CBE 30357

10/24/17

TOPICS

1) REVIEW OF RATIONALE USED TO SIMPLIFY PROBLEMS FOR WHICH $\vec{v} \cdot \vec{\nabla} \vec{v} \neq 0$

- USE NONDIMENSIONALIZATION TO SCALE TERMS WHICH BECOME $O(1)$.

- LOOK AT VALUE OF PARAMETER, Re TO SIMPLIFY EQUATION

$$Re\left(\vec{v}^* \cdot \vec{\nabla}^* \vec{v}^*\right) = -\vec{\nabla}^* p^* + \nabla^{*2} \vec{v}^*$$

$Re \to 0$, SOLVE: $\vec{\nabla}^* p^* = \nabla^{*2} \vec{v}^*$, $\vec{\nabla}^* \cdot \vec{v}^* = 0$

2) EXAMINE SOME $Re \rightarrow 0$ FLOWS TO SEE PHYSICAL OUTCOME

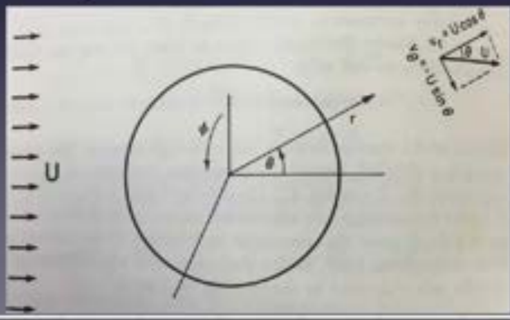
- QUALITATIVELY DIFFERENT THAN HIGH Re ... WHICH IS PROBABLY THE RANGE OF OUR EXPERIENCE.

3) HOW TO SOLVE FLOW PAST A SPHERE FOR $Re \rightarrow 0$

4) SOME FURTHER DISCUSSION OF PARTICULATE FLOWS IN YOUR LUNGS

Simple flow of interest

- Flow of liquid past a stationary sphere (which is the same as single particle moving (perhaps by gravity) through an otherwise quiescent liquid (or gas).
- Here is drawing from Denn's book:



Non-Zero Terms

TABLE 3.4

(Continued)

Spherical coordinates

r direction

$$\rho\left[\cancel{\frac{\partial v_r}{\partial t}} + \boxed{v_r\frac{\partial v_r}{\partial r}} + \boxed{\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}} + \cancel{\frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial\phi}}^{\,0} - \frac{\boxed{v_\theta^2} + \cancel{v_\phi^2}^{\,0}}{r}\right] = \boxed{-\frac{\partial p}{\partial r}} + \cancel{\rho g_r}$$
$$+ \mu\left[\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_r}{\partial r}\right)} + \boxed{\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_r}{\partial\theta}\right)} + \cancel{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial\phi^2}}\ \boxed{- \frac{2v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta} - \frac{2}{r^2}v_\theta\cot\theta} - \cancel{\frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}}\right]$$

θ direction

$$\rho\left(\cancel{\frac{\partial v_\theta}{\partial t}} + \boxed{v_r\frac{\partial v_\theta}{\partial r}} + \boxed{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}} + \cancel{\frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}}^{\,0} + \boxed{\frac{v_\theta v_r}{r}} - \cancel{\frac{v_\phi^2\cot\theta}{r}}^{\,0}\right) = \boxed{-\frac{1}{r}\frac{\partial p}{\partial\theta}} + \mu\left[\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right)}\right.$$
$$\left.+\boxed{\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_\theta}{\partial\theta}\right)} + \cancel{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial\phi^2}}^{\,0} + \boxed{\frac{2}{r^2}\frac{\partial v_r}{\partial\theta} - \frac{v_\theta}{r^2\sin^2\theta}} - \cancel{\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\phi}{\partial\phi}}\right] + \cancel{\rho g_\theta}$$

φ direction

$$\rho\left(\cancel{\frac{\partial v_\phi}{\partial t}} + \cancel{v_r\frac{\partial v_\phi}{\partial r}} + \cancel{\frac{v_\theta}{r}\frac{\partial v_\phi}{\partial\theta}} + \cancel{\frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}} + \cancel{\frac{v_\phi v_r}{r}} + \cancel{\frac{v_\theta v_\phi}{r}\cot\theta}\right) = \boxed{-\frac{1}{r\sin\theta}\frac{\partial p}{\partial\phi}}^{\,0} + \mu\left[\cancel{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right)}\right.$$
$$\left.+\cancel{\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_\phi}{\partial\theta}\right)} + \cancel{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial\phi^2}} + \cancel{\frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial\phi}} - \cancel{\frac{v_\phi}{r^2\sin^2\theta}} + \cancel{\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\theta}{\partial\phi}}\right] + \cancel{\rho g_\phi}$$

L.H.S.

$\vec{v}\cdot\vec{\nabla}\vec{v} \neq 0$

NON DIMENSIONALIZATION

DEFINE:

$v_r^* = \frac{v_r}{U_0}$ $\qquad v_\theta^* = \frac{v_\theta}{U_0}$

$r^* = \frac{r}{R}$ $\qquad r = r^* R$

WE NEED P^*

$P_c \equiv \frac{\mu U_0}{R} \quad \therefore P^* = \frac{P}{\mu U_0/R}$

FOR ALL TERMS ...

$$\underset{\mathcal{O}(1)}{Re\left(\vec{V}^* \cdot \vec{\nabla}^* \vec{V}^*\right)} = \underset{\mathcal{O}(1)}{-\vec{\nabla} p^*} + \underset{\mathcal{O}(1)}{\nabla^{*2} \vec{V}^*}$$

THEN IF $Re \to 0$ WE GET

$$\boxed{\vec{\nabla}^* p^* = \nabla^{*2} \vec{V}^*}$$

STOKES EQUATION

LINEAR PDES SO THAT WITH SOME WORK, THESE CAN BE <u>SOLVED</u>!!

“Correlations: Mass transfer **or** heat transfer”

