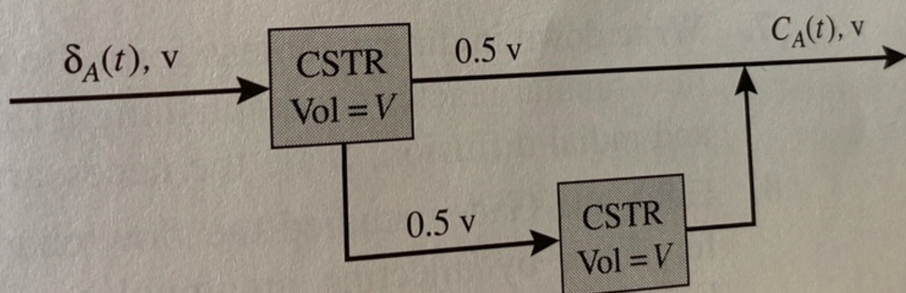
Exercises for Chapter 8

1. Find the residence time distribution, that is, the effluent concentration of tracer A after an impulse input at $t = 0$, for the following system of equivolume CSTRs with a volumetric flow rate of liquid into the system equal to v:

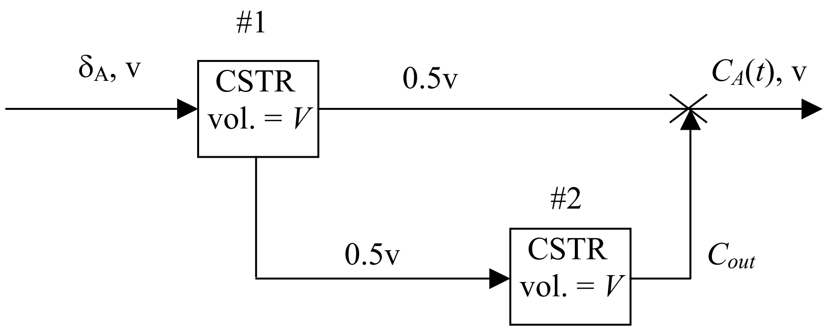


How does the RTD compare to that of a single CSTR with volume $2V$?
... of plug flow and continuous stirred

Chapter 8

Exercise 1:

Find the effluent concentration of a tracer after in impulse input for the following system:



A balance on CSTR #1 yields the following concentration of the tracer as a function of time (equation (h) in example 8.2.1):

$$C(t) = \frac{n}{\mathrm{v}}\frac{\exp(-\frac{t}{\tau})}{\tau} \qquad (1)$$

Next, write a balance on CSTR #2 (accumulation = in – out)

$$V\frac{dC_{out}}{dt} = 0.5\mathrm{v}C - 0.5\mathrm{v}C_{out} \qquad (2)$$

Substitute equation (1) into equation (2):

$$\frac{dC_{out}}{dt} + \frac{0.5}{\tau}C_{out} = \frac{0.5n}{\tau^2\mathrm{v}}\exp(-\frac{t}{\tau}) \qquad (3)$$

Solve this first-order differential equation (with the initial condition $C_{out}(0) = 0$)

$$C_{out} = \frac{n}{\mathrm{v}\tau}\exp(-\frac{0.5t}{\tau}) - \frac{n}{\mathrm{v}\tau}\exp(-\frac{t}{\tau}) \qquad (4)$$

Write a mass balance at point X (in = out)

$$0.5\mathrm{v}C + 0.5\mathrm{v}C_{out} = \mathrm{v}C_A \qquad (5)$$

Therefore,

$$C_A = 0.5C + 0.5C_{out} \tag{6}$$

After substituting equations (1) and (4) into equation (6),

$$C_A = \frac{n}{2\mathrm{v}\tau}\exp\left(-\frac{0.5t}{\tau}\right)$$

The RTD, $E(t)$, can be found from the following equation:

$$E(t) = \frac{C_A}{\int_0^\infty C_A dt} \tag{7}$$

Therefore,

$$E(t) = \frac{\exp\left(-\frac{0.5t}{\tau}\right)}{2\tau} \tag{8}$$

For a single CSTR with a volume equal to 2V, define

$$V' = 2V$$

$$\tau' = \frac{2V}{\mathrm{v}} = 2\tau$$

Therefore,

$$C_A = \frac{n}{2\mathrm{v}\tau}\exp(-\frac{t}{2\tau}) \tag{9}$$

The concentration of a tracer (C_A) for a single CSTR with $V' = 2V$ is the same as the first system. Therefore, the RTD is the same also.

Exercise 2:

The RTD of a PFR is τ_p (not a function of time). Therefore, it does not matter if the PFR is first or the CSTR is first. The RTD of either of these systems is that of a CSTR shifted by τ_p units later.

4. Calculate the mean concentration of A at the outlet ($z = L$) of a laminar flow, tubular reactor ($\overline{C}_A^L$) accomplishing a second-order reaction (kC_A^2), and compare the result to that obtained from a PFR when $[(C_A^0 kL)/u] = 1$. Referring to Example 8.1.1, is the deviation from PFR behavior a strong function of the reaction rate expression (i.e., compare results from first- and second-order rates)?

$$< C_A > = \int_0^\infty C_A(t)E(t)dt \qquad (14)$$

After substituting equations (13) and (8) into equation (14), the mean concentration of A in the outlet is:

$$< C_A > = \frac{C_A^o}{(1+k\tau)^3} \qquad (15)$$

This result confirms that the RTD analysis will yield the same result as a material balance on the system (Example 3.4.3).

