

CBE 30357

10/12/17

TOPICS

1) REVIEW: FLOW IN RECTANGULAR CHANNEL

ELLIPTIC PDE: SOLUTION BY SEPARATION OF VARIABLES.

- YOU SHOULD BE ABLE TO USE RESULT...

2) REVIEW "HYDRAULIC DIAMETER"

- NON-CIRCULAR CROSS-SECTION
- ALLOWS EXTENDING WHAT YOU ALREADY KNOW

3) EQUATIONS THAT DESCRIBE:

UNIFORM FLOW PAST
A STATIONARY SPHERE

4) ANALYSIS OF NAVIER-STOKES
EQUATIONS BY "DIMENSIONAL
ANALYSIS"

5) FALL BREAK 😊

RECTANGULAR CHANNEL CROSS SECTION: $h \times w$

x direction

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right] + \rho g_x \quad (3.3.26a)$$

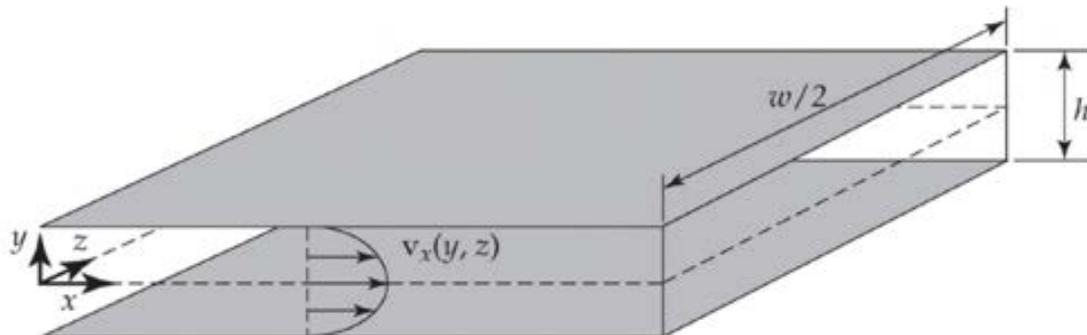
y direction

$$\rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu \left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right] + \rho g_y \quad (3.3.26b)$$

z direction

$$\rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right] + \rho g_z \quad (3.3.26c)$$

Figure 3.7 Flow in a rectangular channel of height h and width w . The section is through the plane $z = 0$.



PEARSON

Transport Phenomena in Biological Systems, Second Edition
George A. Truskey, Fan Yuan, and David F. Katz

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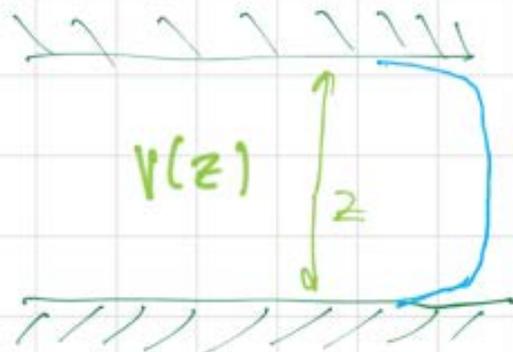
$$0 = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right)$$

POISSON'S EQ

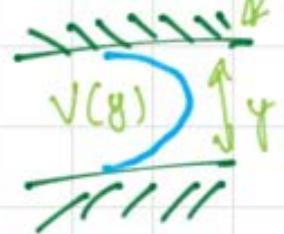
SOLVE BY SEPARATION OF VARIABLES: PP 138-139 TEXT EIGENVALUE PROBLEM

and the solution for the velocity field is

$$v_x(y, z) = \frac{\Delta p h^2}{8\mu L} \left(1 - \frac{4y^2}{h^2} \right) - \frac{\Delta p h^2}{8\mu L} \sum_{n=0}^{\infty} \frac{32(-1)^n \cosh((2n+1)\pi z/h) \cos((2n+1)\pi y/h)}{(2n+1)^3 \pi^3 \cosh((2n+1)\pi w/2h)}$$



SIMILAR
SHAPE
TO
 $v_x(y)$



**BOTTOM
WALL
SHEAR
STRESS:**

$$\tau_{yx} = \frac{\mu}{w} \int_{-w/2}^{w/2} \frac{\partial v_x}{\partial y} \Big|_{y=-h/2} dz$$

$$= \frac{\Delta p h}{2L} \left[1 - 16 \frac{h}{w} \sum_{m=0}^{\infty} \frac{(-1)^m \tanh(2m+1)\pi \frac{w}{2h}}{(2m+1)^3 \pi^3} \right]$$

ASPECT RATIO

$$\tau_{yx} \approx \left(\frac{\partial p}{\partial x} \right) h \left(\frac{1}{2} - .25 \varepsilon \right)$$

↑
BOTTOM
WALL,
DOES NOT
INCLUDE SIDES
 $\varepsilon \equiv \frac{h}{w}$

HYDRAULIC DIAMETER



USE d_H FOR "DIAMETER"
IN STANDARD CORRELATIONS

$$d_H = \frac{4 \text{ AREA}}{\text{PERIMETER}}$$

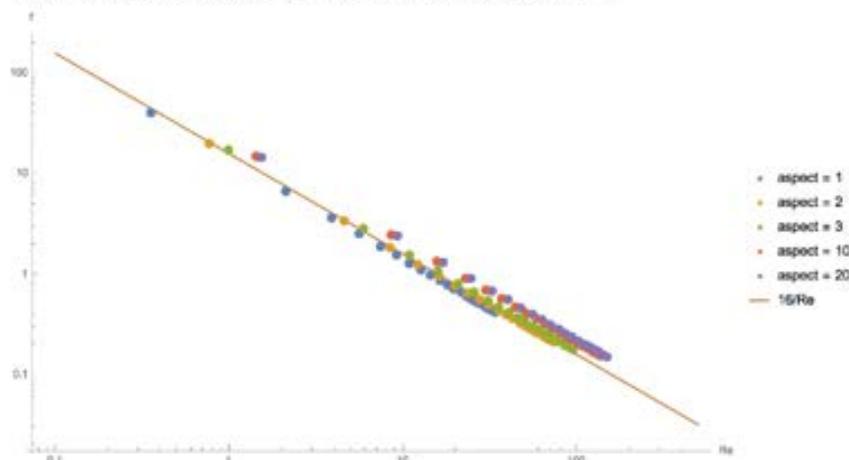

$$Re = \frac{\rho d_H V}{\mu} \quad f = \frac{\Delta P d_H}{2L \rho V^2}$$

$$f = \frac{16}{Re}, \quad f = 0.079 Re^{-0.25}$$

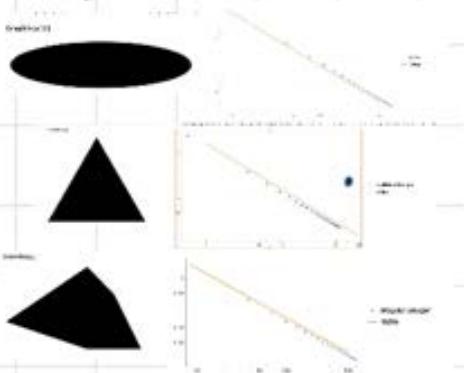
LAMINAR FLOW: RECTANGULAR CROSS-SECTION

THIS WORKS WELL

```
ListLogLogPlot[{ff1, ff2, ff3, ff10, ff20, ff16Re},
PlotLegends -> {"aspect = 1", "aspect = 2", "aspect = 3", "aspect = 10", "aspect = 20", "16/Re"},
Joined -> {False, False, False, False, False, True}, AxesLabel -> {"Re", "f"}]
```

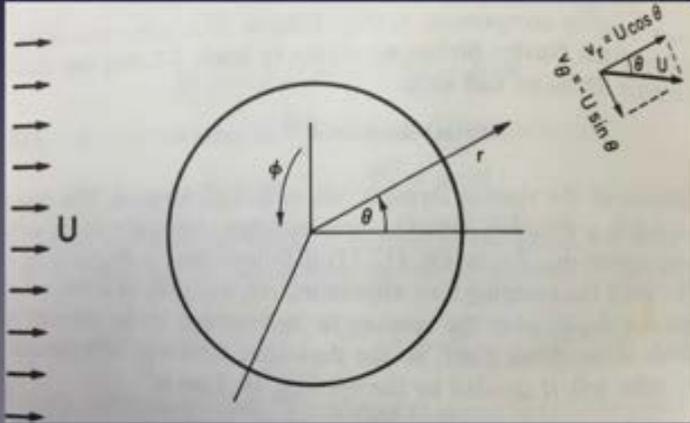


You can see that for aspect ratio = 1, 2, 3... 5, the hydraulic diameter works well.