Table 3.3 Mass transfer† for simple situations

Fluid motion	Range of conditions	Equation	Ref.
1. Inside circular pipes	Re = 4000–60 000 Sc = 0.6–3000	$j_D = 0.023\ \mathrm{Re}^{-0.17}$ $\mathrm{Sh} = 0.023\ \mathrm{Re}^{0.83}\ \mathrm{Sc}^{1/3}$	41, 52
	Re = 10 000 – 400 000 Sc > 100	$j_D = 0.0149\ \mathrm{Re}^{-0.12}$ $\mathrm{Sh} = 0.0149\ \mathrm{Re}^{0.88}\ \mathrm{Sc}^{1/3}$	44
2. Unconfined flow parallel to flat plates‡	Transfer begins at leading edge $\mathrm{Re}_x < 50\ 000$	$j_D = 0.664\ \mathrm{Re}_x^{-0.5}$	32
	$\mathrm{Re}_x = 5 \times 10^5 – 3 \times 10^7$ Pr = 0.7–380	$\mathrm{Nu} = 0.037\ \mathrm{Re}_x^{0.8}\ \mathrm{Pr}_0^{0.43}\left(\frac{\mathrm{Pr}_0}{\mathrm{Pr}_i}\right)^{0.25}$	
	$\mathrm{Re}_x = 2 \times 10^4 – 5 \times 10^5$ Pr = 0.7–380	Between above and $\mathrm{Nu} = 0.0027\ \mathrm{Re}_x\ \mathrm{Pr}_0^{0.43}\left(\frac{\mathrm{Pr}_0}{\mathrm{Pr}_i}\right)^{0.25}$	
3. Confined gas flow parallel to a flat plate in a duct	$\mathrm{Re}_e = 2600–22\ 000$	$j_D = 0.11\ \mathrm{Re}_e^{-0.29}$	
4. Liquid film in wetted-wall tower, transfer between liquid and gas	$\frac{4\Gamma}{\mu} = 0–1200$, ripples suppressed	Eqs. (3.18)–(3.22)	
	$\frac{4\Gamma}{\mu} = 1300–8300$	$\mathrm{Sh} = (1.76 \times 10^{-5})\left(\frac{4\Gamma}{\mu}\right)^{1.506} \mathrm{Sc}^{0.5}$	
5. Perpendicular to single cylinders	Re = 400–25 000 Sc = 0.6–2.6	$\frac{k_G p_t}{G_M}\mathrm{Sc}^{0.56} = 0.281\ \mathrm{Re}'^{0.4}$	5
	$\mathrm{Re}' = 0.1–10^5$ Pr = 0.7–1500	$\mathrm{Nu} = (0.35 + 0.34\ \mathrm{Re}'^{0.5} + 0.15\ \mathrm{Re}'^{0.58})\ \mathrm{Pr}^{0.3}$	16, 21, 42
6. Past single spheres	Sc = 0.6–3200 $\mathrm{Re}''\ \mathrm{Sc}^{0.5} = 1.8–600\ 000$	$\mathrm{Sh} = \mathrm{Sh}_0 + 0.347(\mathrm{Re}''\ \mathrm{Sc}^{0.5})^{0.62}$ $\mathrm{Sh}_0 = \begin{cases} 2.0 + 0.569(\mathrm{Gr}_D\ \mathrm{Sc})^{0.250} & \mathrm{Gr}_D\ \mathrm{Sc} < 10^8 \\ 2.0 + 0.0254(\mathrm{Gr}_D\ \mathrm{Sc})^{0.333}\ \mathrm{Sc}^{0.244} & \mathrm{Gr}_D\ \mathrm{Sc} > 10^8 \end{cases}$	55
7. Through fixed beds of pellets§	Re″ = 90–4000 Sc = 0.6	$j_D = j_H = \frac{2.06}{\varepsilon}\mathrm{Re}''^{-0.575}$	
	Re″ = 5000–10 300 Sc = 0.6	$j_D = 0.95 j_H = \frac{20.4}{\varepsilon}\mathrm{Re}''^{-0.815}$	4, 23, 64
	Re″ = 0.0016–55 Sc = 168–70 600	$j_D = \frac{1.09}{\varepsilon}\mathrm{Re}''^{-2/3}$	
	Re″ = 5–1500 Sc = 168–70 600	$j_D = \frac{0.250}{\varepsilon}\mathrm{Re}''^{-0.31}$	

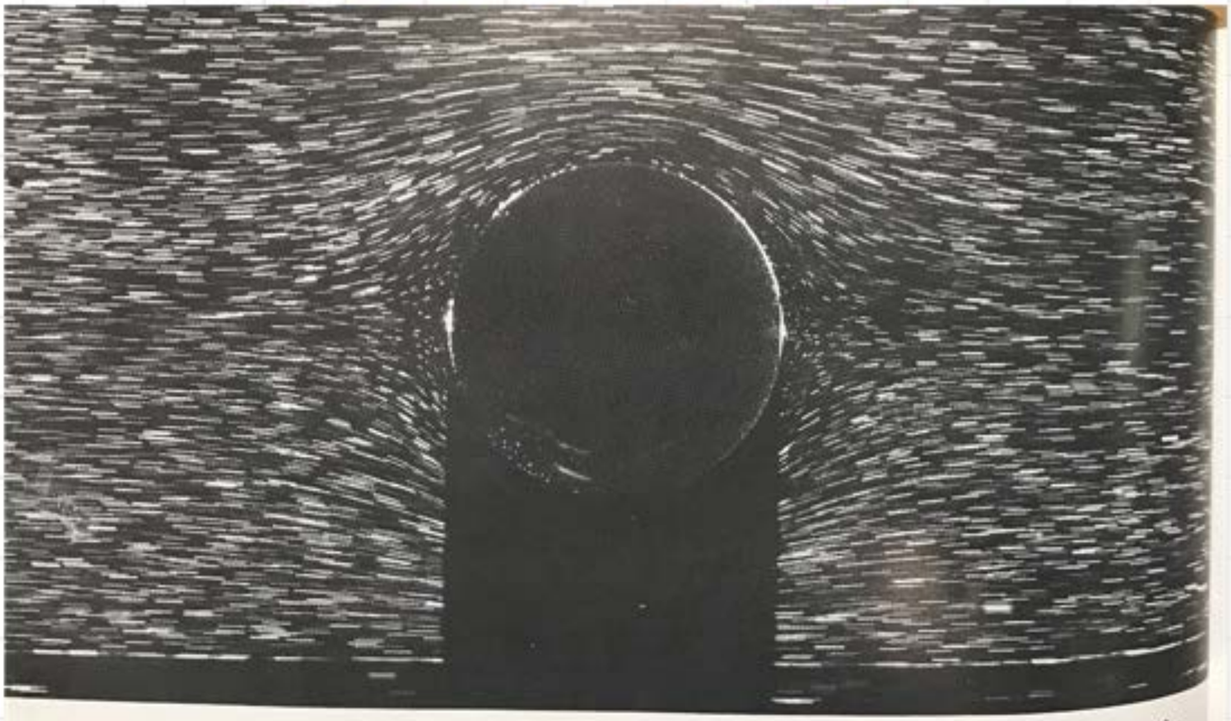
† Average mass-transfer coefficients throughout, for constant solute concentrations at the phase surface. Generally, fluid properties are evaluated at the average conditions between the phase surface and the bulk fluid. The heat-mass-transfer analogy is valid throughout.

‡ Mass-transfer data for this case scatter badly but are reasonably well represented by setting $j_D = j_H$.

§ For fixed beds, the relation between ε and d_p is $a = 6(1-\varepsilon)/d_p$, where a is the specific solid surface, surface per volume of bed. For mixed sizes [58]

$$d_p = \frac{\sum_{i=1}^{n} n_i d_{pi}^3}{\sum_{i=1}^{n} n_i d_{pi}^2}$$

LOW Re FLOWS ....



8. Sphere moving through a tube at $R=0.10$, relative motion. A free sphere is falling steadily down the axis of a tube of twice its diameter filled with glycerine. The camera is moved with the speed of the sphere to show the flow relative to it. The photograph has been rotated to show flow from left to right. Tiny magnesium cuttings are illuminated by a thin sheet of light, which casts a shadow of the sphere. *Coutanceau 1968*

NO WAKE

SYMMETRIC: WHICH WAY IS FLOW GOING?

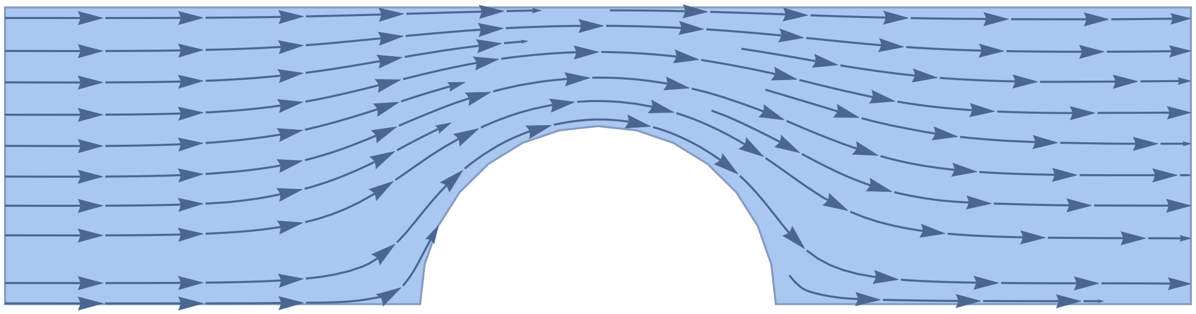
. Uniform flow past a circular cylinder at $R=0.16$. hat the flow is from left to right can scarcely be deduced om the streamline pattern, because in the limit of zero eynolds number the flow past a solid body is reversible, nd hence symmetric about a symmetric shape. It resembles superficially the pattern of potential flow but the disturbances to the uniform stream c more slowly. The flow of water is shown b dust. *Photograph by Sadatoshi Taneda*

