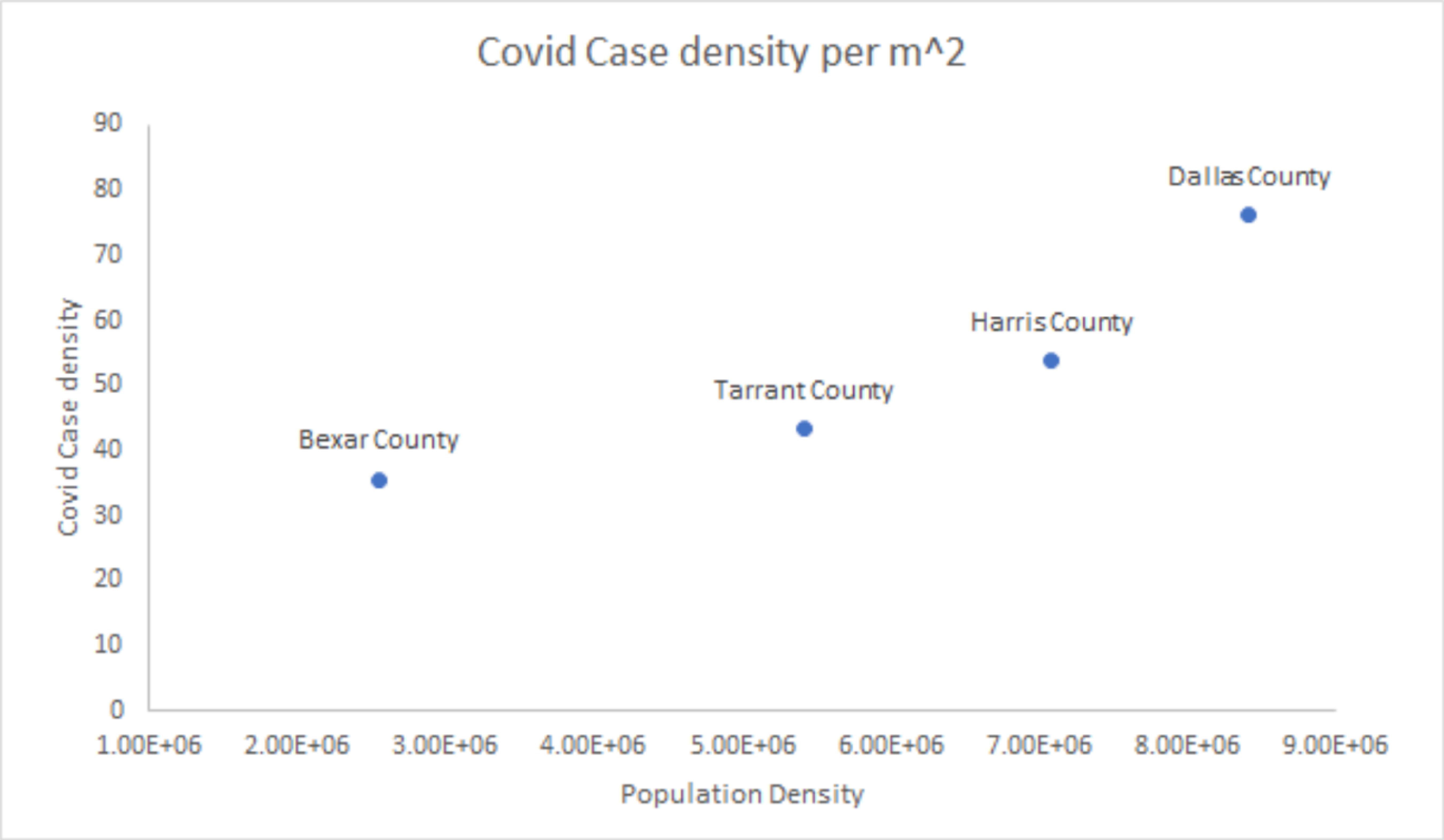
1.

population density = population / area

covid case density = covid case / area

example: 4 counties from Texas



Natural constant β only considers the first part of SIR model that convert uninfected to infected but didn't considers the recover. R0 considers both.

2. A → uninfected people

B → infected people

$r = \beta C_A C_B$

$A + B \rightarrow 2B$

$\frac{dC_A}{dt} = -\beta C_A C_B$ $\frac{dC_B}{dt} = \beta C_A C_B$

$C_A \cdot Area = C_{TOTAL} \cdot Area - C_B \cdot Area$

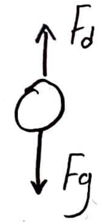
$C_A = C_{TOT} - C_B$ (C_{TOT} = densiry of total population, which is a constant)

use equation above

plot C_B vs. t

If the case have peaked or are peaking, this means our model failed, since we didn't take recover into consideration.

Problem #3 - Stokes Law



F_d - force of viscosity (Stokes drag)
F_g - excess force due to weight and buoyancy

$F_d = 6\pi\mu_f R V$ ① μ_f - viscosity of fluid
R - radius of spherical object
V - flow velocity

The excess force for the terminal velocity is due to the difference between the weight and buoyancy.