Simple flow of interest

- Flow of liquid past a stationary sphere (which is the same as single particle moving (perhaps by gravity) through an otherwise quiescent liquid (or gas)).
- Here is drawing from Denn's book:



Non-Zero Terms

TABLE 3.4

(Continued)

Spherical coordinates

r direction

$$\rho \left[\cancel{\frac{\partial v_r}{\partial t}} + \cancel{v_r \frac{\partial v_r}{\partial r}} + \cancel{\frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta}} + \cancel{\frac{v_\phi}{r \sin \theta} \frac{\partial v_r}{\partial \phi}} + \cancel{\frac{v_\theta^2}{r}} + \cancel{\frac{v_\phi^2}{r}} \right] = \cancel{\frac{\partial p}{\partial r}} + \cancel{\rho g_r} + \mu \left[\cancel{\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial v_r}{\partial r} \right)} + \cancel{\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial v_r}{\partial \theta} \right)} + \cancel{\frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v_r}{\partial \phi^2}} - \cancel{\frac{2 v_r}{r^2}} - \cancel{\frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta}} - \cancel{\frac{2}{r^2} v_\theta \cot \theta} - \cancel{\frac{2}{r^2 \sin^2 \theta} \frac{\partial v_\phi}{\partial \phi}} \right]$$

L.H.S.

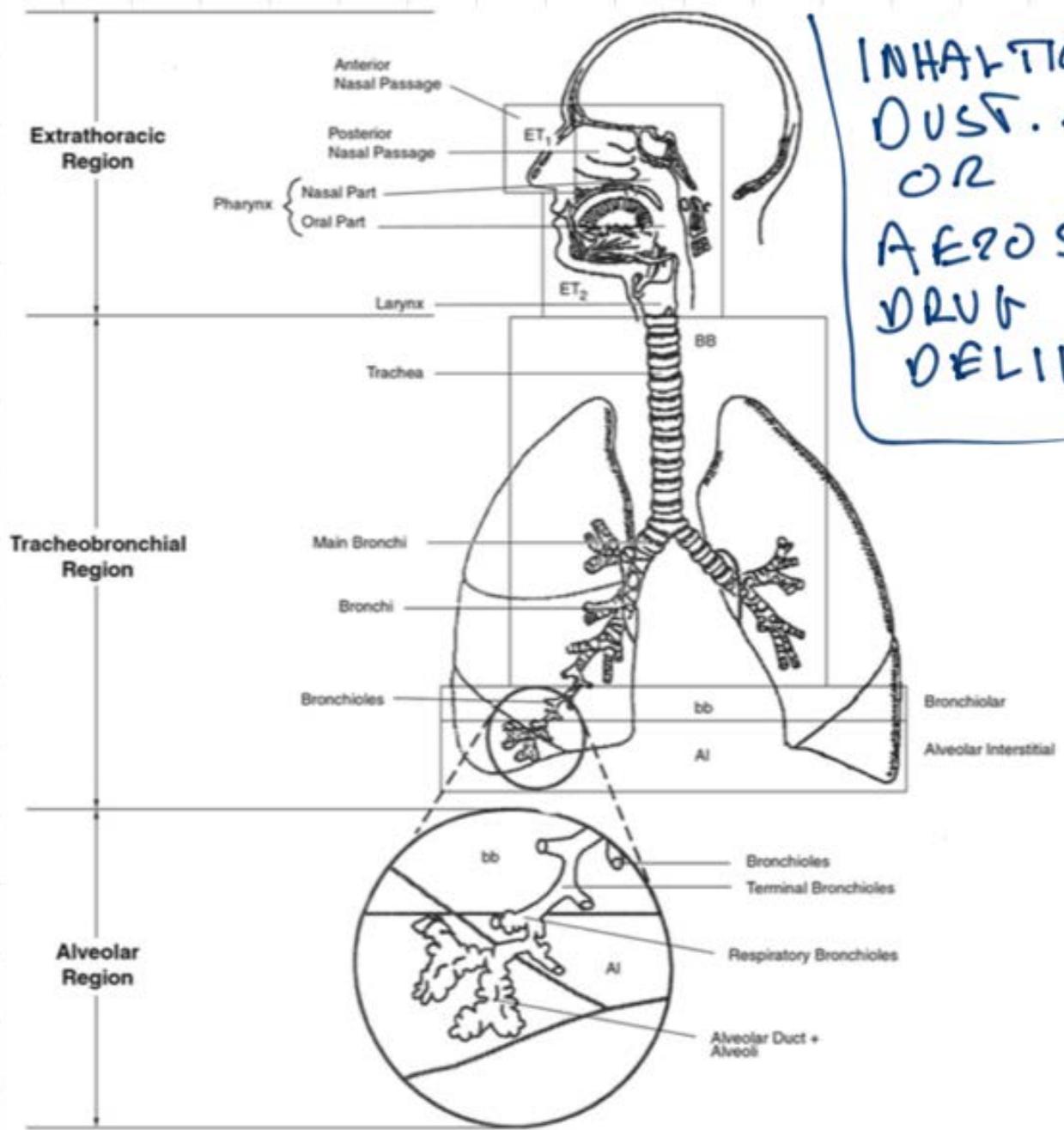
$$\vec{v} \cdot \nabla \vec{v} \neq 0$$

theta direction

$$\rho \left(\cancel{\frac{\partial v_\theta}{\partial t}} + \cancel{v_r \frac{\partial v_\theta}{\partial r}} + \cancel{\frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta}} + \cancel{\frac{v_\phi}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi}} + \cancel{\frac{v_\theta v_r}{r}} - \cancel{\frac{v_\phi^2 \cot \theta}{r}} \right) = \cancel{\frac{1}{r} \frac{\partial p}{\partial \theta}} + \mu \left[\cancel{\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial v_\theta}{\partial r} \right)} + \cancel{\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial v_\theta}{\partial \theta} \right)} + \cancel{\frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v_\theta}{\partial \phi^2}} + \cancel{\frac{2}{r^2} \frac{\partial v_r}{\partial \theta}} - \cancel{\frac{v_\theta}{r^2 \sin^2 \theta}} - \cancel{\frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\phi}{\partial \phi}} \right] + \cancel{\rho g_\theta}$$

phi direction

$$\rho \left(\cancel{\frac{\partial v_\phi}{\partial t}} + \cancel{v_r \frac{\partial v_\phi}{\partial r}} + \cancel{\frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta}} + \cancel{\frac{v_\phi}{\sin \theta} \frac{\partial v_\phi}{\partial \phi}} + \cancel{\frac{v_\phi v_r}{r}} + \cancel{\frac{v_\theta v_\phi}{r} \cot \theta} \right) = \cancel{\frac{1}{r \sin \theta} \frac{\partial p}{\partial \phi}} + \mu \left[\cancel{\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial v_\phi}{\partial r} \right)} + \cancel{\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial v_\phi}{\partial \theta} \right)} + \cancel{\frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v_\phi}{\partial \phi^2}} + \cancel{\frac{2}{r^2 \sin \theta} \frac{\partial v_r}{\partial \phi}} - \cancel{\frac{v_\theta}{r^2 \sin^2 \theta}} + \cancel{\frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\theta}{\partial \phi}} \right] + \cancel{\rho g_\phi}$$



INHALATION OF
DUST...
OR
AEROSOL
DRUG
DELIVERY

TORTUOUS PATH, PARTICLES
STICK TO WALL: CLEARED
BY CILIA.
LONG PATH: ONLY LAST ~5
BRANCHES ABSORB
OR, COULD BREATHE BACK OUT.

AEROSOL DRUG DELIVERY

Review Article

The Transport and Deposition of Nanoparticles in Respiratory System by Inhalation

Huiting Qiao,^{1,2} Wenyong Liu,¹ Hongyu Gu,¹ Daifa Wang,¹ and Yu Wang¹

¹Key Laboratory for Biomechanics and Mechanobiology of Ministry of Education, School of Biological Science and Medical Engineering, Beihang University, Beijing 10091, China

²School of Biomedical Engineering, Science and Health Systems, Drexel University, Philadelphia, PA 19104, USA

Correspondence should be addressed to Yu Wang, wangyu@bmae.edu.cn

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The inhaled nanoparticles have attracted more and more attention, since they are more easily to enter the deep part of respiratory system. Some nanoparticles were reported to cause pulmonary inflammation. The toxicity of nanoparticles depends not only on its

topical respiratory treatment it is best to use particles with an MMAD between 0.5 and 5 μm . This is what is known as the breathable fraction of an aerosol.⁶



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Journal of Controlled Release

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Review

Pulmonary drug delivery by powder aerosols

Michael Yifei Yang, John Gar Yan Chan, Hak-Kim Chan*

Advanced Drug Delivery Group, Faculty of Pharmacy (A121), University of Sydney, Sydney, NSW 2006, Australia



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ABSTRACT

The efficacy of pharmaceutical aerosols relates to its deposition in the clinically relevant regions of the lungs, which can be assessed by in vivo lung deposition studies. Dry powder formulations are popular as devices are por-



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Review

Influence of particle size on regional lung deposition – What evidence is there?

Thiago C. Carvalho^a, Jay I. Peters^b, Robert O. Williams III^{a,*}

^aDivision of Pharmaceutics, College of Pharmacy, University of Texas at Austin, Austin, TX, USA

^bDepartment of Medicine, Division of Pulmonary Diseases/Critical Care Medicine, The University of Texas Health Science Center at San Antonio, San Antonio, TX, USA

ARTICLE INFO

Article history:
Received 6 October 2010

ABSTRACT

The unders
of inhalati

Deposition of Inhaled Particles in the Human Respiratory Tract and Consequences for Regional Targeting in Respiratory Drug Delivery

Joachim Heyder

GSF-National Research Center for Environment and Health, Institute for Inhalation Biology, Neuherberg/Munich, Germany

Particle behavior in the human respiratory tract is well understood and can be used to (1) estimate particle deposition in all regions of the respiratory tract for any regional deposit of any pattern

(decreasing respiratory rate). Total diffusional deposition therefore decreases with increasing particle size up to about 1 μm

IN GENERAL, WE CAN'T GET
AN ANALYTICAL SOLUTION TO
A NONLINEAR P.D.E.

PERHAPS, SOME SIMPLIFICATIONS
COULD BE JUSTIFIED....

HOW CAN THIS BE DONE?

- 1) WE NEED A PROCEDURE
THAT ESTIMATES (BRACKETS)
THE MAGNITUDE OF
INDIVIDUAL TERMS
- 2) FROM THIS, WE CAN GO A
STEP FURTHER,
NONDIMENSIONALIZE EQ.

3) WHEN THIS IS DONE,
A PARAMETER "APPEARS"
REYNOLDS NUMBER.

4) IN THE LIMIT $Re \rightarrow 0$
L.H.S. TERMS CAN BE
NEGLECTED....

WE CAN SOLVE SOME
PROBLEMS

HOW TO ESTIMATE (RELATIVE) SIZE OF THESE TERMS

$$\rho v_n \frac{\partial v_n}{\partial x} \approx \rho v_n \frac{\Delta v_n}{\Delta x}$$

v_n	\sim	U_0	} ABOUT THIS IS BIG
Δv_n	\sim	U_0	
Δx	\sim	<u>2,3...</u> R	

THUS

$$\rho v_n \frac{\partial v_n}{\partial x} \approx \# \rho \frac{U_0^2}{R}$$

HOW ABOUT VISCOUS TERM?

$$\mu \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial v_r}{\partial r}$$

✓ ↓ √

ONE PIECE

$$\mu \frac{\partial^2 v_r}{\partial r^2} = \mu \frac{\partial}{\partial r} \left(\frac{\partial v_r}{\partial r} \right)$$

$$\approx \mu \frac{1}{\Delta r} \left(\frac{\Delta v}{\Delta r} \Big|_1 - \frac{\Delta v}{\Delta r} \Big|_2 \right)$$

$$\approx \frac{\mu}{R} \left(\frac{u_0}{R} \right)$$

$$= \mu \frac{u_0}{R^2}$$

TAKE
RATIO

$$\frac{\rho u_0^2 / R}{\mu u_0 / R^2} = \frac{\rho u_0 R}{\mu} = Re$$

WE SURMISE THAT IF
 $Re \ll 1$, WE CAN
NEGLECT INERTIA
TERMS RELATIVE TO
VISCOS TERMS!!

IF SO: EQUATIONS WILL BE
LINEAR.

WE CAN FORMALIZE THIS
BY NONDIMENSIONALIZING
OUR EQUATIONS

EACH TERM IS "COMPARED" TO SCALES
OF PROBLEM.

LENGTH SCALE: R

VELOCITY SCALE: U_0

TIME SCALE: R/U_0

DEFINE:

$$v_r^* = \frac{v_r}{U_0}$$

$$v_\theta^* = \frac{v_\theta}{U_0}$$

$$r^* = \frac{r}{R}$$

$$r = r^* R$$

SUBSTITUTE DEPENDENT VARIABLES

USE CHAIN RULE FOR DERIVATIVES

$$\frac{\partial}{\partial r} = \frac{\partial}{\partial r^*} \frac{\partial r^*}{\partial r} = \left(\frac{\partial r^*}{\partial r} \right) \frac{\partial}{\partial r^*}$$

\downarrow
 $1/R$

$$\begin{aligned} \frac{\partial V_n}{\partial r} &= \frac{\partial r^*}{\partial r} \frac{\partial}{\partial r^*} (U_0 V_n^*) \\ &= \frac{U_0}{R} \frac{\partial V_n^*}{\partial r^*} \\ \frac{\partial^2 V_n}{\partial r^2} &= \left(\frac{\partial r^*}{\partial r} \right)^2 U \frac{\partial^2 V_n^*}{\partial r^{*2}} \\ &= \frac{U}{R^2} \frac{\partial^2 V_n^*}{\partial r^{*2}} \end{aligned}$$

WHEN YOU GET DONE

$$V_n^* \frac{\partial V_n^*}{\partial r^*} = 0(1)$$

$$\frac{\partial^2 V_n^*}{\partial r^{*2}} = 0(1)$$

MOST
IMPORTANT
POINT!!



WE NEED p^*

$$P_0 \equiv \frac{\mu U_0}{R} \quad \therefore p^* = \frac{P}{\mu U_0 / R}$$

FOR ALL TERMS ...

$$\operatorname{Re} \left(\underbrace{\vec{\nabla}^* \cdot \vec{\nabla}^* \vec{V}^*}_{\mathcal{O}(1)} \right) = - \underbrace{\vec{\nabla}^* p^*}_{\mathcal{O}(1)} + \underbrace{\nabla^{*2} \vec{V}^*}_{\mathcal{O}(1)}$$

THEN IF $Re \rightarrow 0$ WE GET

$$\vec{\nabla}^* p^* = \nabla^{*2} \vec{V}^*$$

STOKES
EQUATION

LINEAR PDE

MANY SOLUTIONS ARE

KNOWN

... YES, BUT ARE THESE USEFUL??

FOR PEARL IN 1960'S
COMMERCIAL

$$Re = \frac{Dv\rho_f}{\mu_f} = \frac{(1.5 \text{ cm})(.5 \text{ cm/s})(1 \text{ g/cm}^3)}{4 \text{ g/cm-s}}$$

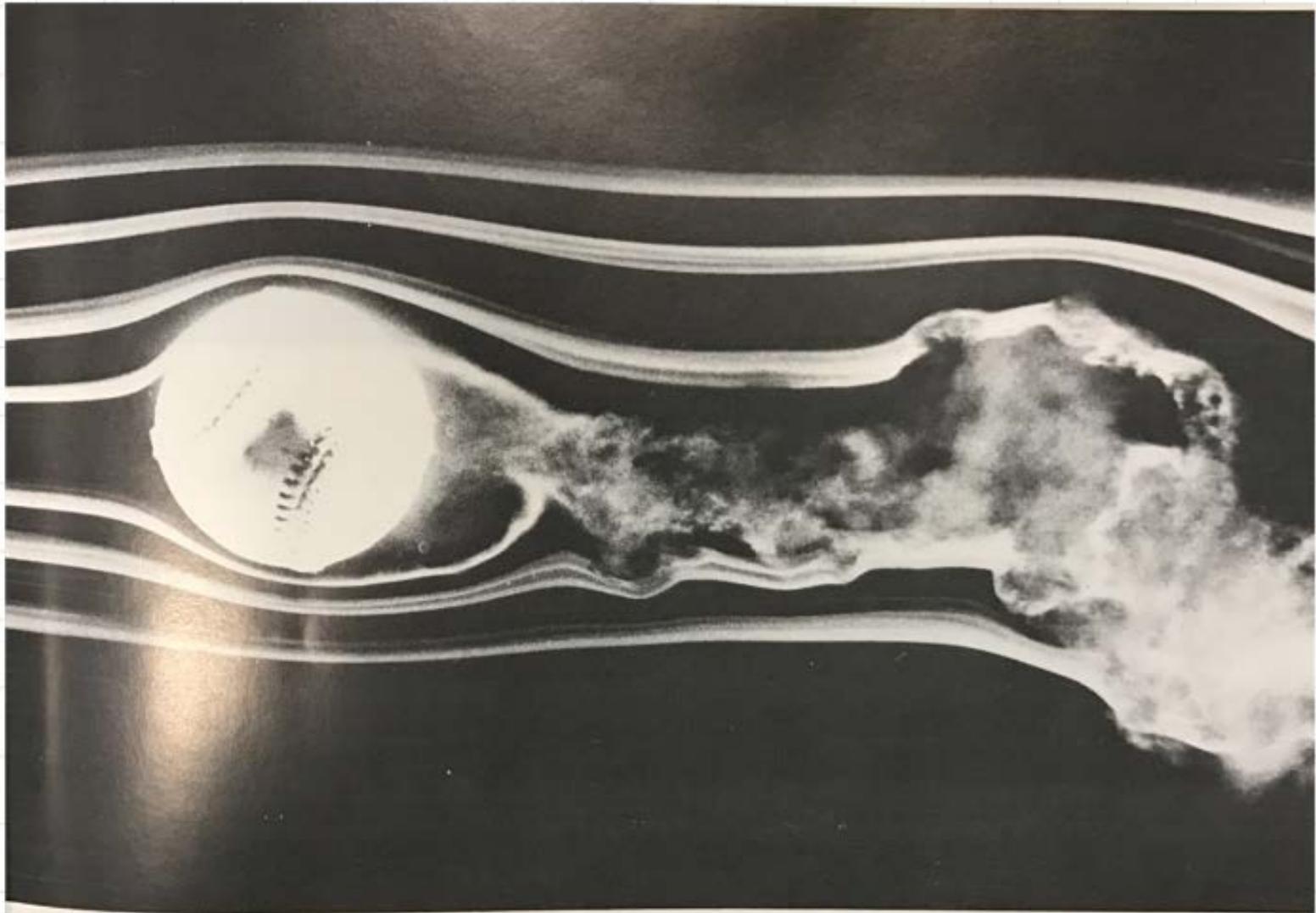
$$Re \approx .2$$

FOR 10 μm PARTICLE
IN AIR

$$Re = \frac{(10^{-3} \text{ cm})(.1 \text{ cm/s})(\frac{1}{890} \text{ g/cm}^3)}{.00018 \text{ g/cm-s}}$$
$$= 6 \times 10^{-3}$$

WHAT WOULD FLOW PAST
A SPHERE AT LOW Re
LOOK LIKE?

NOT THIS:

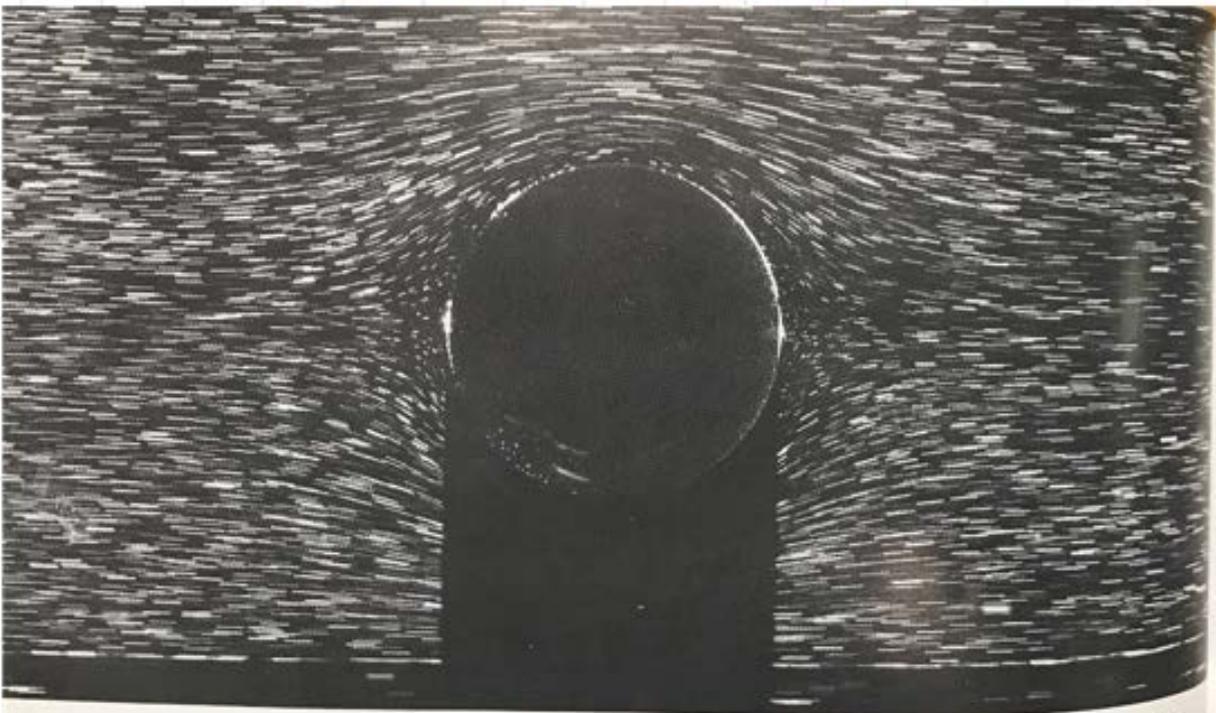


66. **Spinning baseball.** The late F. N. M. Brown devoted many years to developing and using smoke visualization in wind tunnels at the University of Notre Dame. Here the

flow speed is about 77 ft/sec and the ball is rotated at 630 rpm. This unpublished photograph is similar to several in Brown 1971. *Photograph courtesy of T. J. Mueller*

