

# HEAT TRANSFER AND HEAT EXCHANGE EQUIPMENT

Mark J McCready  
University of Notre Dame  
Indiana, USA  
July 25, 2017

# IMPERIAL FLOWSHEET

Heat exchangers

- CO<sub>2</sub> LOOP
- N<sub>2</sub> LOOP
- MEA LOOP

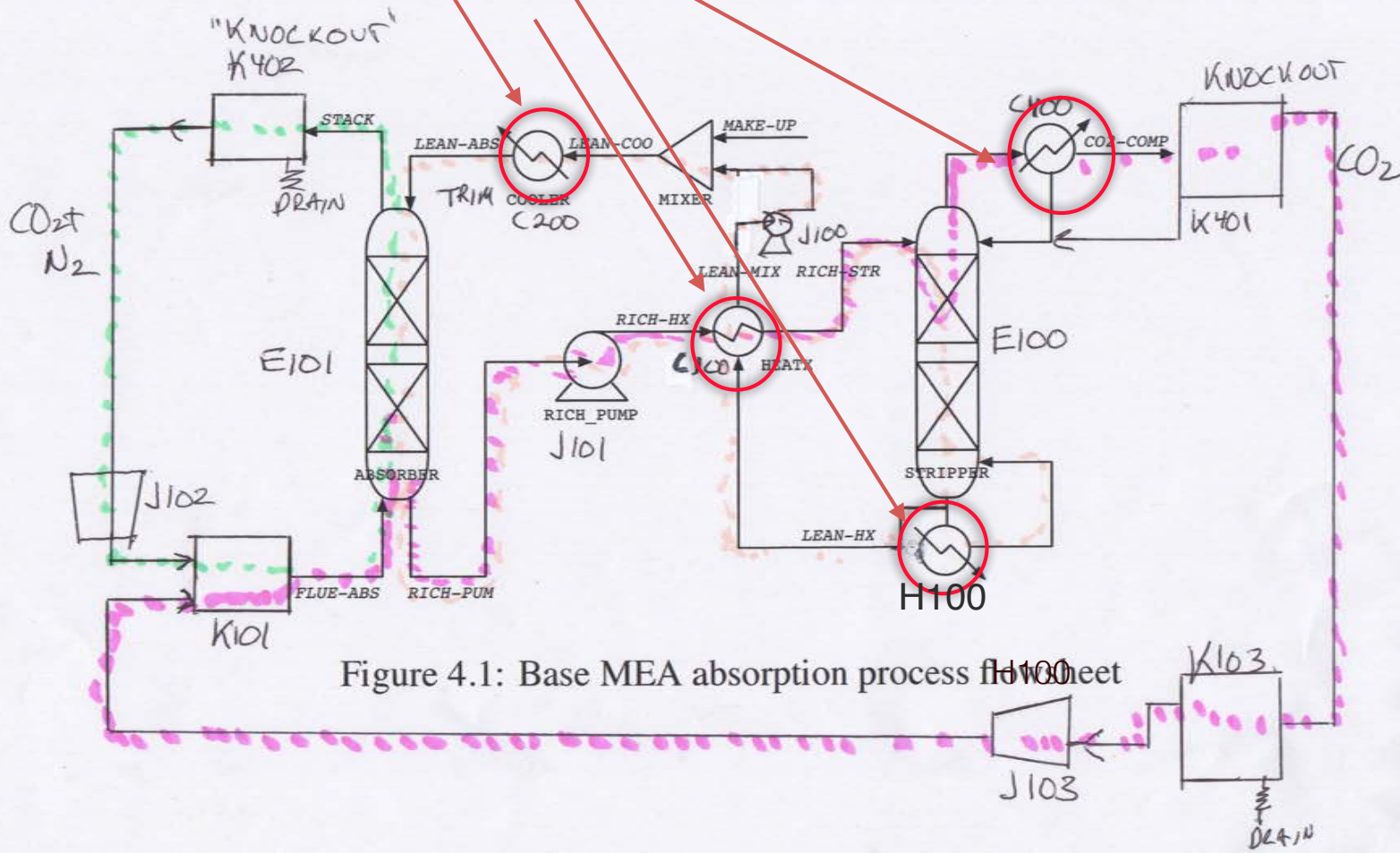


Figure 4.1: Base MEA absorption process flowsheet

# IMPERIAL HEAT EXCHANGERS

C100



H100

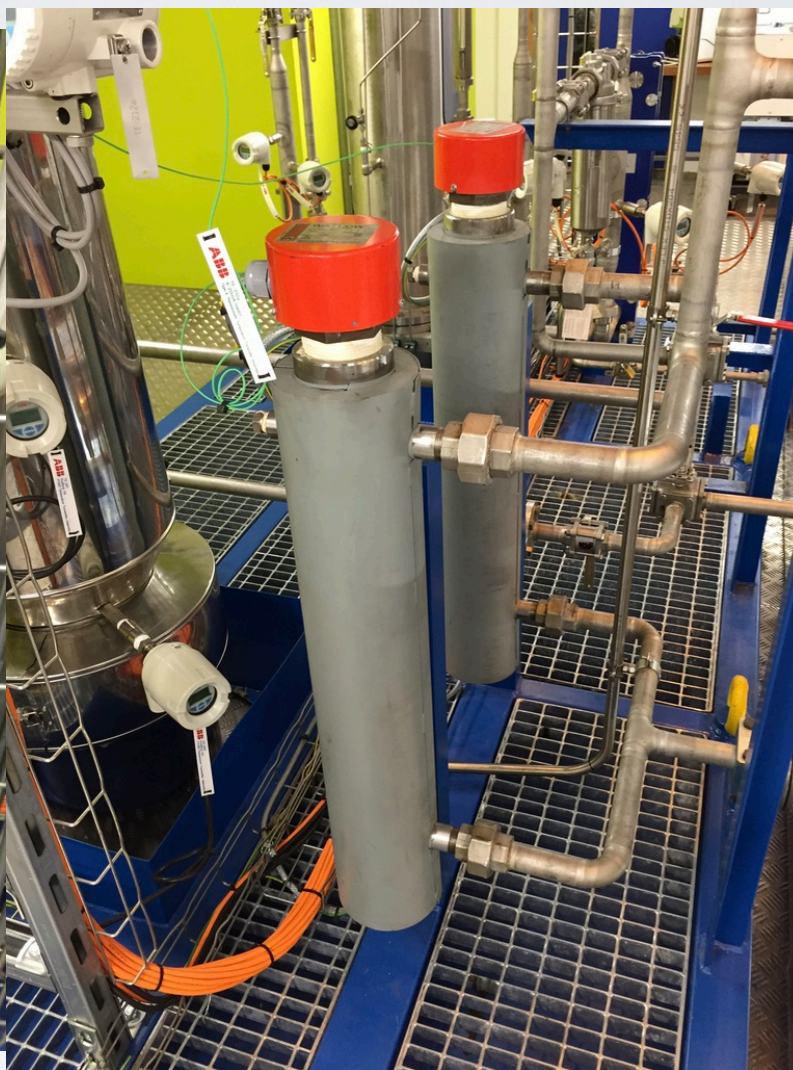


# IMPERIAL HEAT EXCHANGERS

C200



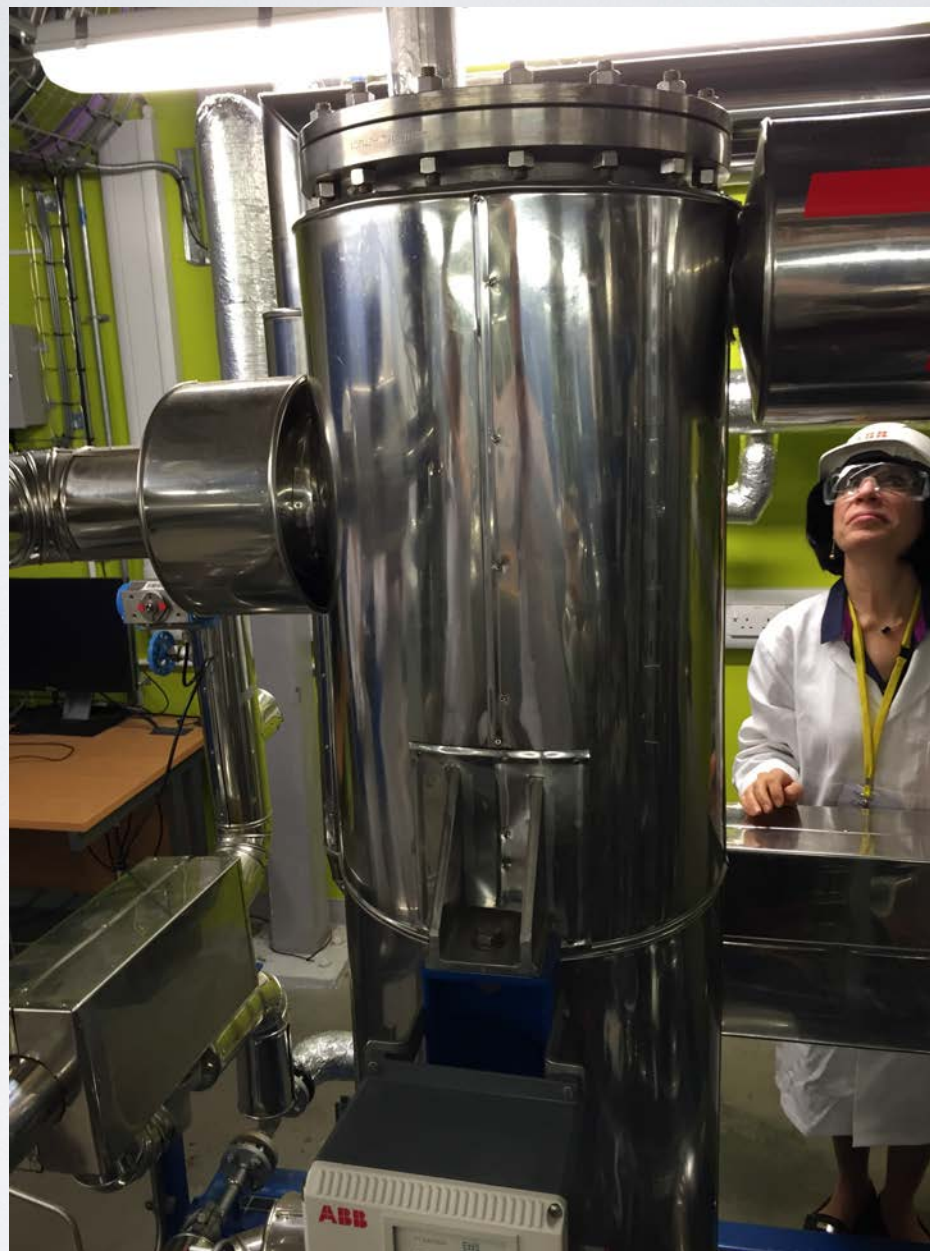
H200



C106



# PROFESSOR SADDAWI INSPECTS THE REBOILER!



H100

# SPIRAL HEAT EXCHANGER



C400

# FLOW SHEET HEAT EXCHANGERS

- Regeneration heat (added to bottom of stripping column)
  - “reboiler”
    - boils the MEA-water mixture and the “steam” strips the CO<sub>2</sub>
      - Steam provides the heat
- Chiller before MEA is fed to absorber
  - “Trim cooler”
    - Chilled water
- “Intercooler”(“clean” and “dirty” streams exchange heat)
  - counter current plate/frame heat exchanger
- “Condenser” on top of stripping column
  - condense water and MEA, let CO<sub>2</sub> pass through to recycle
    - Chilled water: Spiral geometry