$\therefore F_g = F_w - F_b$ ②

$F_w = mg = m_{water}\, g = \rho_{water} V_{water}\, g \xrightarrow{V_{water} = V_{sphere}} = \rho_{water} V_{sphere}\, g$

③ $F_w = \rho_{water} \frac{4}{3}\pi R^3 g$; $V_{sphere} = \frac{4}{3}\pi R^3$

$F_b = \rho_f\, g\, V_{displaced} \xrightarrow{V_{displaced} = V_{sphere}} = \rho_f\, g\, V_{sphere}$

$F_b = \rho_f\, g\, \frac{4}{3}\pi R^3$

$\therefore$ Substitute ③ and ④ in ②

$F_g = \rho_{water} \frac{4}{3}\pi R^3 g - \rho_f\, g \frac{4}{3}\pi R^3 = (\rho_{water} - \rho_f)\, g\, \frac{4}{3}\pi R^3$ ⑤

@ equilibrium → $F_{net} = 0$

$F_{net} = 0 = F_d - F_g = 6\pi\mu R V - (\rho_{water} - \rho_f)\, g\, \frac{4}{3}\pi R^3 = 0$ ⑥

Solve for V:

$$\boxed{V = \frac{2}{9}\frac{(\rho_{water} - \rho_f)}{\mu}\, g R^2}$$ ⑦ → terminal velocity of water drops

$\mu_{air} = 1.81 \times 10^{-5}\ kg/(m \cdot s)$

$\rho_{air} = 1.225\ kg/m^3$

$\rho_{water} = 1000\ kg/m^3$

$g = 9.81\ m/s^2$

$R_1 = 10 \times 10^{-6}\ m$

$R_2 = 1 \times 10^{-6}\ m$

$R_3 = 1 \times 10^{-7}\ m$

Relative potential:

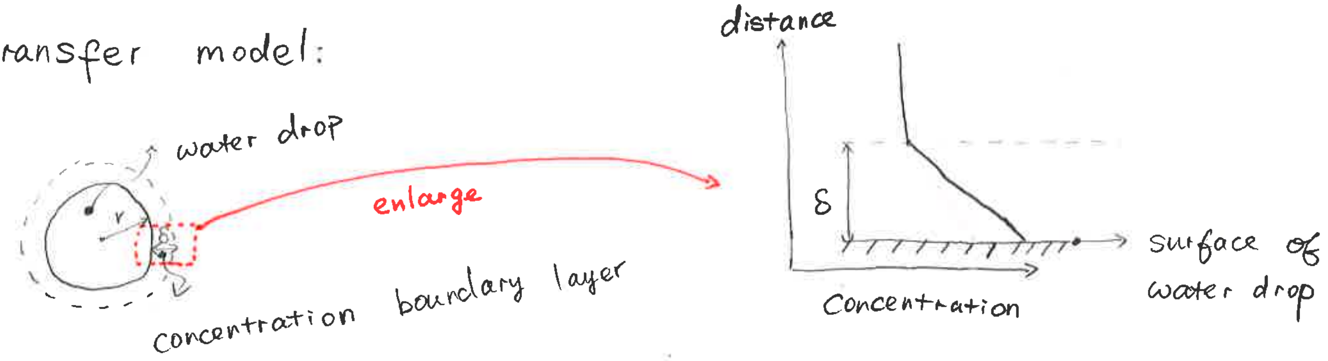
* A particle must be small enough that it has no significant inertia so that it can exactly follow the main air flow all the way to the branches of the lungs. It also needs to have a settling velocity small enough so that it would not settle more than a few microns (100 μm) during the ~1-2 seconds trip to alveoli. However, since these particles will still have a low diffusivity, and hence would not diffuse very far compared to the size of the alveoli in the ~2 that they could be in this region, if they are big enough to have some slight gravity settling, then they might be favored to hit the walls.

Fabricated by TA Weikai Cao

Problem # 4 Drop evaporation

The mass transfer occurs when the concentration of gaseous H_2O at the drop surface doesn't equal to the concentration of gaseous H_2O existing in the bulk air. Therefore, the water drop will evaporate if the air humidity does not reach saturation at a certain temperature.

Mass transfer model:



The water molecules at the surface diffuses through the boundary layer to the bulk due to the concentration difference. As a result, Rate of decrease of mass of drop = Rate of mass flux across the boundary layer.

mass of drop

surface area of drop

concentration of bulk water. ($\frac{\text{mass}}{\text{liter}}$)

$$\frac{dM}{dt} = -k\,A\,(C^* - C_0)\,\frac{1}{M_w}$$

mass transfer coefficient (unit: $\frac{\text{length}}{\text{time}}$)

Also:

$$M = \frac{4}{3}\pi r^3 \cdot \rho$$

$$Sh = \frac{kd}{D} \Rightarrow k = \frac{D\,Sh}{2r}$$

So $$\cancel{4\pi}\rho\,\cancel{r^2}\frac{dr}{dt} = -\frac{D\,Sh}{2r}\cdot\cancel{4\pi}\cancel{r^2}\cdot(C^* - C_0)/M_w$$

$$-r\,dr = \frac{D\,Sh\cdot(C^* - C_0)}{2\rho M_w}\,dt$$

Integrate the equation above, $-\int_R^0 r\,dr = \frac{D\,Sh\cdot(C^* - C_0)}{2\rho M_w}\int_0^T dt$

Fabricated by TA Weikai Cao

$$\rightarrow \quad \frac{1}{2} R^2 = \frac{D\, Sh\, (C^* - C_0)}{2 \rho M_w} \cdot T$$

$$\therefore \quad T = \frac{\rho M_w R^2}{D\, Sh \cdot (C^* - C_0)}$$

concentration of gaseous water at the surface

Concentration of gaseous water in the bulk

* Actually, the concentration of gaseous water at the surface is regarded as the saturated concentration at a certain temperature.