LOW Re FLOWS ...

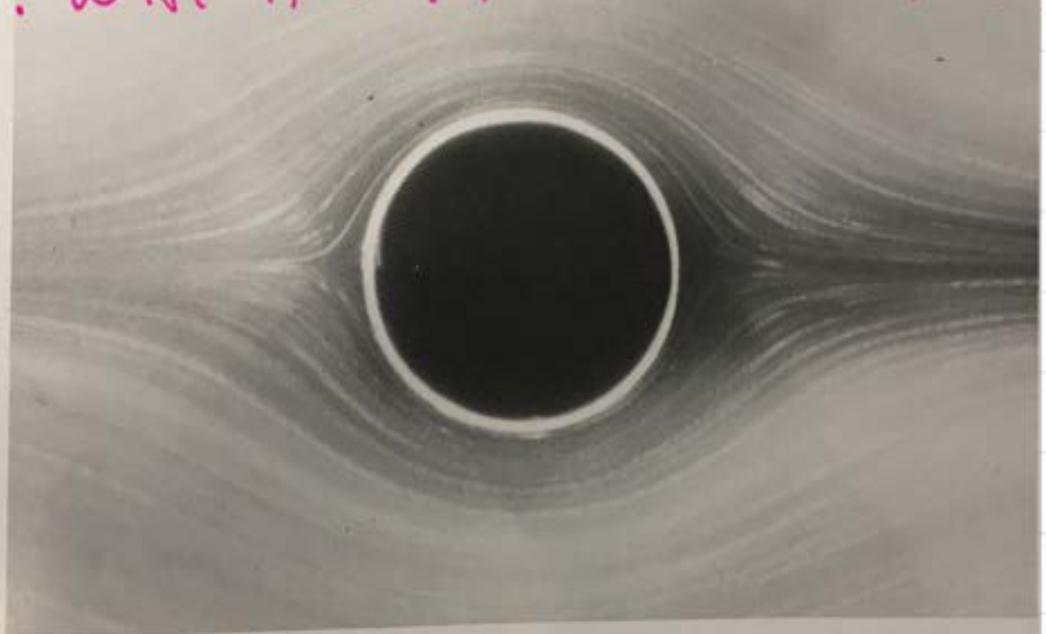
NO
WAKE



8. Sphere moving through a tube at $R=0.10$, relative motion. A free sphere is falling steadily down the axis of a tube of twice its diameter filled with glycerine. The camera is moved with the speed of the sphere to show the flow rel-

ative to it. The photograph has been rotated to show flow from left to right. Tiny magnesium cuttings are illuminated by a thin sheet of light, which casts a shadow of the sphere. *Coutanceau 1968*

SYMMETRIC: WHICH WAY IS FLOW GOING?

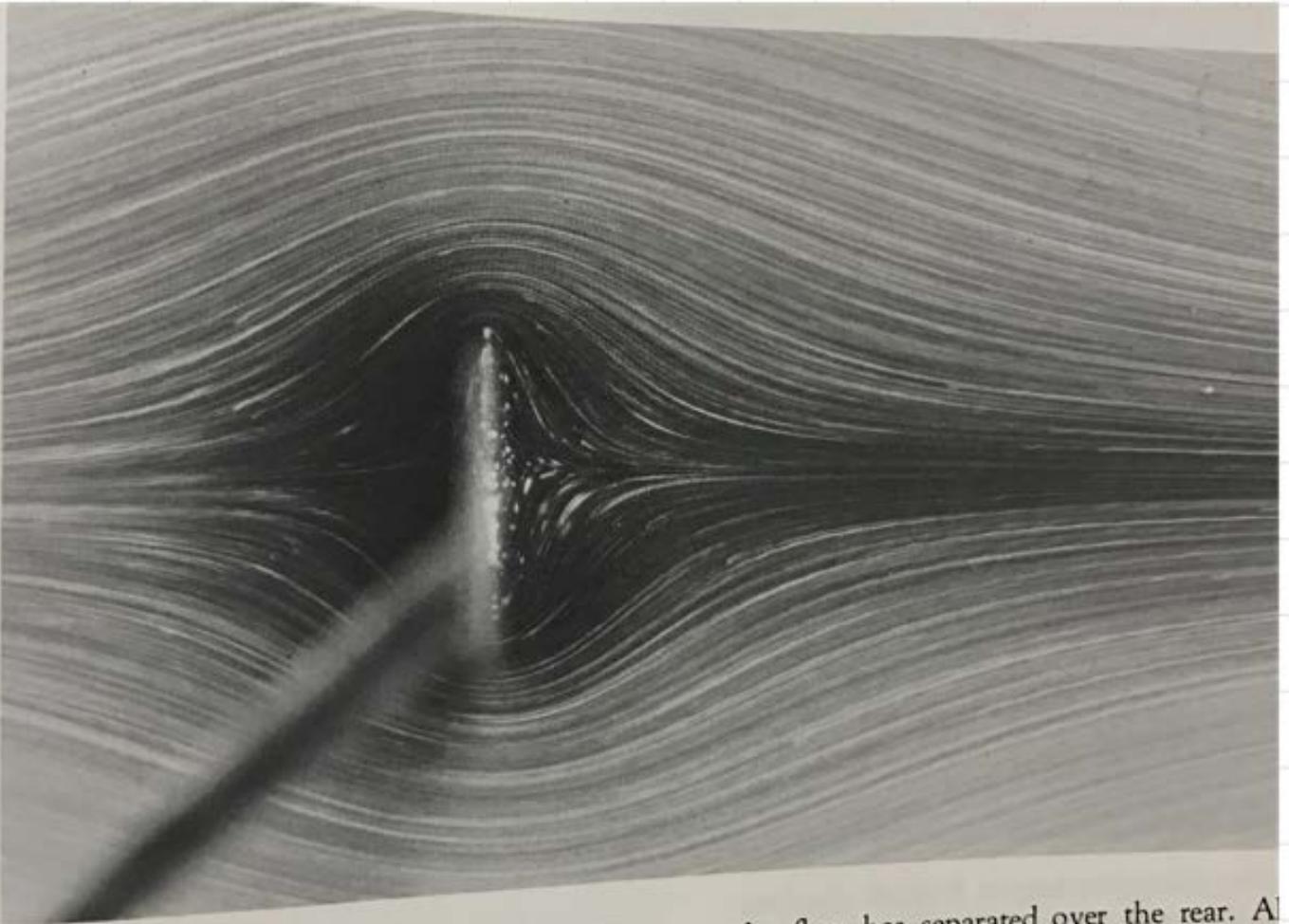


9. Uniform flow past a circular cylinder at $R=0.16$. That the flow is from left to right can scarcely be deduced from the streamline pattern, because in the limit of zero Reynolds number the flow past a solid body is reversible, and hence symmetric about a symmetric shape. It resem-

bles superficially the pattern of potential flow but the disturbances to the uniform stream come more slowly. The flow of water is shown by dust. *Photograph by Sadatoshi Taneda*

Low Re :

EVEN A FLAT PLATE
HAS NO WAKE



Uniform flow normal to a plate at $R=0.334$. The streamline pattern is still almost symmetric fore-and-aft at higher Reynolds number. It is possible, however, that

the flow has separated over the rear. A shows the flow of glycerine. Taneda 1968

BESIDES OLD TV
COMMERCIALS WHERE
DO THESE FLOWS OCCUR?