Exercise 4:

For a PFR accomplishing a second-order reaction, the material balance is:

$$u\frac{dC_A}{dz} = -kC_A^2 \qquad (1)$$

Non-dimensionalize the material balance:

Let $y = {C_A}/{C_A^o}$ and Z = z/L

Therefore, equation (1) becomes

$$\frac{dy}{dZ} = -\alpha y^2 \text{ where } \alpha = \frac{kLC_A^o}{u} \qquad (2)$$

The boundary conditions are: y = 1 at Z = 0, and y = 0.5 at Z = 1 (average concentration at the outlet is half of that at the inlet).

$$y = \frac{1}{1+\alpha Z} \quad \text{(where } \alpha = 1\text{)} \qquad (3)$$

For a second-order reaction taking place in a laminar flow tubular reactor, the average concentration and the concentration at any radial position are the following:

$$\overline{C_A} = \frac{\int_0^{\bar{r}_t} C_A(\bar{r})\bar{u}(\bar{r})2\pi \bar{r}d\bar{r}}{\int_0^{\bar{r}_t} \bar{u}(\bar{r})2\pi \bar{r}d\bar{r}} \tag{4}$$

$$\bar{u}(\bar{r})\frac{dC_A}{dz} = -kC_A^2,\ z = 0 \text{ at } C_A = C_A^o \tag{5}$$

Notice that equation (5) is the material balance on a PFR with second-order reaction. The solution is given in equation (3).

The velocity profile for laminar flow in a tubular reactor is the following:

$$\bar{u}(\bar{r}) = 2u_{\max}\left[1-\left(\bar{r}/\bar{r}_t\right)^2\right] \tag{6}$$

where $\bar{r}_t$ is the radius of the tubular reactor.

After substitution of equations (6) and (3) into equation (4) and letting $\rho_r = \frac{\bar{r}}{\bar{r}_t}$, the denominator is the following:

$$\int_0^1 2u_{\max}\left(1-\rho_r^2\right)2\pi \bar{r}_t^2 \rho_r d\rho_r = \pi \bar{r}_t^2 u_{\max} \tag{7}$$

In order to evaluate the numerator, $C_A(\bar{r})$ must be determined from equation (3):

$$C_A(\bar{r}) = \frac{C_A^o}{1+\frac{kC_A^o z}{u(\bar{r})}} \tag{8}$$

The velocity profile is determined from equation (6), and the equation is again non-dimensionalized and the concentration is evaluated at *L*. The result is the following:

$$\text{numerator} = 4\pi u_{\max} C_A^o \bar{r}_r^2 \int_0^1 \frac{\rho_r(1-\rho_r^2)d\rho_r}{1+\left(\frac{1}{2(1-\rho_r^2)}\right)} \tag{9}$$

After solving the integral,

$$\text{numerator} = 8\pi u_{\max} C_A^o \overline{r_t}^2 (0.0687)$$

Therefore, the average concentration of A at the outlet of the reactor is the following:

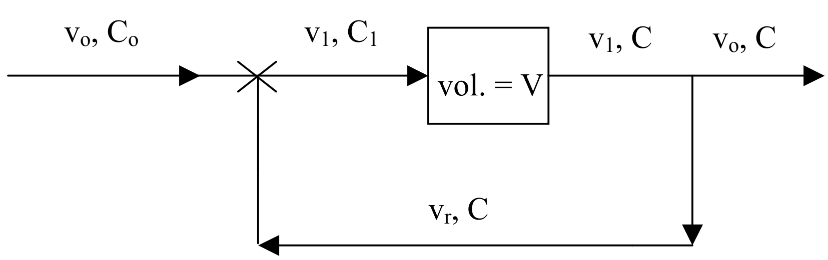
$$\overline{C_A} = \frac{8\pi C_A^o (0.0687) u_{\max} \overline{r_t}^2}{\pi u_{\max} \overline{r_t}^2} = 8(C_A^o)(0.0687) = 0.55 C_A^o$$

Therefore, the conversion (for a second-order reaction) is 45% in a laminar flow tubular reactor whereas it is 50% in a PFR.

As shown in example 8.1.1, the conversion in a laminar flow tubular reactor for a first-order reaction is 55.7% and in a PFR it is 63.2%. Therefore, the deviation from PFR is not a large function of the reaction-rate expression.

Exercise 5:

The following recycle reactor can represent the Berty reactor.



A material balance at the mixing point marked with an X, is the following:

$$\mathrm{v}_o C_o + \mathrm{v}_r C = \mathrm{v}_1 C_1 \qquad (1)$$

Additionally,

$$\mathrm{v}_o + \mathrm{v}_r = \mathrm{v}_1 \qquad (2)$$

If $\alpha = \frac{\mathrm{v}_r}{\mathrm{v}_o}$, then equations (1) and (2) can be combined and the result is the following: