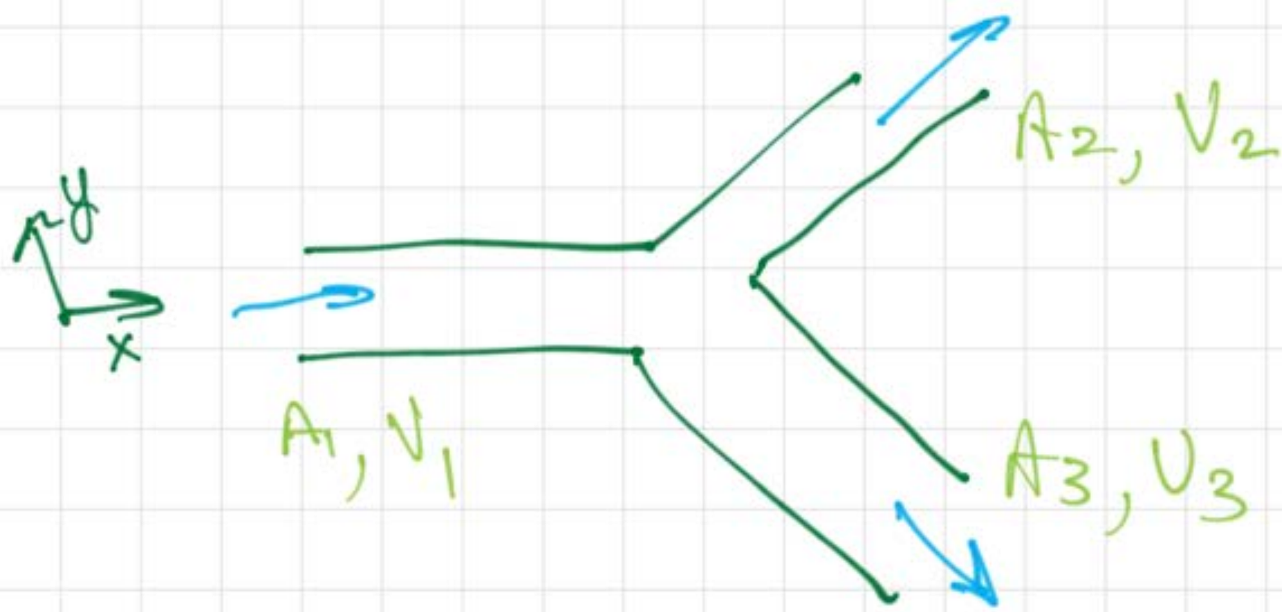
$$\int_V \vec{\nabla} \cdot \rho \vec{v} \, dV = \int_S \rho (\vec{v} \cdot \vec{n}) \, dS$$

DIVERGENCE THEOREM

THIS ALLOWS US TO KEEP TRACK OF MASS INSIDE BY WATCHING INFLOWS AND OUT FLOWS

$$\therefore \quad \frac{dm}{dt} = -\int \rho(\bar{v}\cdot\bar{n})\,ds$$

LOOK AT A SIMPLE CASE FOR WHICH WE KNOW ANSWER

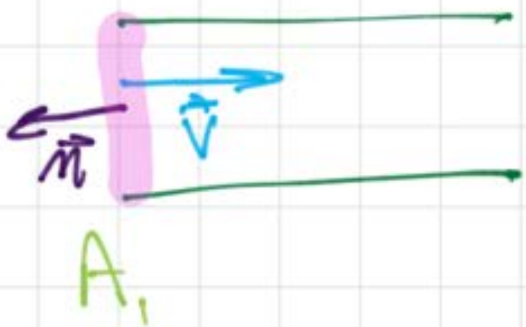


PICK S.S. $\frac{dm}{dt} = 0$

INTEGRAL IS NON-ZERO ONLY AT OPENINGS WHERE $\bar{v}$ IS NON-ZERO

INFLOW AT 1

$$-\int_{\text{AREA 1}} \rho(\bar{V}\cdot\bar{n})\,dS$$



$\bar{V}\cdot\bar{n} = -V_x$

TAKE ρ = CONST

$$-\int_{A_1} \rho(\bar{V}\cdot\bar{n})\,dS = -\rho\int_{A_1} -V_x\,dS$$

$$= \rho \langle V\rangle_1 A_1$$

RATE OF MASSFLOW IN $= \rho\, Q_1$

DENSITY

VOLUMETRIC FLOW RATE

@ 2 $\vec{v} \cdot \bar{n}$ IS NOT JUST V_x OR V_y, BUT IT IS ALL OF THE 'V'

$$-\int_{A_2} \rho \, \vec{v} \cdot \vec{n} \, ds = -\int_{A_2} \rho \, V_2 \, ds$$

NO "−" SIGN

$$= -\rho \langle V \rangle_2 A_2$$

$$= -\rho \, Q_2$$

@ 3 $$-\int_{A_3} \rho (\vec{v} \cdot \vec{n}) \, ds = \rho \langle V \rangle_3 A_3$$

$$= Q_3 A_3$$

$\therefore$

$$0 = \rho \langle v \rangle_1 A_1 - \rho \langle v \rangle_2 A_2 - \rho \langle v \rangle_3 A_3$$

$$\frac{dm}{dt} = \rho \sum_{\substack{i \\ \text{IN}}} \langle v \rangle_i A_i - \rho \sum_{\substack{j \\ \text{OUT}}} \langle v \rangle_j A_j$$

USEFUL IN THIS FORM.

FOR TURBULENT IN PIPES & DUCTS, VELOCITY PROFILE IS USUALLY "FLAT" ENOUGH TO JUST WRITE "V_1" AS A SINGLE VALUE.

MOMENTUM CONSERVATION

MORE CARE AND EFFORT IS NEEDED !!

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v}\right) = -\vec{\nabla}p + \vec{\nabla}\cdot\vec{\tau} + \rho\vec{g}$$

START THIS WITH THE SAME VOLUME INTEGRAL

$$\int_V \rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v}\right)dV =$$

$$\int_V \left(\vec{\nabla}p + \vec{\nabla}\cdot\vec{\tau} + \rho\vec{g}\right)dV$$

SAME AS $\dfrac{d\vec{P}}{dt} = \sum \vec{F}$

WE SELECTIVELY CHOOSE EITHER LEAVING A VOLUME INTEGRAL OR CONVERTING TO A SURFACE INTEGRAL

$$\int_V \rho \frac{\partial \bar{v}}{\partial t} dV = \frac{d}{dt} \int_V \rho \bar{v} \, dV$$

RATE OF CHANGE OF ALL MOMENTUM IN V

$$\int_V \rho (\bar{v} \cdot \bar{\nabla} \bar{v}) dV = \int_S \bar{v} \rho (\hat{n} \cdot \bar{v}) ds$$

ALLOWS TRACKING OF MOMENTUM IN AND OUT

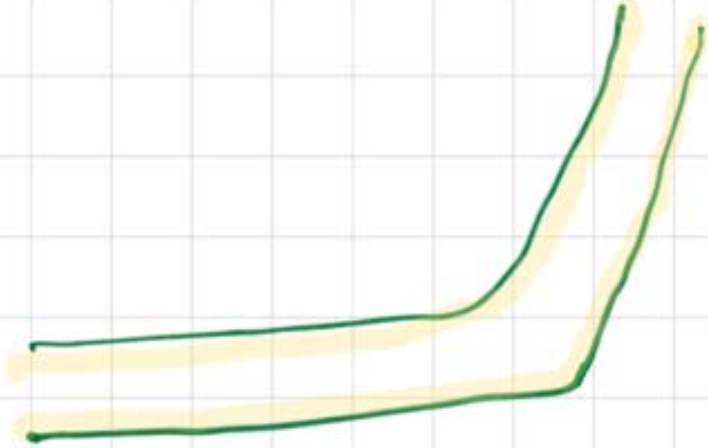
$$\int_V \vec{\nabla} \cdot \bar{\bar{\tau}} \, dV = \int_S \underbrace{\vec{n} \cdot \bar{\bar{\tau}} \, dS}$$

THIS INTEGRAL CONTAINS ALL OF THE VISCOUS STRESS ACTING ON THE WALLS. WE USUALLY DON'T ACTUALLY EVALUATE THIS

$$\int_V -\vec{\nabla} p \, dV = -\int_S p \, \vec{n} \, dS$$

THIS INTEGRAL CONTAINS THE PRESSURE ACTING ON ALL WALLS AND OPEN SURFACES

WE WILL EVALUATE IT ONLY ON OPEN SURFACES!!



$$\int_{\text{SOLID SURFACES}} (\bar{m}\cdot\bar{c})\,ds$$

$$-\int_{\text{SOLID SUFACE}} p\,\bar{n}\,ds$$

SET EQUAL TO $\bar{F}$

$\bar{F}$ IS THE TOTAL VECTOR ACTION OF THE SOLID WALLS ON THE FLUID, WE GET THIS FROM THE OTHER TERMS

$$\int_V \rho \vec{g}\, dV = m\vec{g}$$

THUS:

$$\frac{d}{dt}\left(\int \rho \vec{v}\, dV \right) + \int_S \vec{V}\rho(\vec{n}\cdot\vec{v})\, dS =$$

$$-\int_S p\,\vec{n}\, dS + \int_S (\vec{n}\cdot\vec{\tau})\, dS + m\vec{g}$$

WE NEED TO CONSIDER HOW WE CAN BEST USE THESE EQUATIONS.

<u>NEED</u>: SOME BODY OF EXPERIENCE

<u>NEED</u>: SIMPLER FORMS

COMMENTS ABOUT FORCE TERMS

PRESSURE

$$-\int_A p\,\bar{n}\,dS$$

USUALLY WE KNOW PRESSURE ONLY AT OPEN SURFACES

SO I DON'T EVEN TRY!!

$$\int_S \bar{n}\cdot\bar{\bar{\tau}}\,dS = \text{TOTAL FORCE ALONG WALLS}$$

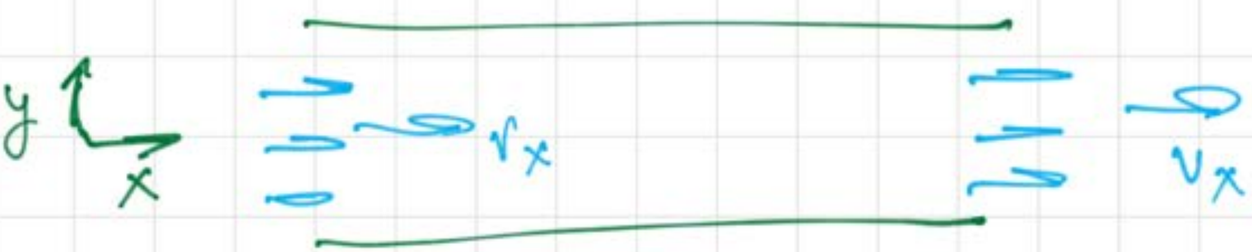
$= F$

FORCE NEEDED TO HOLD FITTING IN PLACE

$$\int \rho \bar{g} \, dV = m\bar{g}$$

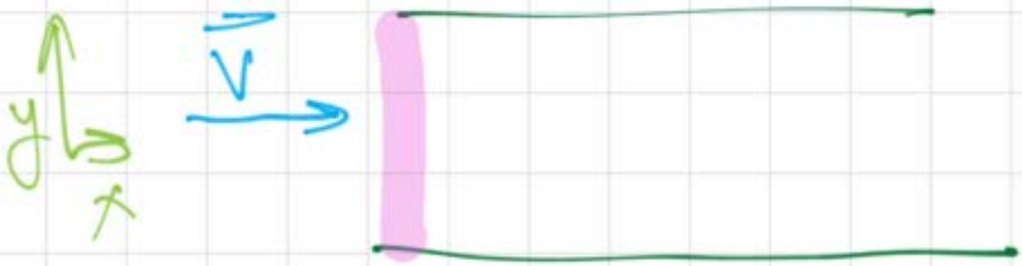
GRAVITY FORCE ON ENTIRE CONTROL VOLUME

WE WILL CONSIDER SOME EXAMPLE SITUATIONS



$$\int_A \underbrace{\bar{V}\rho}_{\frac{\text{MOMENTUM}}{\text{VOLUME}}} \underbrace{(\bar{V}\cdot\bar{n})}_{\text{ACTUAL INFLOW OR OUTFLOW VELOCITY (SCALAR)}} dS$$

$$= \rho \langle v_x v_x \rangle_1 A_1 - \rho \langle v_x v_x \rangle_2 A_2$$

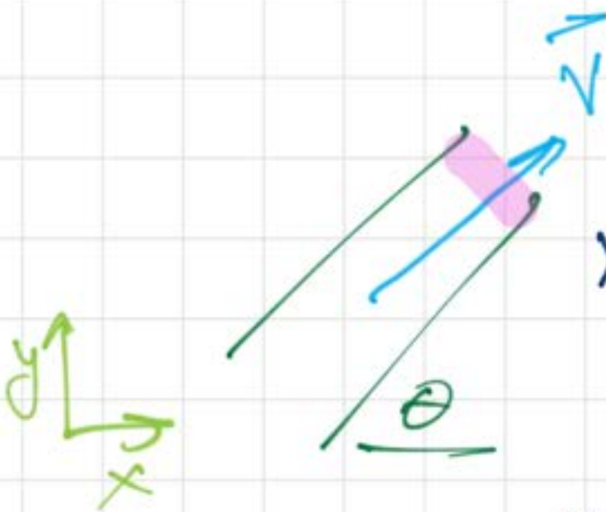


$$\int \vec{v}\,\rho\,(\vec{v}\cdot\vec{n})\,dS = -\int \rho\,\vec{V}\;v_x\,dS$$

V IS ALSO ONLY: V_x

$$-\rho\langle V_x V_x\rangle_1 A_1$$

$$-\int \rho\; V_x V_x\,dS = -\rho\langle V_x V_x\rangle_1 A_1$$



$$x \rightarrow \int \rho\,|\vec{v}|\cos\theta\,(\vec{v}\cdot\vec{n})\,dS$$
$$= \rho\,V_x\,Q$$

$$y \rightarrow \int \rho\,|\vec{v}|\sin\theta\,(\vec{v}\cdot\vec{n})\,dS$$
$$= \rho\,V_y\,Q$$

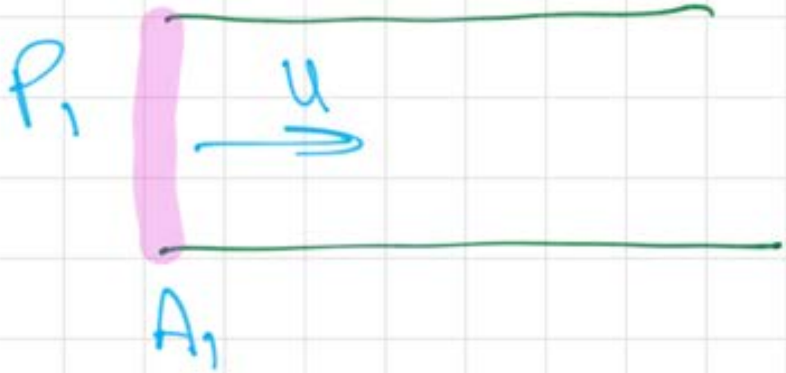
$\rho V_1 A_1 = \rho V_2 A_2$

$$x \quad 0 = -\rho \langle V_x V_x \rangle_1 A_1 + \rho V_x Q \Big|_2$$

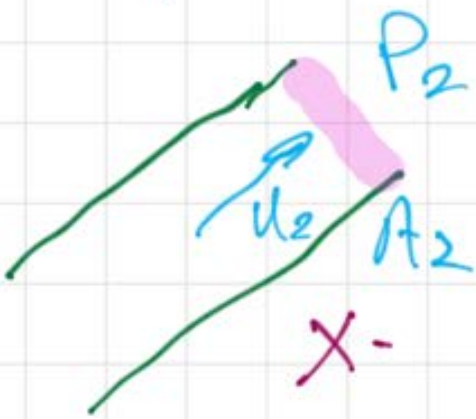
$$= -\rho V_1^2 A_1 + \rho V_2 \cos\theta \, V_2 A_2$$

$$y \quad 0 = 0 + \rho V_y Q \Big|_2$$

$$= 0 + \rho V_2 \sin\theta \, V_2 A_2$$

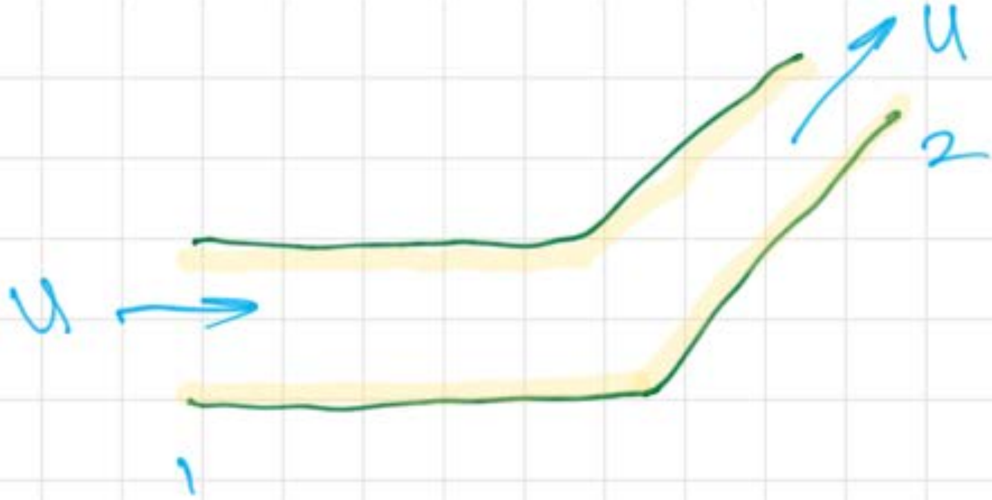


$$x - \quad -\int_{A_1} p\bar{n}\,ds = P_1A_1$$



$$x - \quad -\int_{A} p\bar{n}\,ds = -P_2\cos\theta A_2$$

$$y - \quad -\int_{A} p\bar{n}\,ds = -P_2\sin\theta A_2$$



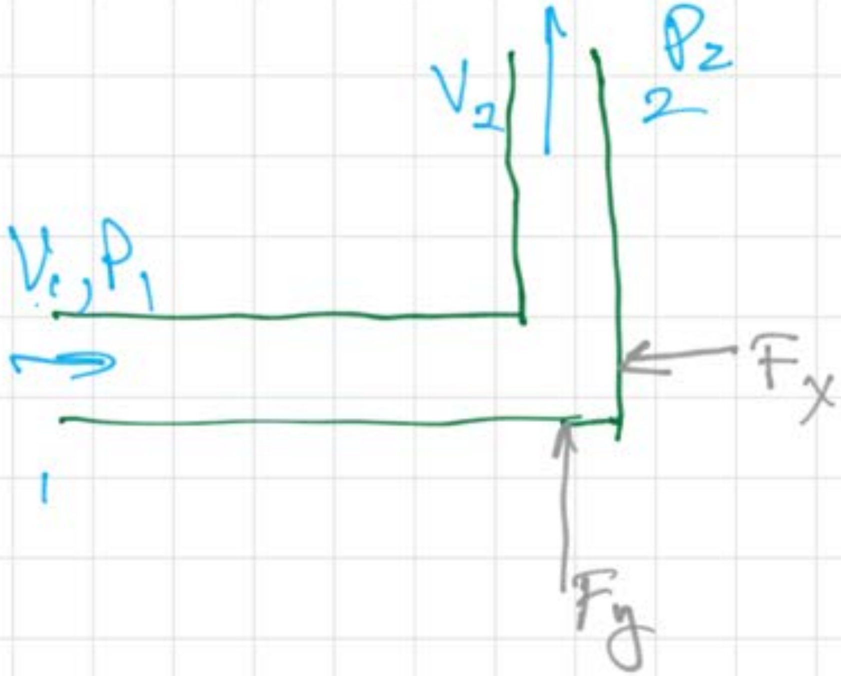
$$\int (\bar{n} \cdot \bar{\tau}) ds = F_x \hat{i} + F_y \hat{j}$$

COMBINATION OF PRESSURE AND SHEAR STRESS
USUALLY CAN'T EVALUATE INTEGRALS EXACTLY

SO: COMPLETE BALANCE

x ~ $-\rho V_1^2 A_1 + \rho V_2^2 \cos\theta A_2 = P_1 A_1 - P_2 A_2 \cos\theta + F_x$

y ~ $0 + \rho V_2^2 \sin\theta A_2 = -P_2 A_2 \sin\theta + F_y$



$x \qquad -\rho V_1^2 A_1 = P_1 A_1 + F_x$

$$F_x = -P_1 A_1 - \rho V_1^2 A_1$$

FORCE MATCHES PRESSURE AND DEFLECTS COMPLETE FLOW

$y \qquad +\rho V_2^2 A_2 = -P_2 A_2 + F_y$

$$F_y = P_2 A_2 + \rho V_2^2 A_2$$