1) PROCESS FLOWS OF SUSPENSIONS

a) FOODS.

b) CONSUMER PRODUCTS

c) COMPOSITE MOLDING

d) SOLID-LIQUID MIXING

e) SOLIDS TRANSPORT

f) FLUIDIZED BED
REACTORS

2) MEDICAL: INHALERS
FOR DRUG DELIVERY

3) ENVIRONMENT:
AEROSOLS, SEDIMENT TRANSPORT

TAKE SPECIFIC EXAMPLE
OF "INHALERS"

PROCESS INVOLVES
EITHER

NEBULIZING A LIQUID
INTO SMALL DROPS

OR DISPERSING SMALL
PARTICLES INTO A
CARRIER GAS

IN EITHER CASE, SOUNDS
LIKE A GREAT IDEA!!

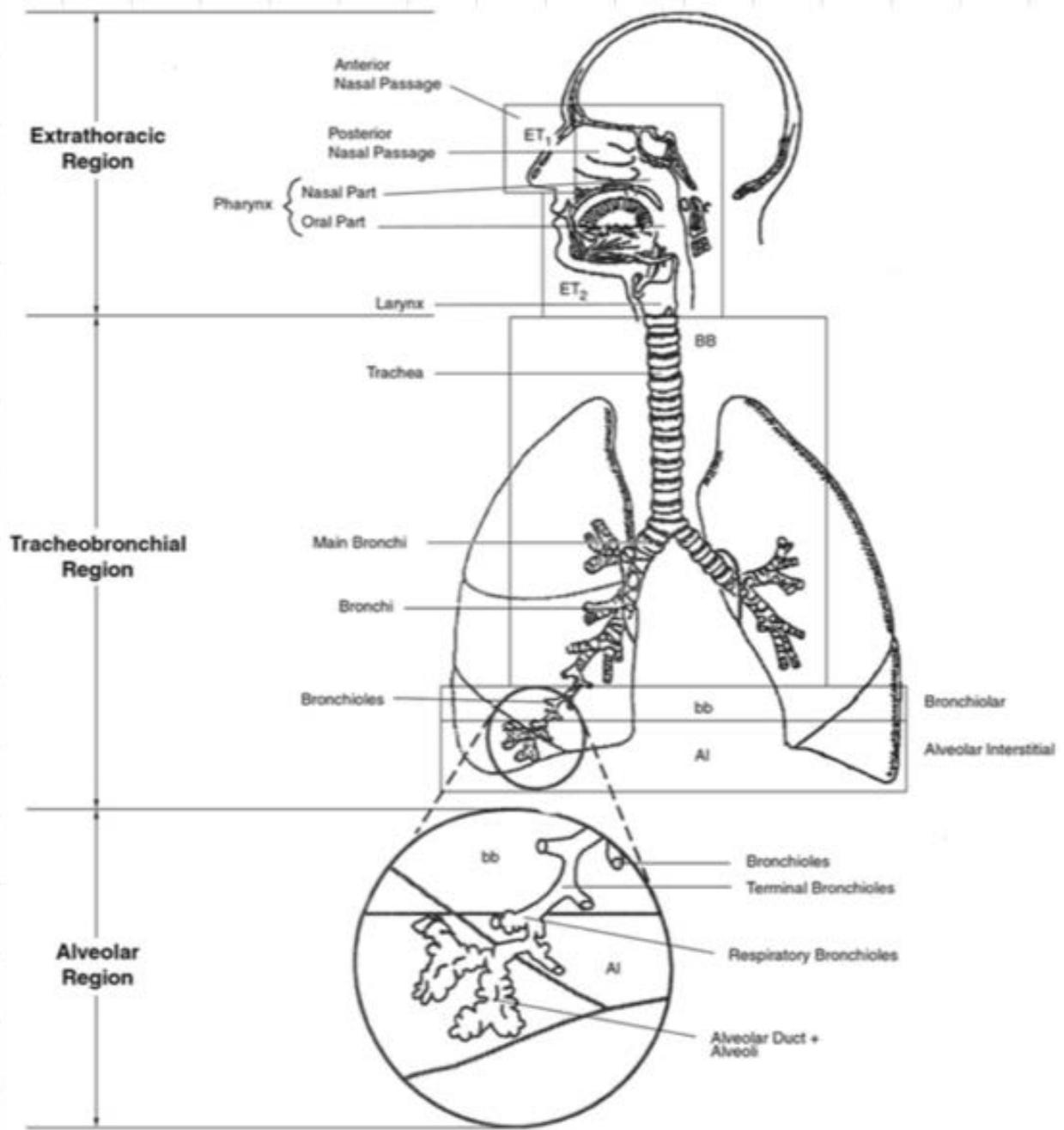
A) YOU DIRECTLY APPLY ANTI-INFLAMMATORY AGENT TO LUNG TISSUE.

B) QUICK PATH TO BLOOD STREAM FOR PHARMACEUTICALS THAT CAN'T EASILY BE DELIVERED BY PILLS.