# HEAT EXCHANGERS

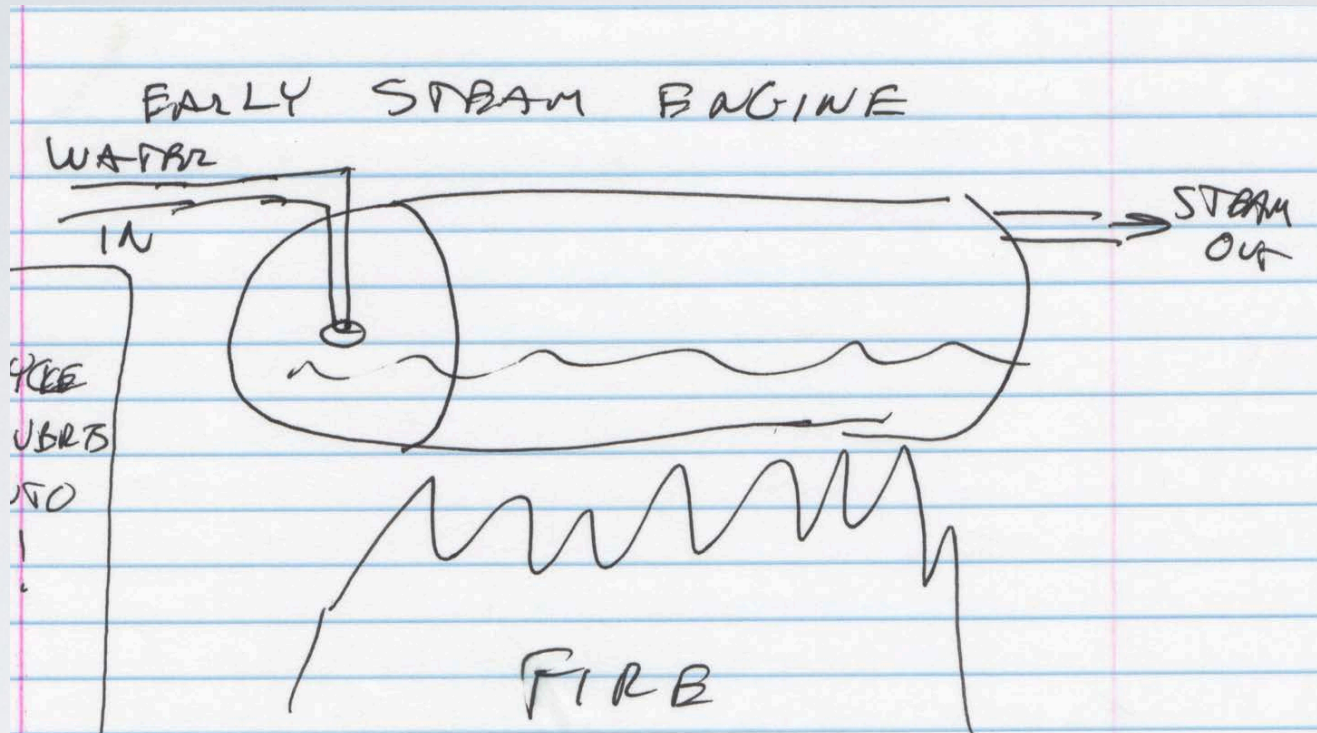
- Two basic “ideas” ... a.k.a. equations:
  - Energy is conserved
    - **First law of thermodynamics**
      - You have already done such calculations!
  - *Rate* of heat transfer will determine the total “transfer area” needed for the heat exchanger
    - Newton’s law of cooling
    - This may be new to you



# HEAT TRANSFER FUNDAMENTALS

- We see that understanding heat transfer is essential to knowing exactly how the process operates.
- Let's see if we can efficiently learn some fundamentals.

# A PRIMITIVE BUT TECHNOLOGICALLY IMPORTANT HEAT EXCHANGER

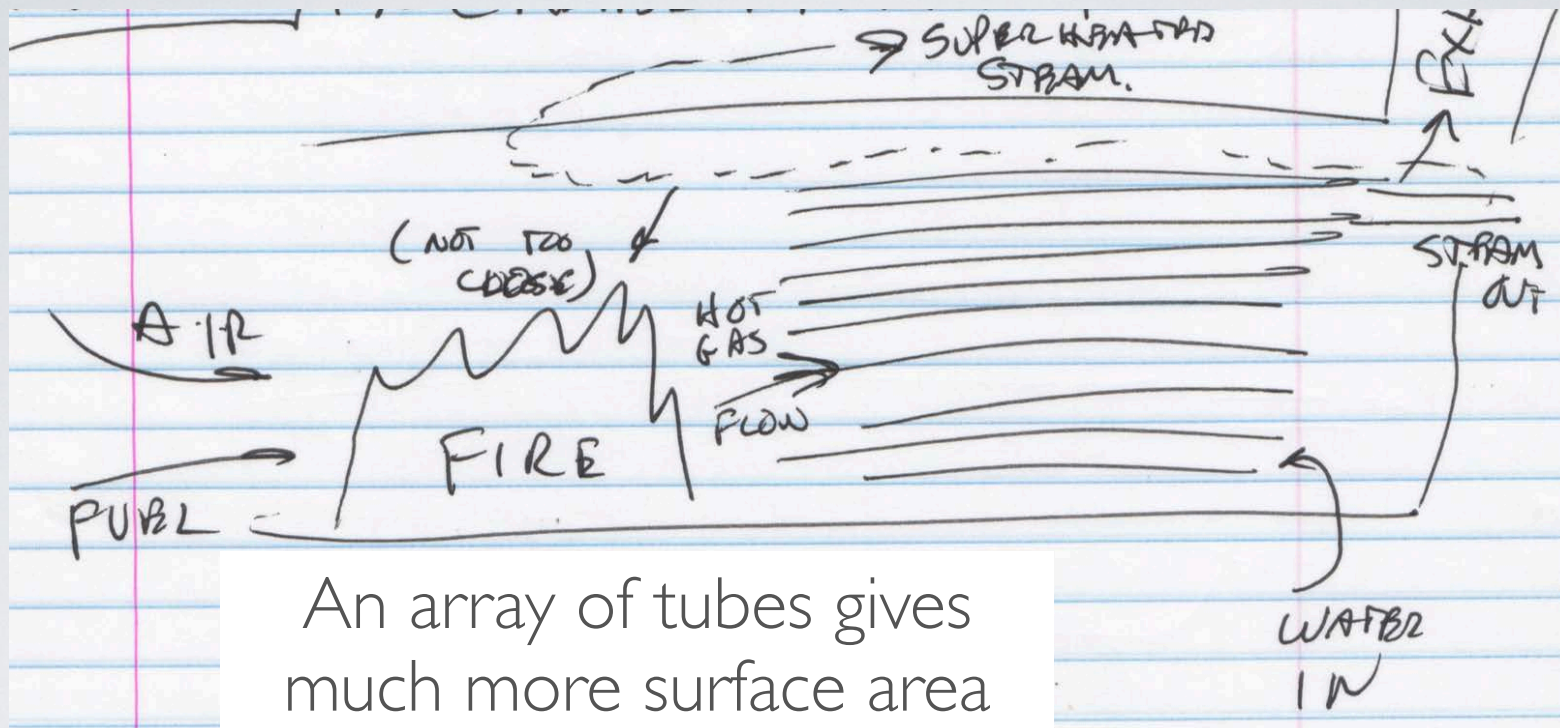


This device could produce saturated steam at a pressure somewhat above 1 atmosphere

We can see that a lot of the heat from the fire is lost and does not heat the water.

We need more contact area between the hot combustion gases and the liquid water

# A BETTER HEAT EXCHANGER



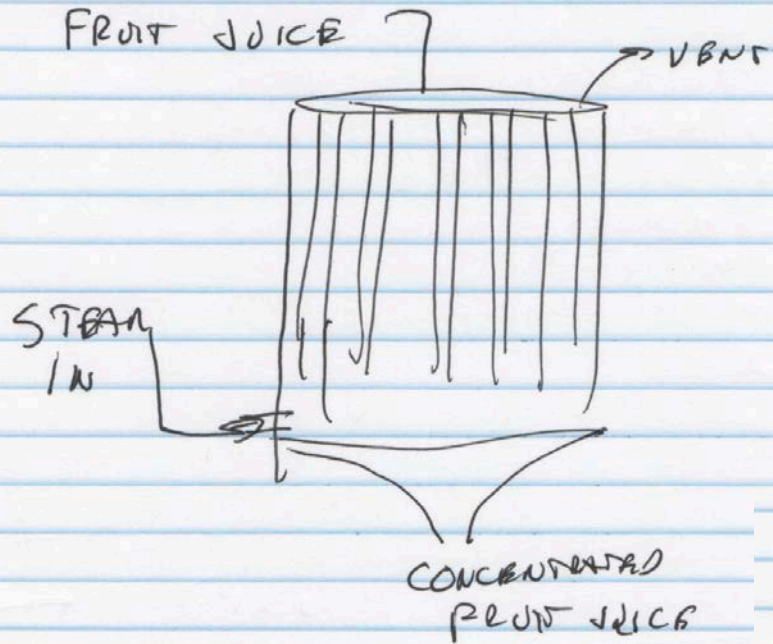
An array of tubes gives much more surface area

A section of tubes that does not have liquid water in it allows the steam to become superheated and hence increases its ability to do work.