Some calculations

- Planar flows from the Notebook: Flow past a circle and square

Flow past an obstacle
There is no wake and the lower plot of the pressure shows that the pressure is constantly decreasing

```
rmf = RegionMember[Ω];
Show[BoundaryDiscretizeRegion[Ω],
 StreamPlot[{xVel[x, y], yVel[x, y]}, {x, 0, 2}, {y, 0, 1/2},
  RegionFunction → Function[{x, y}, rmf[{x, y}]], AspectRatio → Automatic], ImageSize → 600]
```



```
pressureplot = RegionMember[Ω];
Show[BoundaryDiscretizeRegion[Ω],
 ContourPlot[{pressure[x, y]}, {x, 0, 2}, {y, 0, 1/2}, RegionFunction → Function[{x, y}, rmf[{x, y}]],
  AspectRatio → Automatic], ImageSize → 600]
```

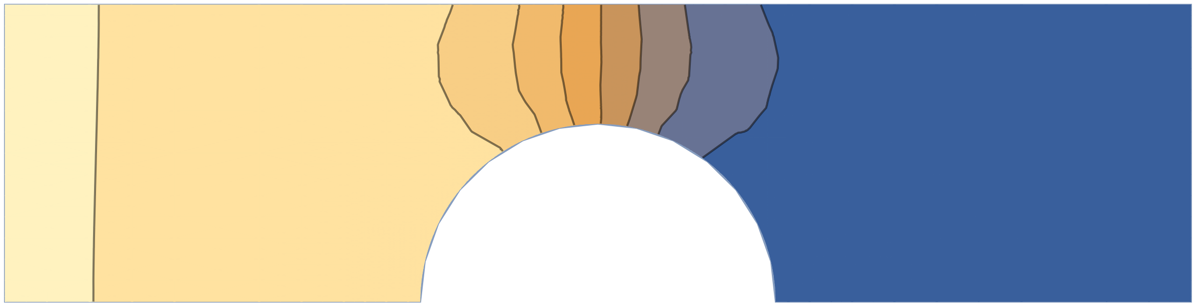
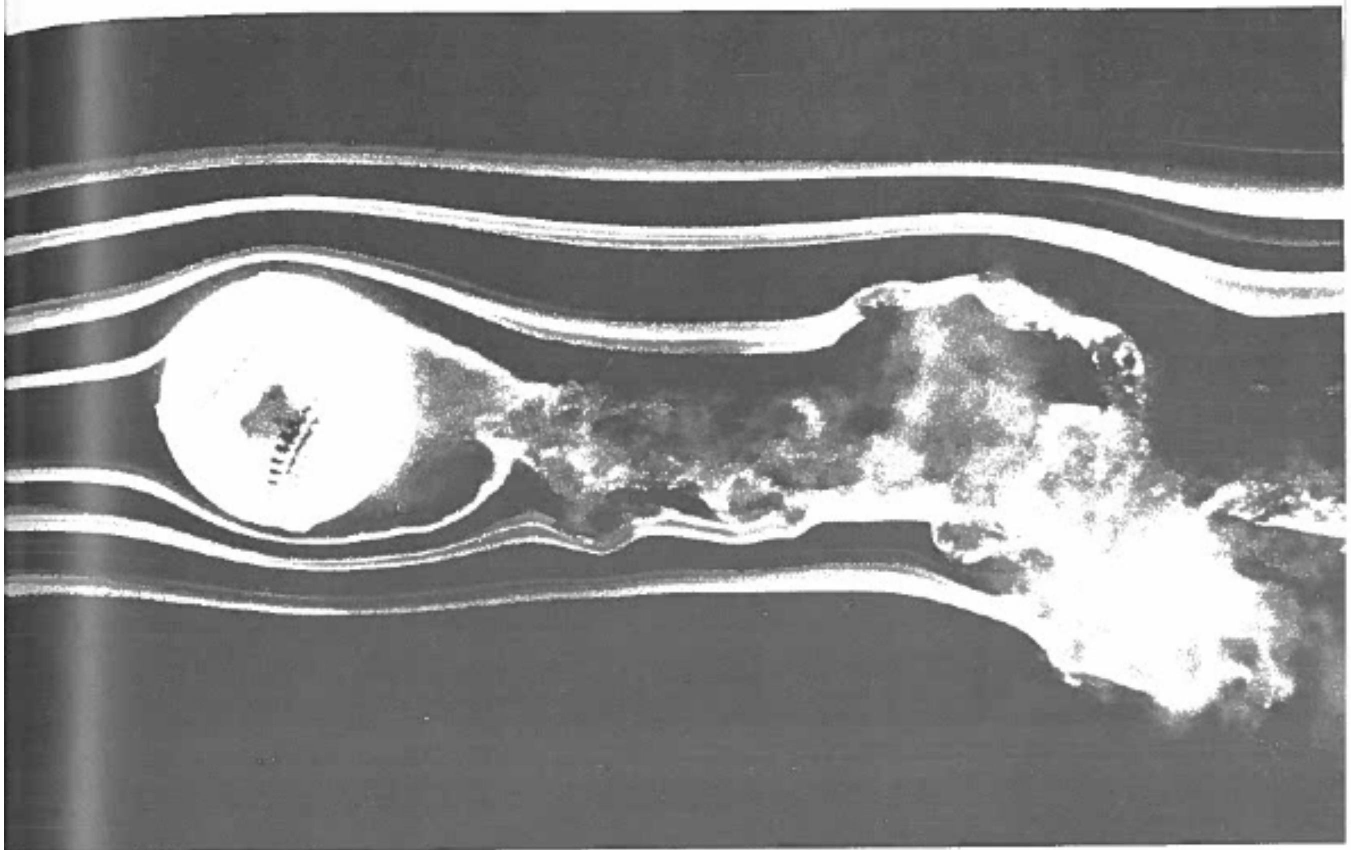
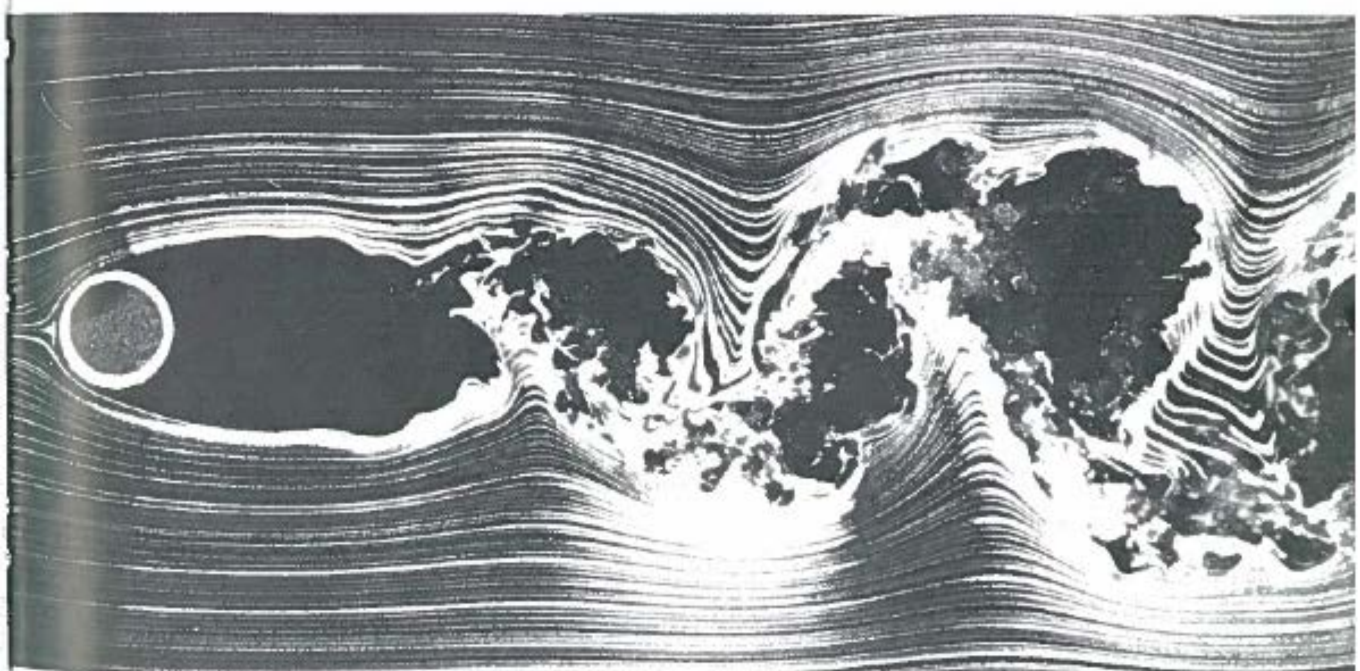


Photo taken at Notre Dame, presumably in a building that used to be SSW of Jordan Hall.