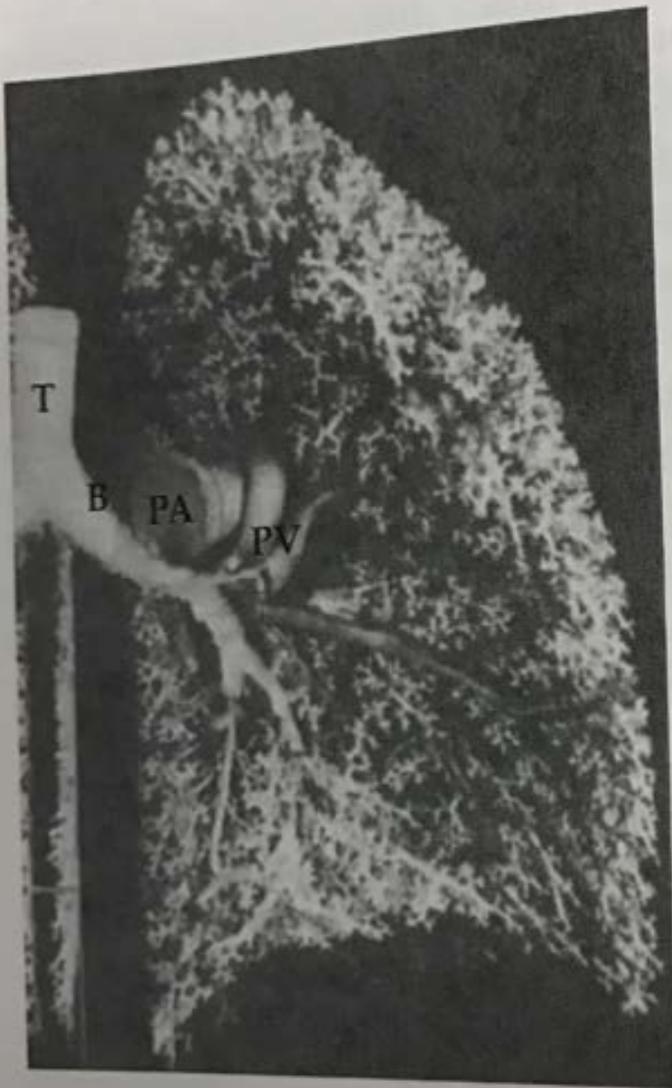
THERE ARE SOME PROBLEMS

1) HUMANS HAVE HAD TO
DEAL WITH AIRBORNE
SOLID PARTICLES ...
ALWAYS

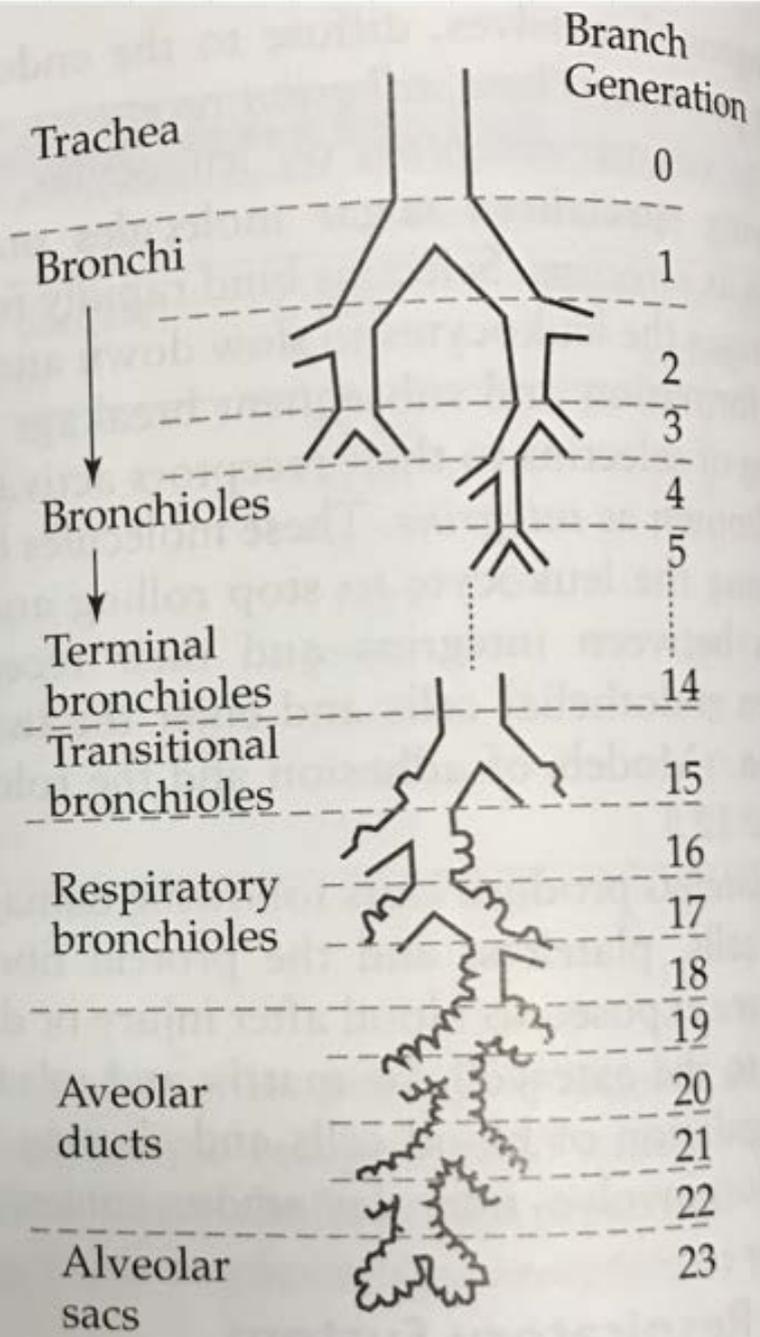
YOU HAVE DEFENSES



TORTUOUS PATH, PARTICLES
 STICK TO WALL: CLEARED
 BY CILIA.
 LONG PATH: ONLY LAST ~5
 BRANCHES ABSORB
 OR, COULD BREATHE BACK OUT.



(a)



(b)

FIGURE 1.13 (a) Cast of a human lung, showing the trachea (T), one bronchus (B), the pulmonary artery (PA), and the pulmonary vein (PV). (b) Schematic of the organization of the airways in the human lung. (From Ref. [13], used with permission.)

TABLE 1.14

Morphometric Data for the Bronchi of a Human Lung

Order ^a	Mean diameter (mm)	Mean length (mm)
0	18.0	120.0
1	12.2	47.6
2	8.3	19.0
3	5.6	7.6
4	4.5	12.7
5	3.5	10.7
6	2.8	9.0
7	2.3	7.6
8	1.86	6.4
9	1.54	5.4
10	1.3	4.6
11	1.09	3.9
12	0.95	3.3
13	0.82	2.7
14	0.74	2.3
15	0.66	2.0
16	0.6	1.65
17	0.54	1.41
18	0.5	1.17
19	0.47	0.99
20	0.45	0.83
21	0.43	0.7
22	0.41	0.59
23	0.41	0.5

Source: Adapted from Ref. [14].

^aThe trachea is called the airway of order 1, and the alveoli are the airways of order 23.

WIDELY USED, BUT SUBJECT OF CONTINUOUS STUDY!

Review Article

The Transport and Deposition of Nanoparticles in Respiratory System by Inhalation

Huiting Qiao,^{1,2} Wenyong Liu,¹ Hongyu Gu,¹ Daifa Wang,¹ and Yu Wang¹

¹Key Laboratory for Biomechanics and Mechanobiology of Ministry of Education, School of Biological Science and Medical Engineering, Beihang University, Beijing 10091, China

²School of Biomedical Engineering, Science and Health Systems, Drexel University, Philadelphia, PA 19104, USA

Correspondence should be addressed to Yu Wang, wangyu@buaa.edu.cn

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The inhaled nanoparticles have attracted more and more attention, since they are more easily to enter the deep part of respiratory system. Some nanoparticles were reported to cause pulmonary inflammation. The toxicity of nanoparticles depends not only on its

topical respiratory treatment it is best to use particles with an MMAD between 0.5 and 5 μm. This is what is known as the breathable fraction of an aerosol.⁶

Journal of Controlled Release. Includes Elsevier logo, ScienceDirect link, and a red circular logo with 'R' and an arrow.

Pulmonary drug delivery by powder aerosols. Michael Yifei Yang, John Gar Yan Chan, Hak-Kim Chan*. Includes a CrossMark logo.

ARTICLE INFO ABSTRACT. The efficacy of pharmaceutical aerosols relates to its deposition in the clinically relevant regions of the lungs, which can be assessed by in vivo lung deposition studies. Dry powder formulations are popular as devices are por-

International Journal of Pharmaceutics. Includes Elsevier logo, ScienceDirect link, and a green book cover image.

Influence of particle size on regional lung deposition – What evidence is there? Thiago C. Carvalho^a, Jay I. Peters^b, Robert O. Williams III^{a,*}. Includes footnotes for ^a and ^b.

Deposition of Inhaled Particles in the Human Respiratory Tract and Consequences for Regional Targeting in Respiratory Drug Delivery. Joachim Heyder. GSF-National Research Center for Environment and Health, Institute for Inhalation Biology, Neuherberg/Munich, Germany. Particle behavior in the human respiratory tract is well understood and can be used to (1) estimate particle deposition in all regions of the respiratory tract for any regional model of any pattern (decreasing respiratory rate). Total diffusional deposition therefore decreases with increasing particle size up to about 1 μm.