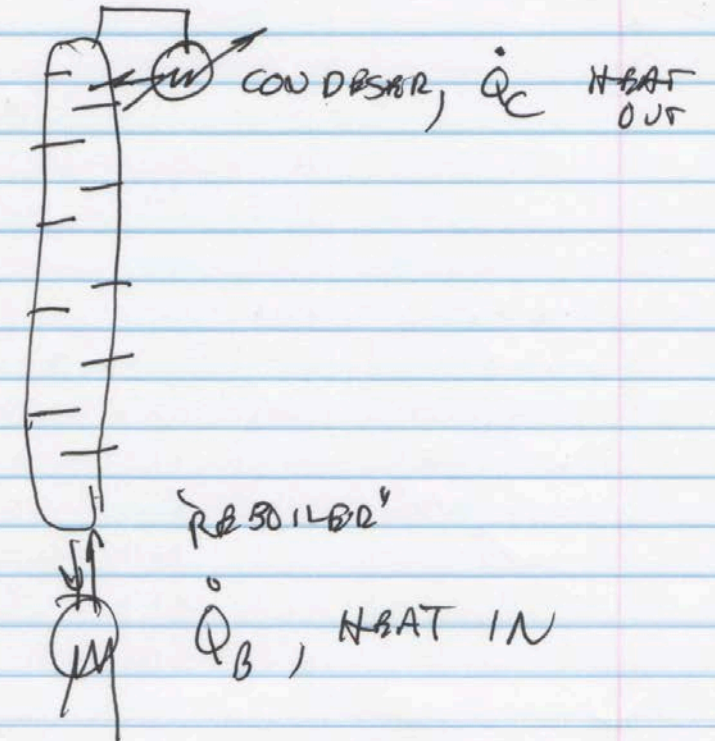
# “BIG BOY” STEAM LOCOMOTIVE



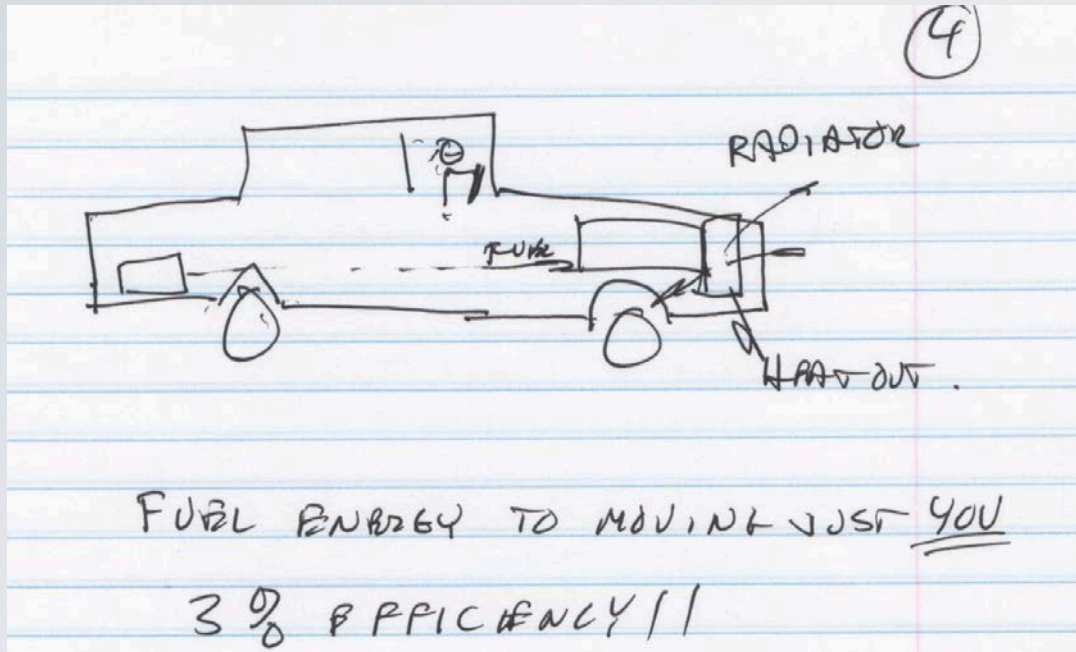
# OTHER TECHNOLOGY EXAMPLES



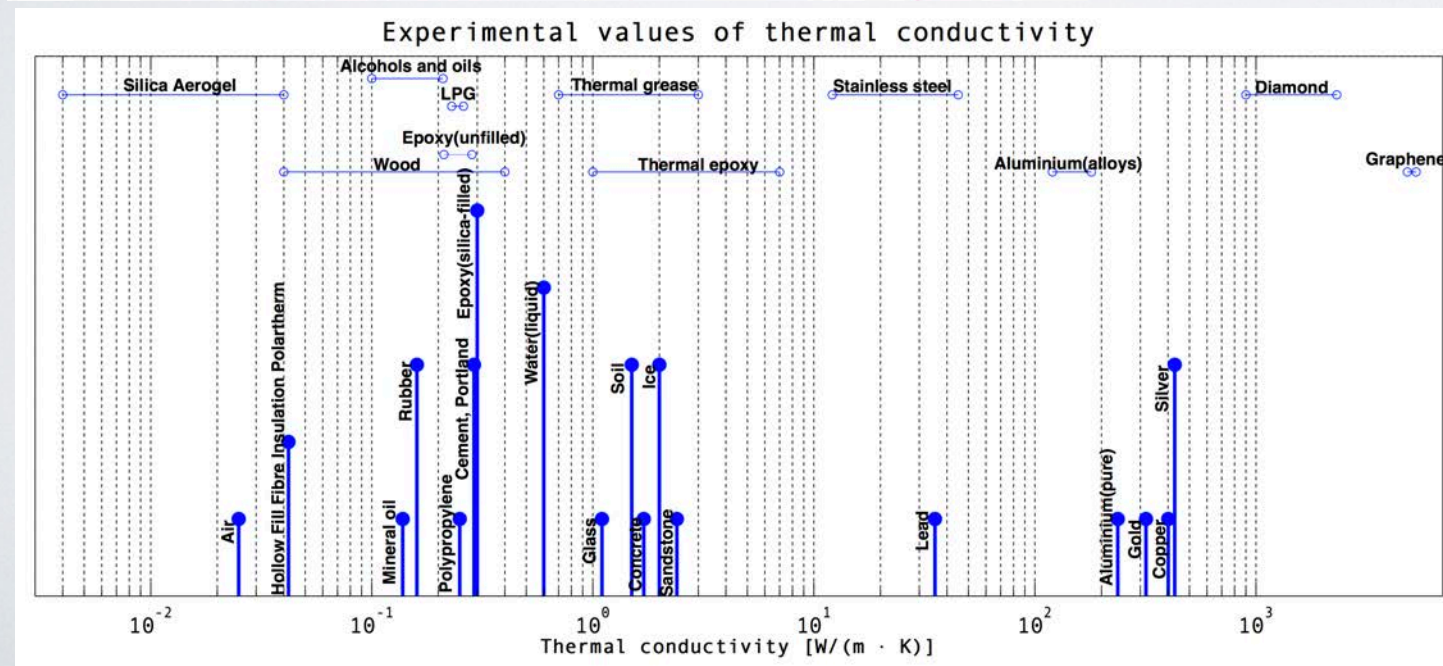
USE  
HEAT TO  
SEPARATE A  
BINARY A+B  
MIXTURE



# A CAR:



- For air exchange note that heat exchangers typically have “fins”. This is because a low density gas is a good insulator, not a good heat transfer fluid!



[https://upload.wikimedia.org/wikipedia/commons/1/1e/Thermal\\_conductivity.svg](https://upload.wikimedia.org/wikipedia/commons/1/1e/Thermal_conductivity.svg)

# MODES OF HEAT TRANSFER

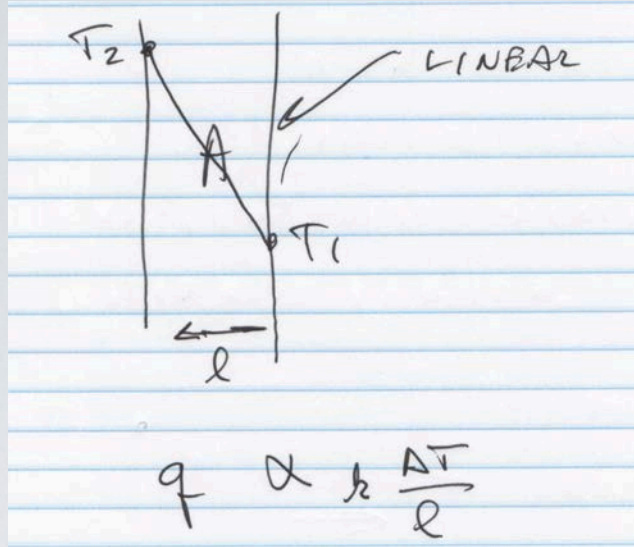
- Radiation
  - Heat transport by electromagnetic waves
- Conduction
  - Transport by molecular/atoms vibrating (for solids). Free electrons for metals. Random molecular motion for gases and liquids.
- Convection
  - Transport by net motion of fluid. (molecular motion that is correlated, not random)

# RADIATIVE HEAT TRANSFER

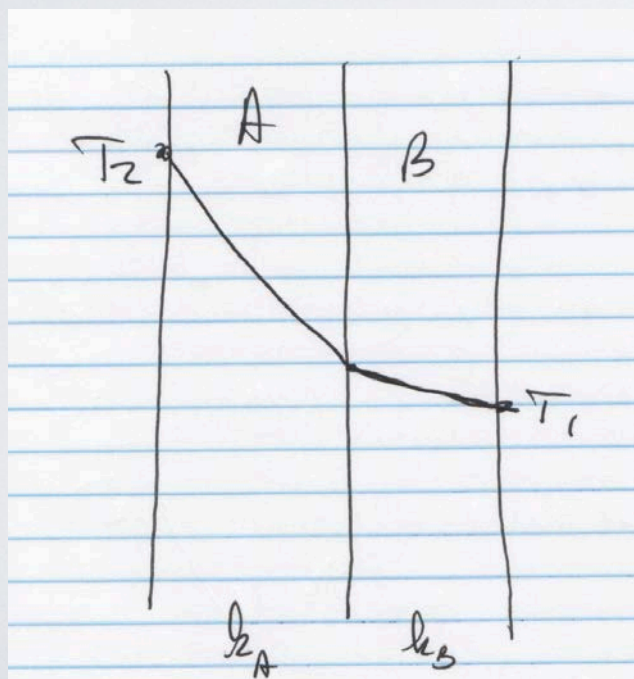
- The third mode of heat transfer is “radiation”. This is transfer of energy through a “transparent” medium by electromagnetic waves.
- The power of temperature in the driving force is “4”, that is
  - $q \sim \epsilon \sigma (T^4 - T_0^4)$ ,  $\epsilon$ , is the “emissivity” and  $\sigma$  is the Stefan-Boltzmann constant .
  - We don't neglect radiation entirely as you can see that the outside of the absorber and stripper are “shiny metal”, for which  $\epsilon \sim .06$ . The emissivity is close to 1 for dark colored, slightly roughened surfaces and exactly 1 for a “black body”.



# STEADY CONDUCTION IN A SOLID

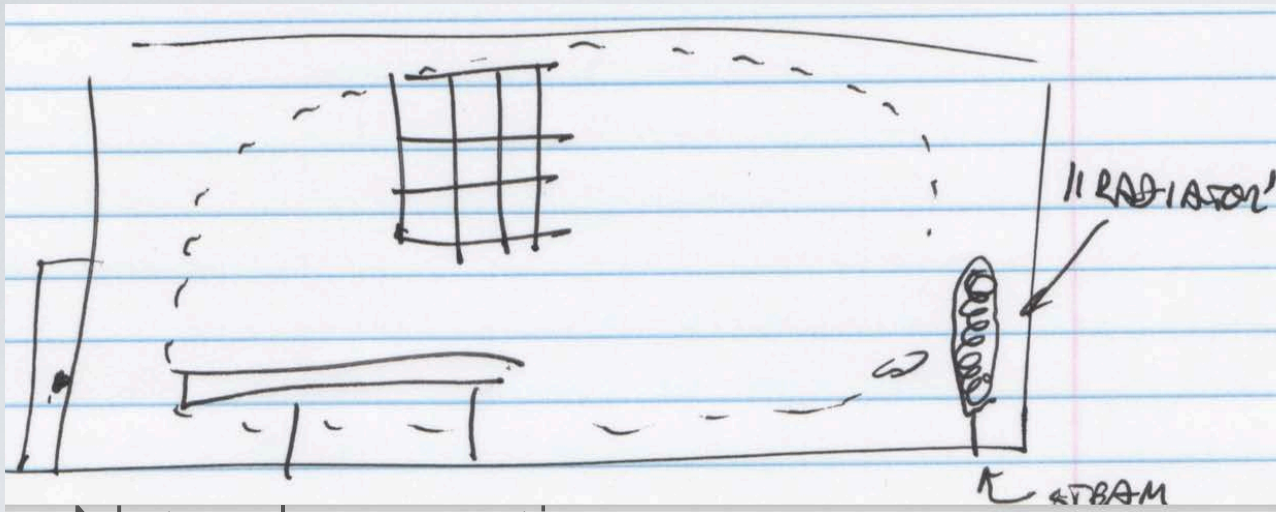


- Fourier's "Law" of heat conduction.