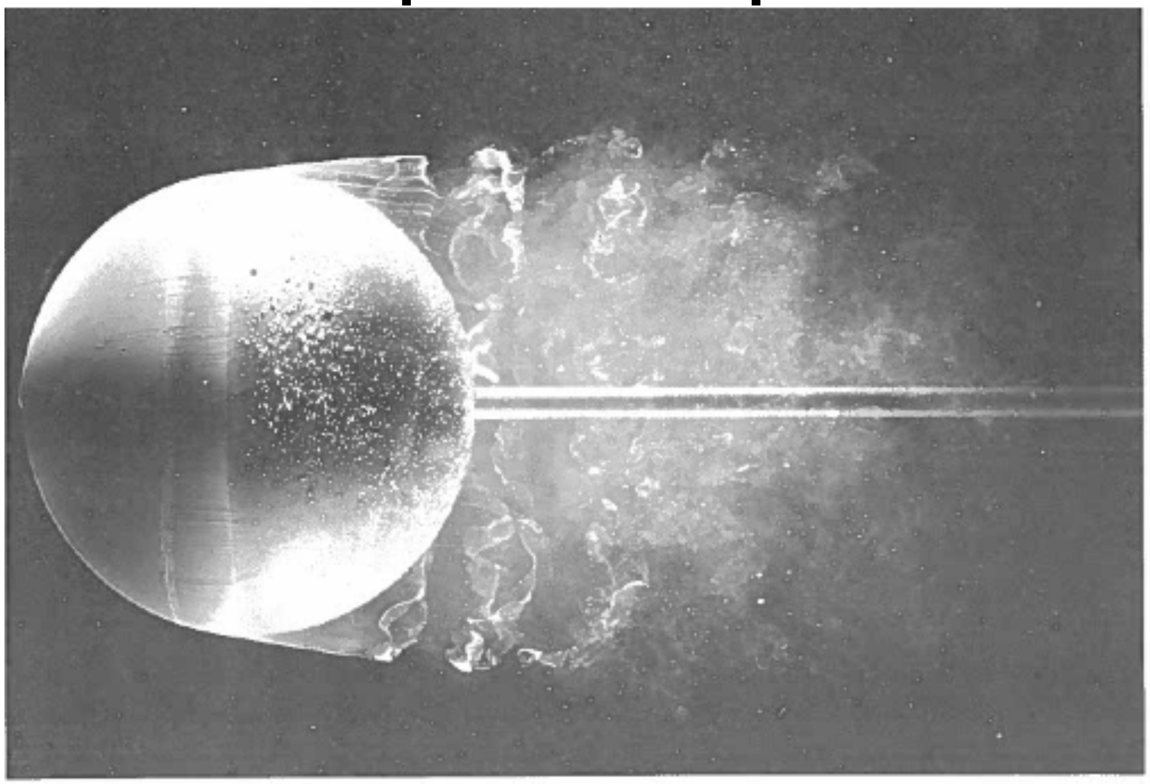
66. **Spinning baseball.** The late F. N. M. Brown devoted many years to developing and using smoke visualization in wind tunnels at the University of Notre Dame. Here the flow speed is about 77 ft/sec and the ball is rotated at 630 rpm. This unpublished photograph is similar to several in Brown 1971. *Photograph courtesy of T. J. Mueller*

Fast flow past a cylinder



48. Circular cylinder at R=10,000. At five times the speed of the photograph at the top of the page, the flow pattern is scarcely changed. The drag coefficient consequently remains almost constant in the range of Reynolds number spanned by these two photographs. It drops later when, as in figure 57, the boundary layer becomes turbulent at separation. *Photograph by Thomas Corke and Hassan Nagib*

Flow past a sphere

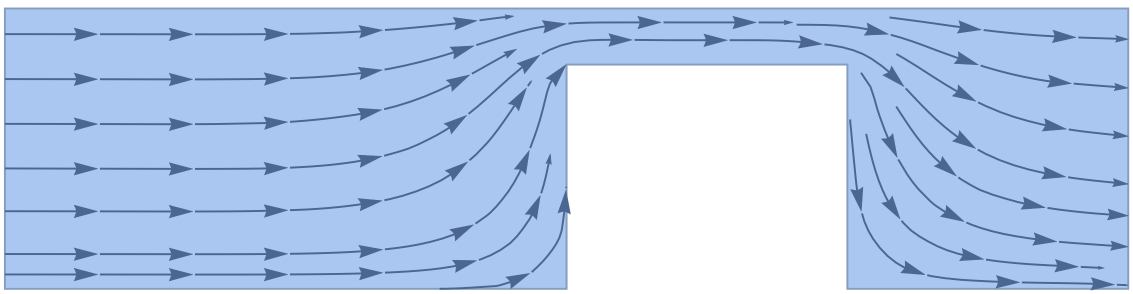


55. **Instantaneous flow past a sphere at $R=15{,}000$.** Dye in water shows a laminar boundary layer separating ahead of the equator and remaining laminar for almost one radius. It then becomes unstable and quickly turns turbulent. *ONERA photograph, Werlé 1980*

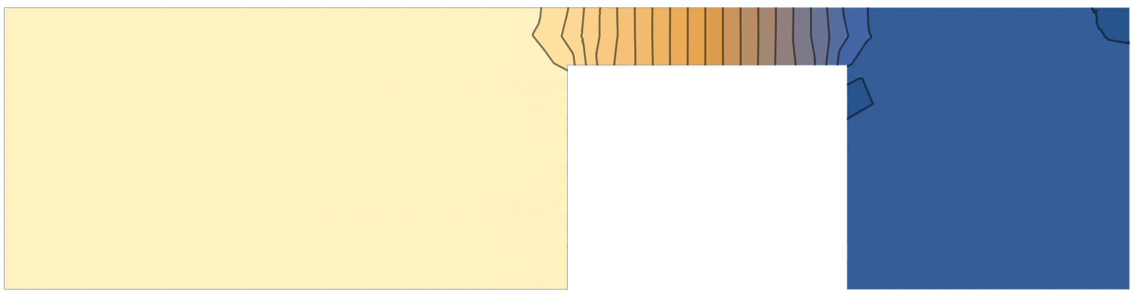
Flow past an obstacle

There is no wake and the lower plot of the pressure shows that the pressure is constantly decreasing

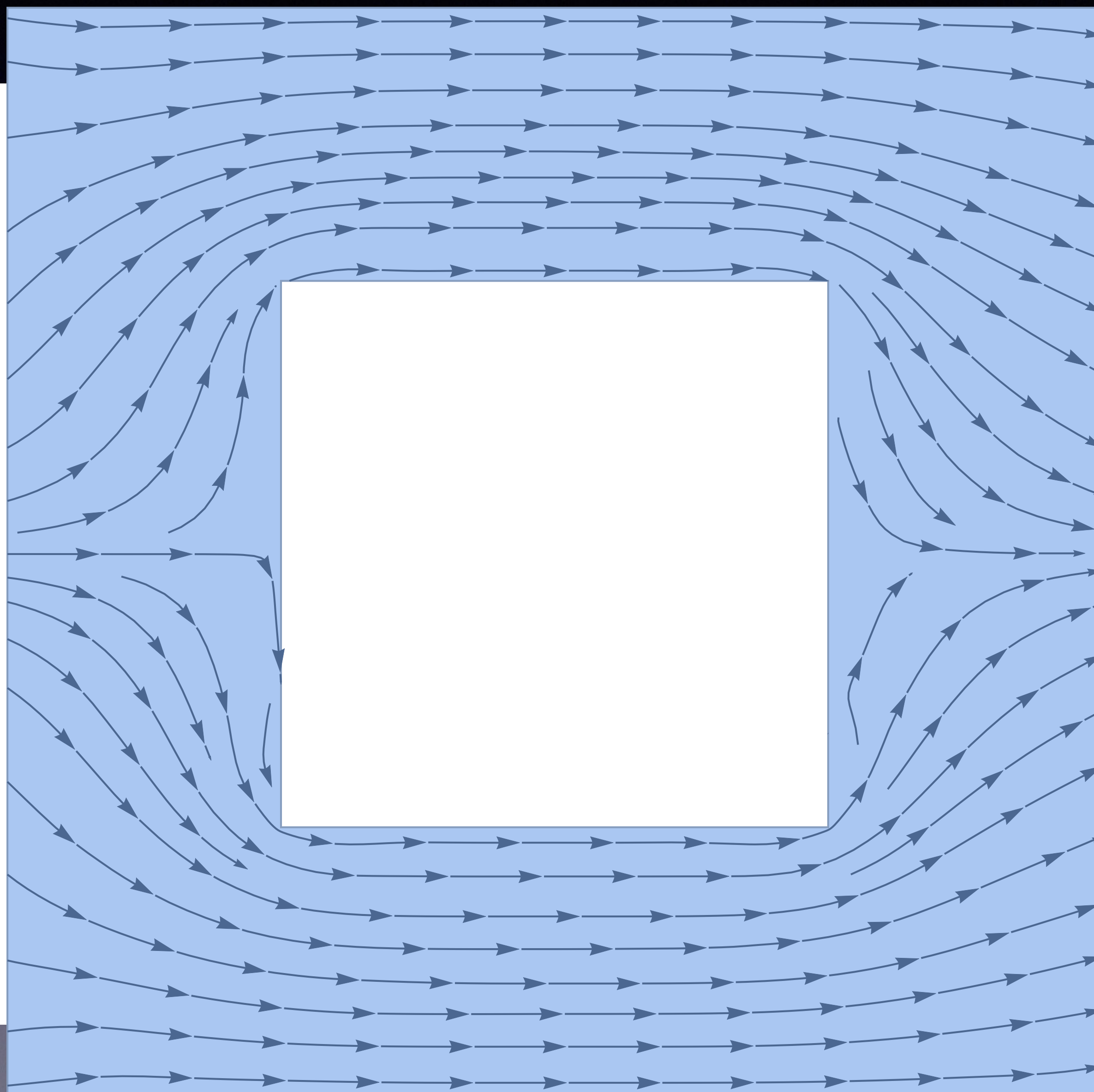
```
rmf = RegionMember[Ω];
Show[BoundaryDiscretizeRegion[Ω],
 StreamPlot[{xVel[x, y], yVel[x, y]}, {x, 0, 2}, {y, 0, 1/2},
  RegionFunction → Function[{x, y}, rmf[{x, y}]], AspectRatio → Automatic], ImageSize → 600]
```



```
rmf = RegionMember[Ω];
Show[BoundaryDiscretizeRegion[Ω],
 ContourPlot[pressure[x, y], {x, 0, 2}, {y, 0, 1/2}, Contours → 20,
  RegionFunction → Function[{x, y}, rmf[{x, y}]], AspectRatio → Automatic], ImageSize → 600]
```



Or this…

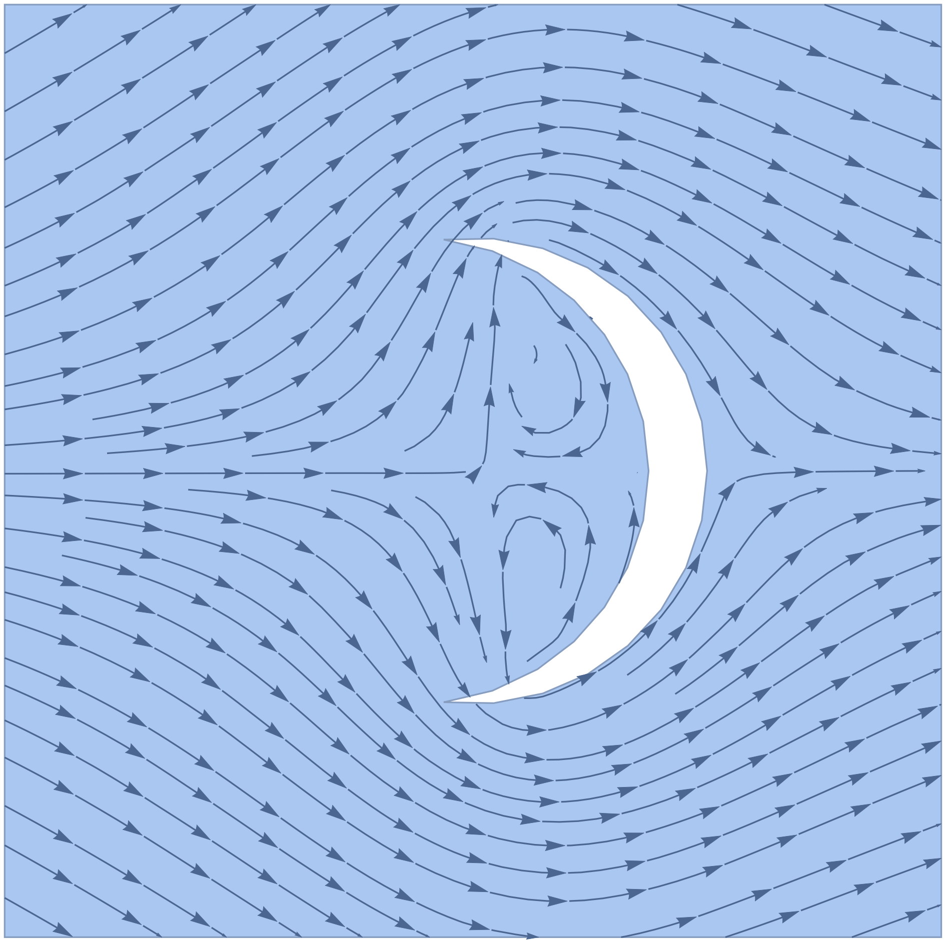


Flow in a “lens”

- The next two figures show flow in a crescent or lens
- The recirculation region occurs on the “inside” but is the same shape for flow from either the right or the left

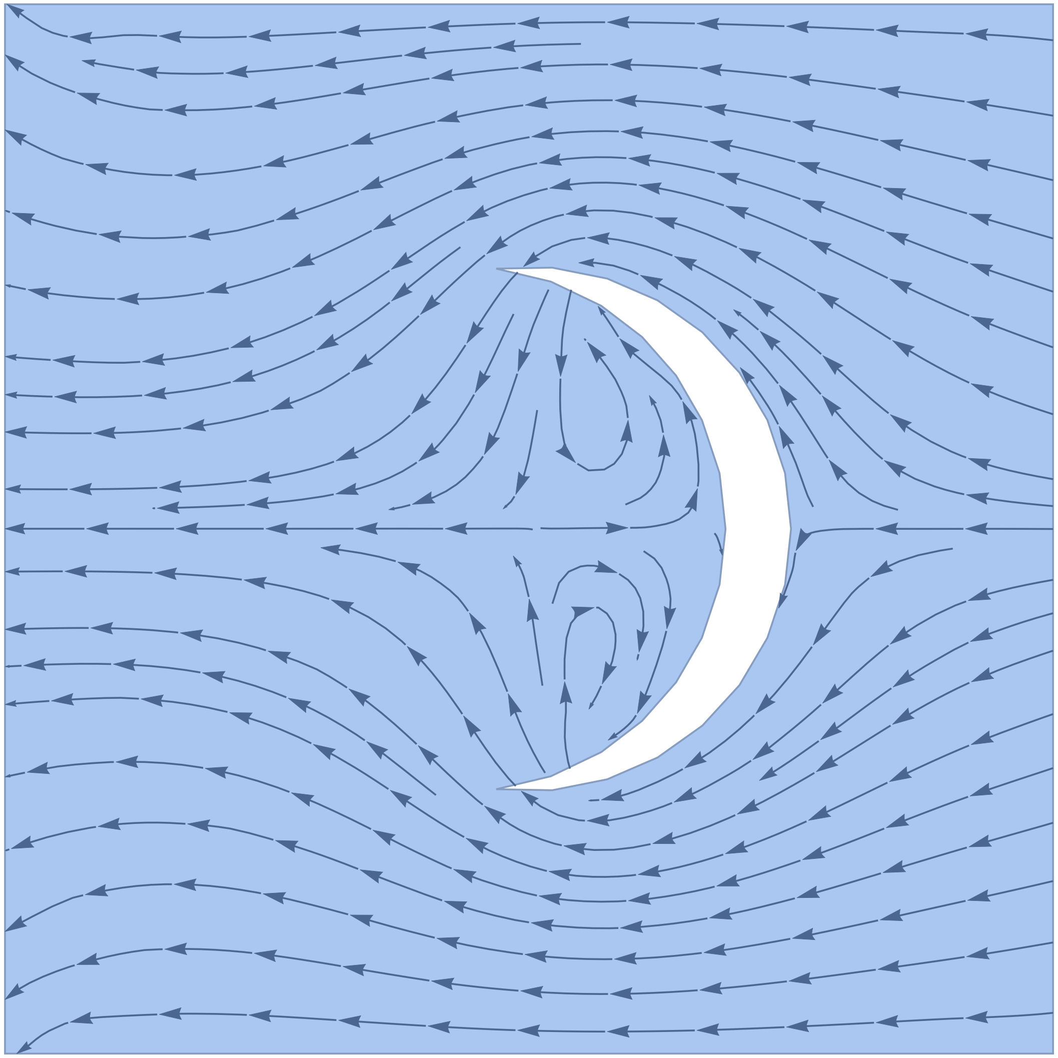
Left to right

```
rmf = RegionMember[Ω];
Show[BoundaryDiscretizeRegion[Ω],
 StreamPlot[{xVel[x, y], yVel[x, y]}, {x, -1, 1}, {y, -1, 1},
  RegionFunction → Function[{x, y}, rmf[{x, y}]], AspectRatio → Automatic], ImageSize → 600]
```



Right to Left!

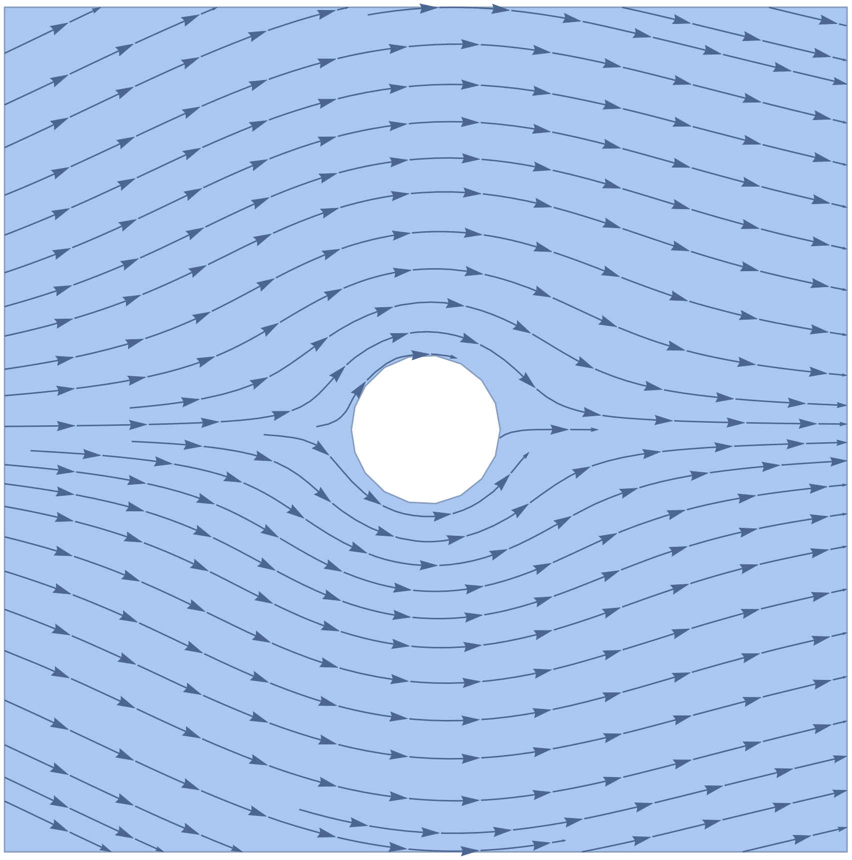
```
rmf = RegionMember[Ω];
Show[BoundaryDiscretizeRegion[Ω],
 StreamPlot[{xVel[x, y], yVel[x, y]}, {x, -1, 1}, {y, -1, 1},
  RegionFunction → Function[{x, y}, rmf[{x, y}]], AspectRatio → Automatic], ImageSize → 600]
```



Flow past a sphere

- Note how the streamlines are displaced outward far from the sphere

```
rmf = RegionMember[Ω];
Show[BoundaryDiscretizeRegion[Ω],
 StreamPlot[{xVel[x, y], yVel[x, y]}, {x, -2, 2}, {y, -2, 2},
  RegionFunction → Function[{x, y}, rmf[{x, y}]], AspectRatio → Automatic], ImageSize → 600]
```



THERE IS A 30 MINUTE VIDEO ON YOUTUBE ABOUT LOW R_e FLOWS

PLEASE WATCH!!

IT "STARS" G. I. TAYLOR

SEARCH: 'LOW REYNOLDS NUMBER FLOWS' IN YOUTUBE

RESULT " 7. LOW-REYNOLDS-NUMBER FLOWS BARRY BELMONT

FOR PEARL IN 1960'S COMMERCIAL

$$Re = \frac{D V \rho_f}{\mu_f} = \frac{(1.5\,cm)(.5\,cm/s)(1\,g/cm^3)}{4\,g/cm\text{-}s}$$

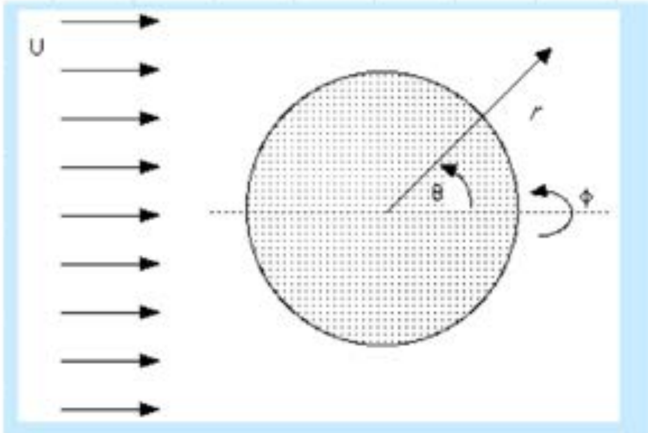
$$Re \simeq .2$$

FOR 10 μm PARTICLE IN AIR

$$Re = \frac{(10^{-3}\,cm)(.1\,cm/s)(\frac{1}{890}\,g/cm^3)}{.00018\,g/cm\text{-}s}$$

$$= 6 \times 10^{-3}$$

HOW DO WE SOLVE THIS PROBLEM?



$$\left\{ \begin{array}{l} \vec{\nabla} \cdot \vec{V} = 0 \\ \vec{\nabla} P = \mu \nabla^2 \vec{V} \end{array} \right\}$$

USE SPHERICAL COORDINATES

- CONTINUITY EQ.
- r-DIRECTION N.S. EQ
- θ-DIRECTION N.S. EQ

$\Big\}$ $Re \Rightarrow 0$

JUST THE R.H.S.'S

BOUNDARY CONDITIONS

$V_r(r=R)=0 \quad V_\theta(r=R)=0$ $\Big\}$ FLUID STICKS TO SPHERE

$V_r(r \Rightarrow \infty) = U\cos\theta$

$V_\theta(r \Rightarrow \infty) = -U\sin\theta$

$\Big\}$ AS $R \Rightarrow \infty$ $\vec{V} \Rightarrow U$

SOLUTION PROCEDURE

SEQUENTIALLY CONSIDER 3 PDE'S USING AN ASSUMED FORM OF SOLUTION

$$u_r \sim f(r)\cos\theta$$
$$u_\theta \sim g(r)\sin\theta$$

FROM B.C.'S

CONTINUITY EQUATION ALLOWS EASY ISOLATION OF

$g(r)$

SUBS THIS TO ELIMINATE $g(r)$ IN U_r + V_θ EQ'S

TAKE CROSS-DERIVATITIVES AND SUBTRACT V_r + V_θ EQ TO ELIMINATE PRESSURE

SOLVE 4TH ORDER EULER ODE.

FIT B.C.'S 👍

DRAG = $6\pi\mu R U$

WE GET: SETTLING VELOCITY

$\Sigma F =$ GRAVITY + BUOYANCY + DRAG

$$\Sigma F = \rho g V - \rho_f g V - 6\pi\mu U R$$

$$= \rho g \frac{4}{3}\pi R^3 - \rho_f g \frac{4}{3}\pi R^3 - 6\pi\mu U R$$

$$6\pi\mu U R = \frac{4}{3}(\rho-\rho_f)\, g\, \pi R^3$$

$$U = \frac{2}{9}\frac{(\rho-\rho_f)\, g\, R^2}{\mu}$$

MATCHES EXPERIMENTS UP TO $Re \sim .8$.

EINSTEIN USED DRAG TO CALCULATE <u>DIFFUSIVITY</u> OF A PARTICLE

$$D = \frac{KT}{6\pi\mu R}$$

PARTICLE DIFFUSIVITY

BOLTZMANN CONSTANT

PARTICLE RADIUS

BESIDES OLD TV COMMERCIALS WHERE DO THESE FLOWS OCCUR?

1) PROCESS FLOWS OF SUSPENSIONS
   a) FOODS.
   b) CONSUMER PRODUCTS
   c) COMPOSITE MOLDING
   d) SOLID-LIQUID MIXING
   e) SOLIDS TRANSPORT
   f) FLUIDIZED BED REACTORS
2) MEDICAL: INHALERS FOR DRUG DELIVERY
3) ENVIROMENT:
   AEROSOLS, SEDIMENT TRANSPORT

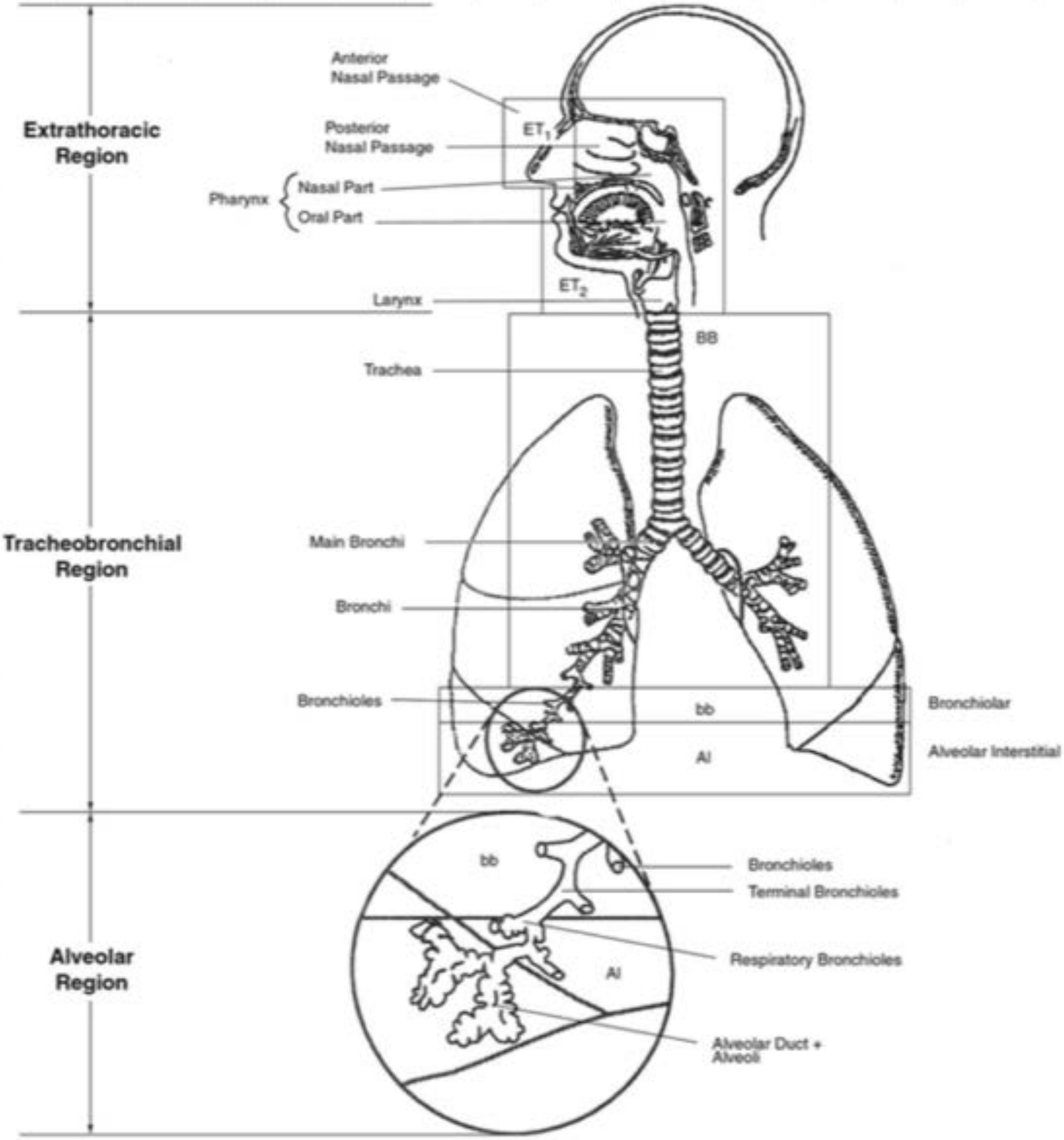
TAKE SPECIFIC EXAMPLE OF "INHALERS"

PROCESS INVOLVES EITHER

NEBULIZING A LIQUID INTO SMALL DROPS

OR DISPERSING SMALL PARTICLES INTO A CARRIER GAS

IN EITHER CASE, SOUNDS LIKE A GREAT IDEA!!



TORTUOUS PATH, PARTICLES
STICK TO WALL: CLEARED
BY CILIA.
LONG PATH: ONLY LAST ~5
BRANCHES ABSORB
OR, COULD BREATH BACK OUT.