SEEMS LIKE IT IS SAFE TO SAY THAT YOU ARE OPTIMIZED FOR THE TRADEOFFS OF EFFICIENT BREATHING VERSUS PROTECTION FROM PARTICLES

The respiratory tract is especially designed, both anatomically and functionally, so that air can reach the most distal areas of the lungs in the cleanest possible condition. Nasal hairs, nasal turbinates, vocal chords, the cilia of the bronchial epithelium, the sneeze and cough reflexes, etc., all contribute to this filtering process. And, on most occasions it is properly done. But human beings are full of paradoxes: an efficient system, designed to avoid certain

particles from penetrating into the lungs, is at the same time used to intentionally deposit drugs in the airways and even for these to reach the alveoli in the best possible condition. It is thus necessary to get around the defense systems by evading reflex arcs, mucus layers, ciliary movements, etc., so that, with the inspiratory flow, the molecules that can improve diseases are deposited in the lungs. A system that evolved over time in order to filter and clean the air should be dodged in order to deposit other substances that we deliberately want to reach the inside of the organism. Without a

PARTICLE CLEARING MECHANISMS

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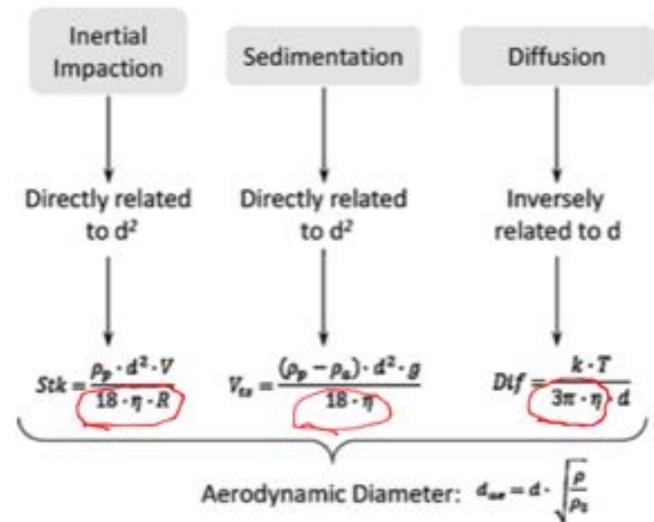
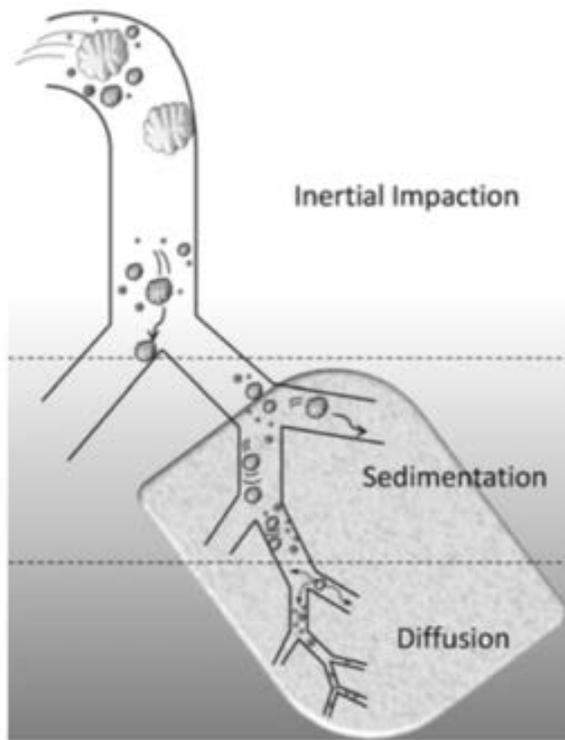
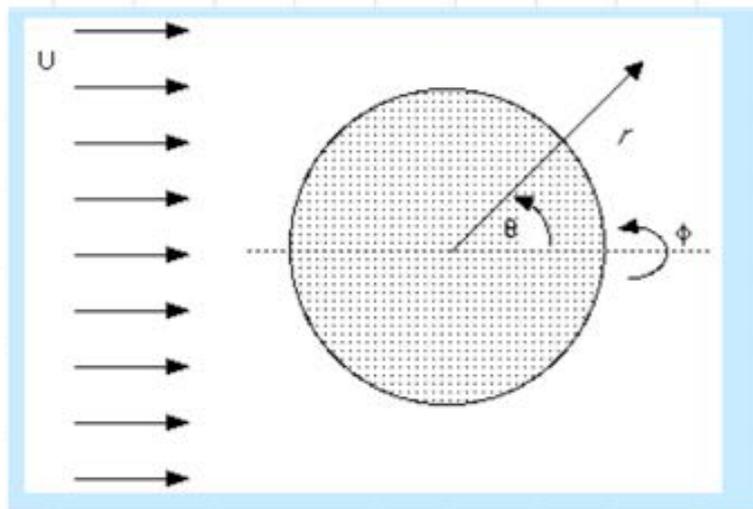


Fig. 2. The influence of particle size on deposition. d : particle diameter; Stk : Stokes number; ρ_p : particle density; V : air velocity; η : air viscosity; R : airway radius; V_{ts} : terminal settling velocity; ρ_a : air density; g : gravitational acceleration; Dif : diffusion coefficient; k : Boltzmann's constant; T : absolute temperature; d_{ae} : aerodynamic diameter; ρ_0 : unity density.

B , mass, m , and velocity, v , according to Eq. (1) (Gonda, 2004):

WHERE DO EQUATIONS COME FROM?

SOLUTION TO NAVIER-STOKES EQUATIONS FOR FLOW PAST A SPHERE: $Re \Rightarrow 0$



USE SPHERICAL COORDINATES

- CONTINUITY EQ.
 - r -DIRECTION N.S. EQ.
 - θ -DIRECTION N.S. EQ.
- } $R \rightarrow 0$
- JUST THE R.H.S.'S

$$v_r(r=R) = 0 \quad v_\theta(r=R) = 0$$

$$v_r(r \rightarrow \infty) = U \cos \theta$$

$$v_\theta(r \rightarrow \infty) = -U \sin \theta$$

SOLUTION PROCEDURE

SEQUENTIALLY CONSIDER
3 PDE'S USING AN
ASSUMED FORM OF SOLUTION

$$\begin{aligned} u_n &\sim f(r) \cos \theta \\ u_\theta &\sim g(r) \sin \theta \end{aligned} \left. \vphantom{\begin{aligned} u_n \\ u_\theta \end{aligned}} \right\} \begin{array}{l} \text{FROM} \\ \text{B.C.'S} \end{array}$$

CONTINUITY EQUATION
ALLOWS EASY ISOLATION OF

$$g(r)$$

SUBS THIS TO ELIMINATE

$$g(r) \text{ IN } u_n \text{ \& } u_\theta \text{ EQ'S}$$

TAKE CROSS-DERIVATIVES
AND SUBTRACT $u_n \text{ \& } u_\theta$ EQ
TO ELIMINATE PRESSURE

SOLVE 4TH ORDER
EULER ODE.

FIT B.C.'S 👍

$$\text{DRAG} = 6\pi\mu R U$$

WE GET: SETTLING VELOCITY

EINSTEIN USED THIS
TO CALCULATE DIFFUSIVITY
OF A PARTICLE

$$D = \frac{kT}{6\pi\mu R}$$

Annotations:

- D : PARTICLE DIFFUSIVITY
- ← kT : BOLTZMANN CONSTANT
- ← R : PARTICLE RADIUS