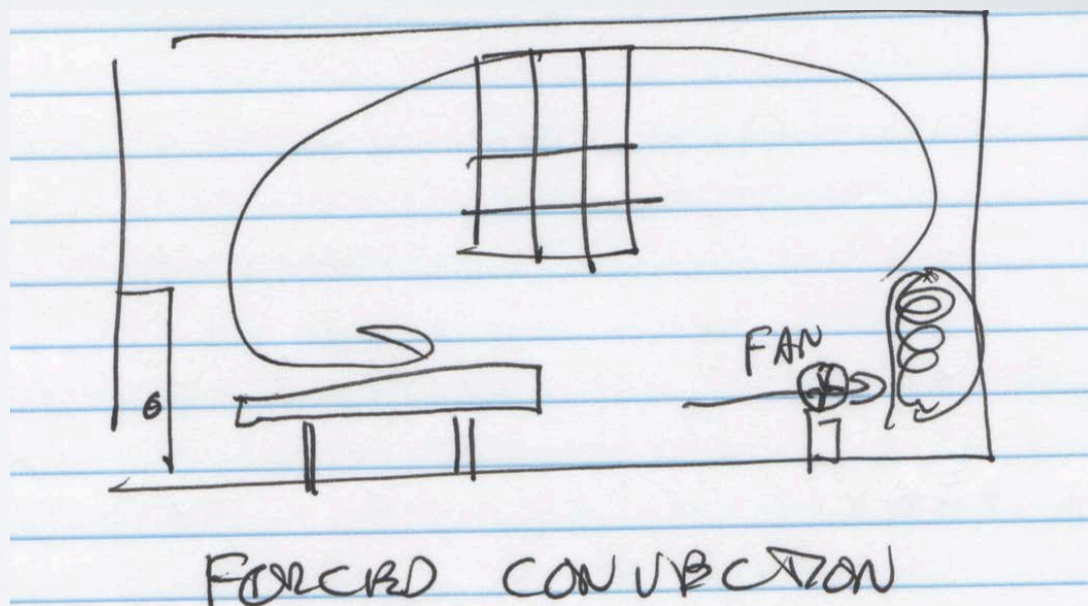


$$q = \text{CONST}$$
$$k_A \frac{\Delta T_A}{l} = k_B \frac{\Delta T_B}{l}$$

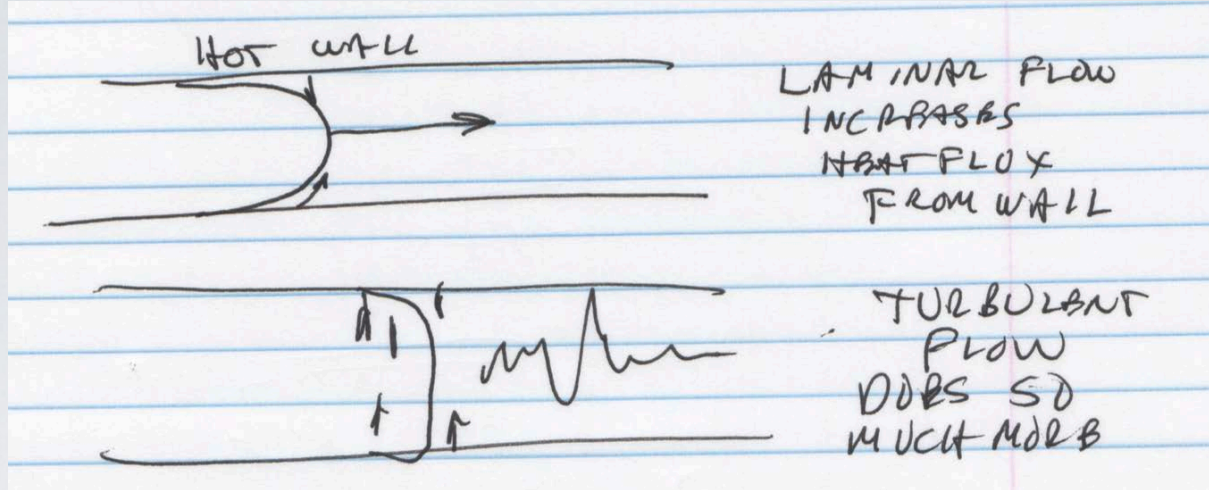
# CONVECTION



Natural convection  
— flow is from  
buoyancy of gas



# TURBULENCE GREATLY INCREASES HEAT TRANSFER



- Note that direction of heat transfer for this problem is the radial coordinate of the pipe.
- We hypothesize a constitutive equation for these convective flow situations (since it does not seem possible to solve the differential equations for turbulent flow...)

$$q' = hA(T_w - T_\infty)$$

$q'$  – heat flux

$h$  – heat transfer coefficient

$A$  – area of contact

$T_w$  – wall temperature

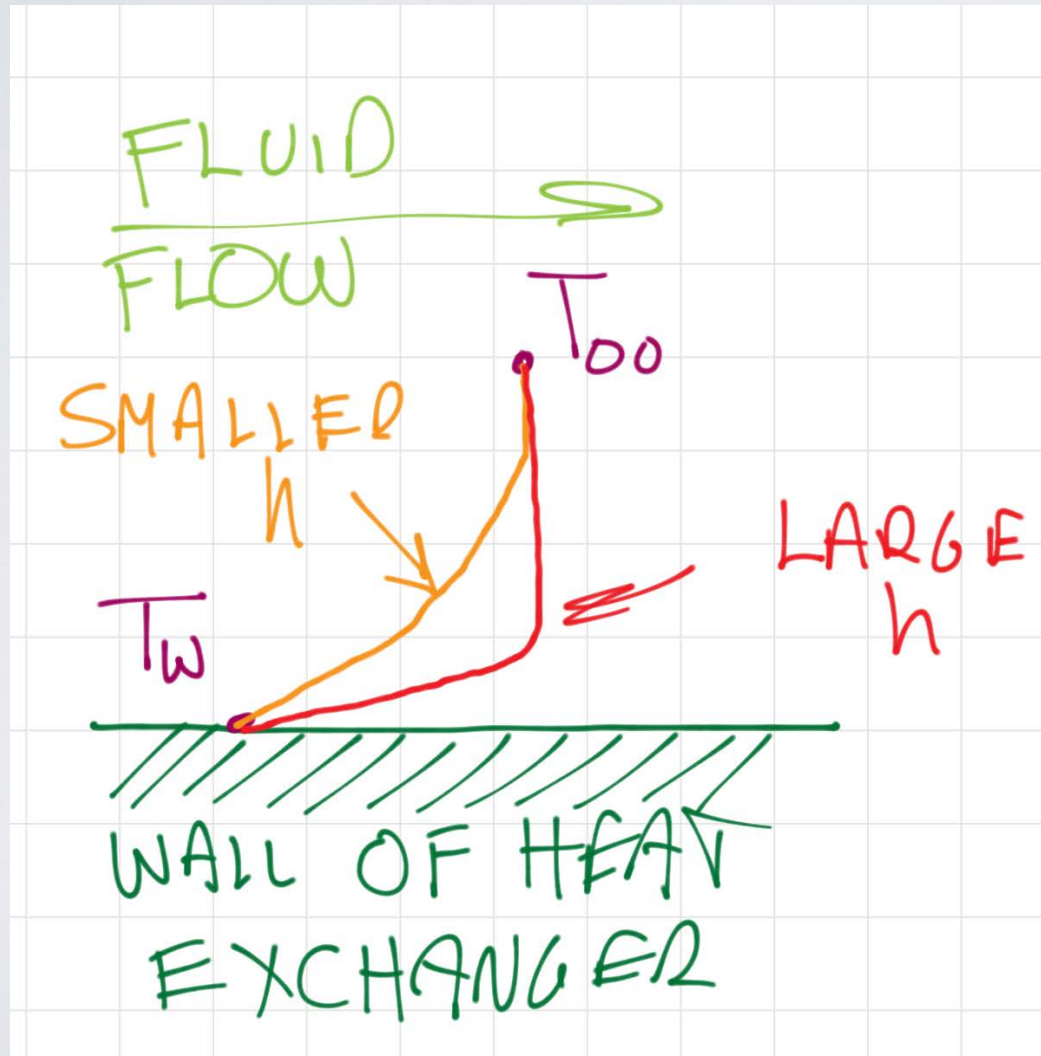
$T_\infty$  – temperature away from wall