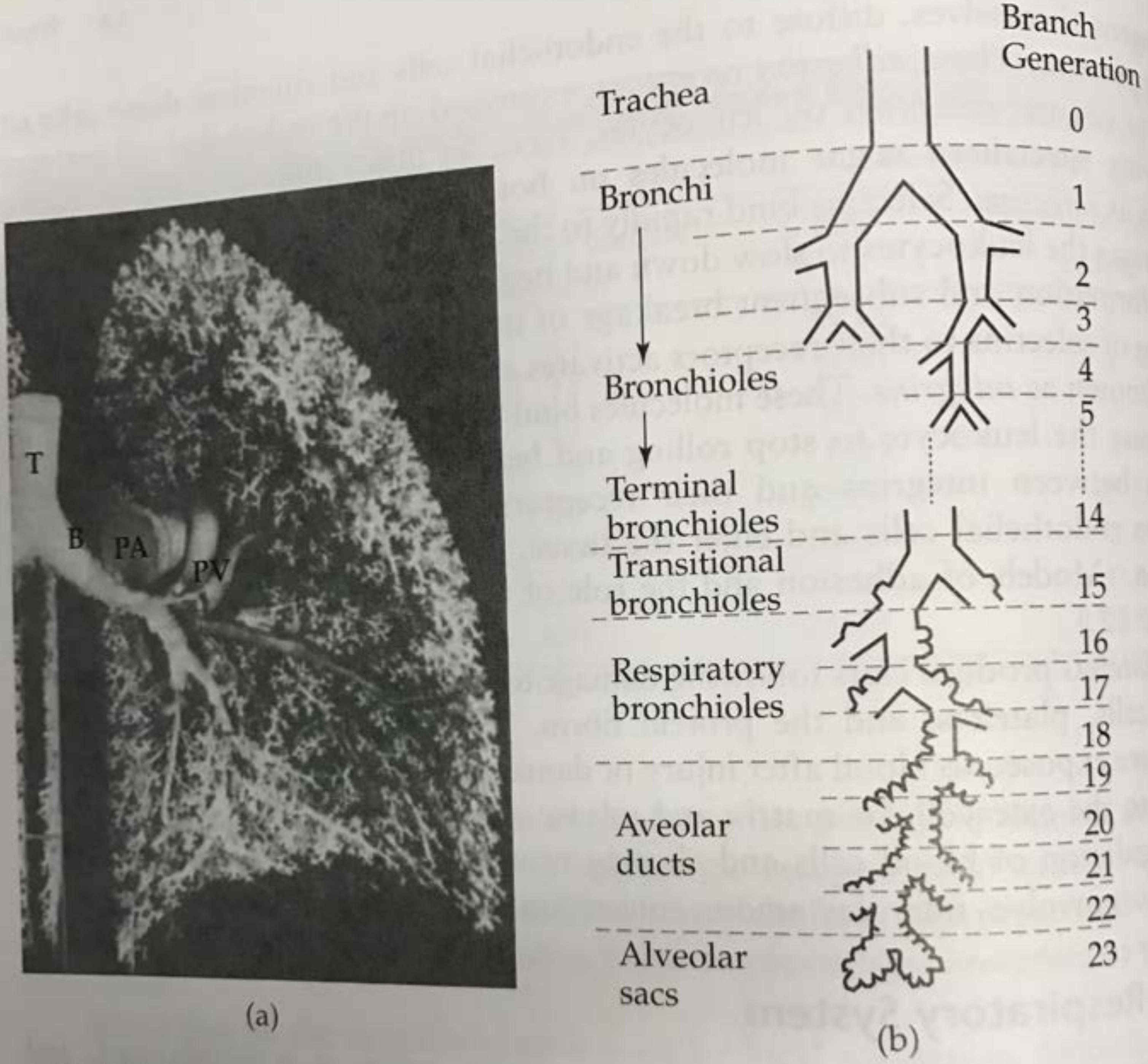


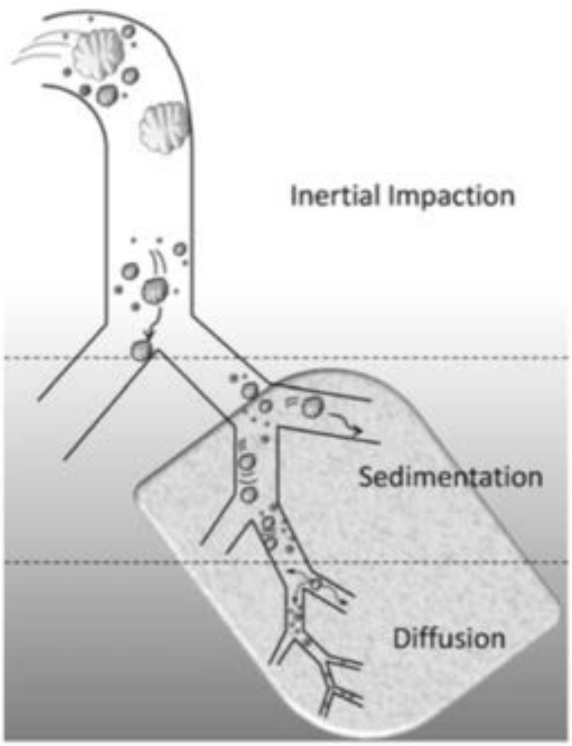
FIGURE 1.13 (a) Cast of a human lung, showing the trachea (T), one bronchus (B), the pulmonary artery (PA), and the pulmonary vein (PV). (b) Schematic of the organization of the airways in the human lung. (From Ref. [13], used with permission.)

SEEMS LIKE IT IS SAFE TO SAY THAT YOU ARE OPTIMIZED FOR THE TRADEOFFS OF EFFICIENT BREATHING VERSUS PROTECTION FROM PARTICLES

The respiratory tract is especially designed, both anatomically and functionally, so that air can reach the most distal areas of the lungs in the cleanest possible condition. Nasal hairs, nasal turbinates, vocal chords, the cilia of the bronchial epithelium, the sneeze and cough reflexes, etc., all contribute to this filtering process. And, on most occasions it is properly done. But human beings are full of paradoxes: an efficient system, designed to avoid certain

particles from penetrating into the lungs, is at the same time used to intentionally deposit drugs in the airways and even for these to reach the alveoli in the best possible condition. It is thus necessary to get around the defense systems by evading reflex arcs, mucus layers, ciliary movements, etc., so that, with the inspiratory flow, the molecules that can improve diseases are deposited in the lungs. A system that evolved over time in order to filter and clean the air should be dodged in order to deposit other substances that we deliberately want to reach the inside of the organism. Without a

PARTICLE CLEARING MECHANISMS



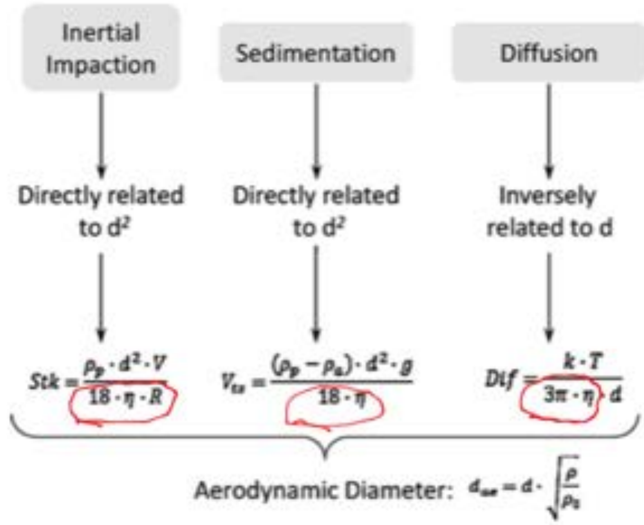


Fig. 2. The influence of particle size on deposition. *d*: particle diameter; *Stk*: Stokes number; ρ_p: particle density; *V*: air velocity; η: air viscosity; *R*: airway radius; V_{ts}: terminal settling velocity; ρ_a: air density; *g*: gravitational acceleration; *Dif*: diffusion coefficient; *k*: Boltzmann's constant; *T*: absolute temperature; d_{ae}: aerodynamic diameter; ρ_0: unity density.

B, mass, *m*, and velocity, *v*, according to Eq. (1) (Gonda, 2004):

WHERE DO EQUATIONS COME FROM ?

SOLUTION TO NAVIER-STOKES EQUATIONS FOR FLOW PAST A SPHERE: $Re \Rightarrow 0$