in the stream

# NEWTON'S LAW OF COOLING

- $Q = hA(T - T_0)$
- This *empirical* equation says that the heat flow from a boundary (say a pipe wall), for a flowing fluid is the product of the temperature difference:  
*Thermodynamic* driving force and
- A variable,  $h$ , the “heat transfer coefficient” that is a function of the intensity of the fluid mixing and the physical properties (e.g., thermal conductivity) of the fluid
  - The underlying physical processes that determine “ $h$ ” are the subject of the courses of *Transport Phenomena*
  - We see that *Thermodynamics* tells us what can occur and *Transport Phenomena* tells us how fast it will occur.

# WALL REGION OF HEAT EXCHANGER: "FORCED CONVECTION"



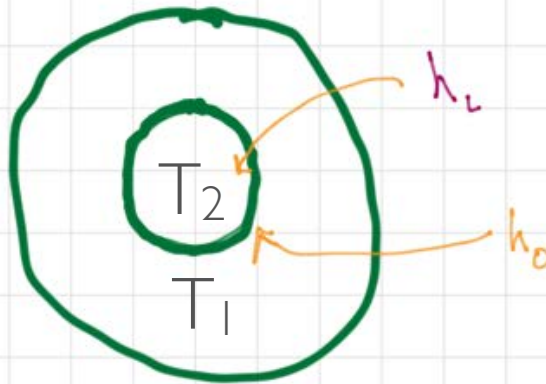
Steeper gradient is associated with faster heat transfer

# ANOTHER BIT OF LINGO: *BOUNDARY-LAYER*

- For process flows, the Reynolds number is usually very much larger than unity and in most cases the flow is turbulent.
- Thus, convection is the dominant mode of heat transfer across most of the pipe.
  - In this region, the temperature changes very little.
- However, because the fluid velocity is “0” at the wall: (no slip), the *convection* near the wall is greatly decreased and hence *conduction* becomes relatively more important.
- This region, near the wall (or potentially at the boundary between two fluid phases) is termed a “boundary-layer”.
  - In this region conductive and convective transport effects are of the same magnitude.
  - Also the temperature gradient is very much larger within the boundary-layer than far away from the boundary where convection is dominating.

# HEAT EXCHANGER

Cross-section



Heat transfer resistance occurs at both inside and outside of inner pipe, hence there is a heat transfer coefficient for each side.

Sum of resistances used to get an overall heat transfer coefficient,  $U$

$$U_i \equiv \frac{Q}{A_i \Delta T}$$

$$\Delta T = T_2 - T_1$$

$$\frac{l}{A_i U_i} = \frac{l}{h_i A_i} + \frac{l}{k \Delta r} + \frac{l}{h_o A_o}$$

outside resistance

inside resistance

$$\frac{l}{U_i} = \frac{l}{h_i} + \frac{A_i}{k \Delta r} + \frac{A_i}{h_o A_i}$$

resistance associated with pipe wall

The resistance of the pipe wall will usually be much smaller than the contributions from the heat transfer coefficients if the pipe is made of a metal, but if we want to be precise we should write it as

$$\frac{A_i \ln \frac{d_{OUTER}}{d_{INNER}}}{2\pi L}$$

Note that all of the areas are  $\pi d L$  for a specified  $L$ .

A  $U$  BASED ON THE OUTSIDE OF THE (INNER) PIPE IS:

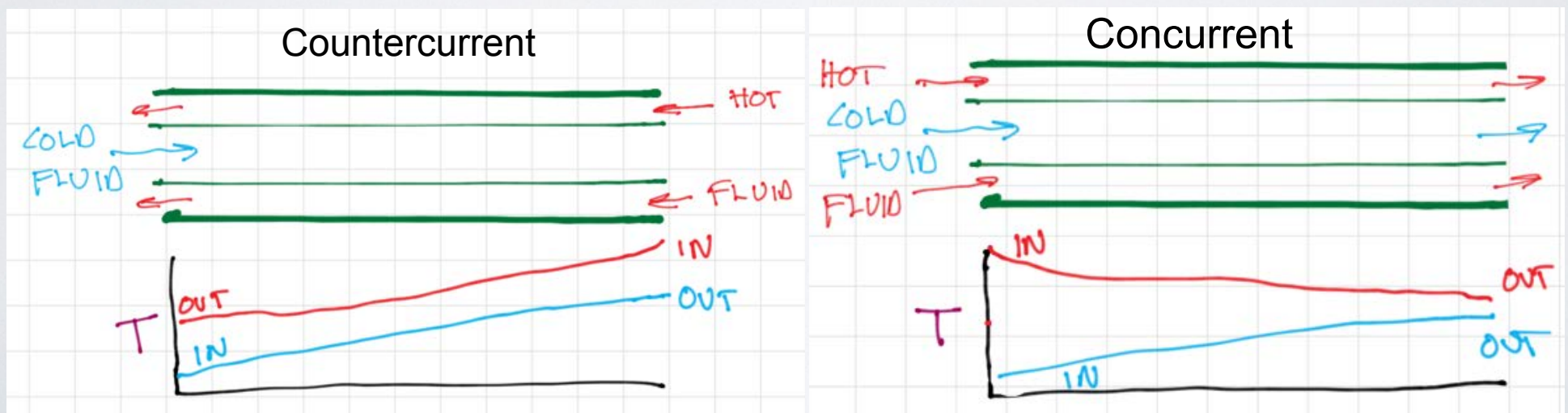
$$U_i \equiv \frac{q}{A_i \Delta T}$$

$$\frac{1}{U_o} = \frac{A_o}{A_i h_i} + \frac{A_o \ln \frac{d_o}{d_i}}{2\pi L k} + \frac{1}{h_o}$$



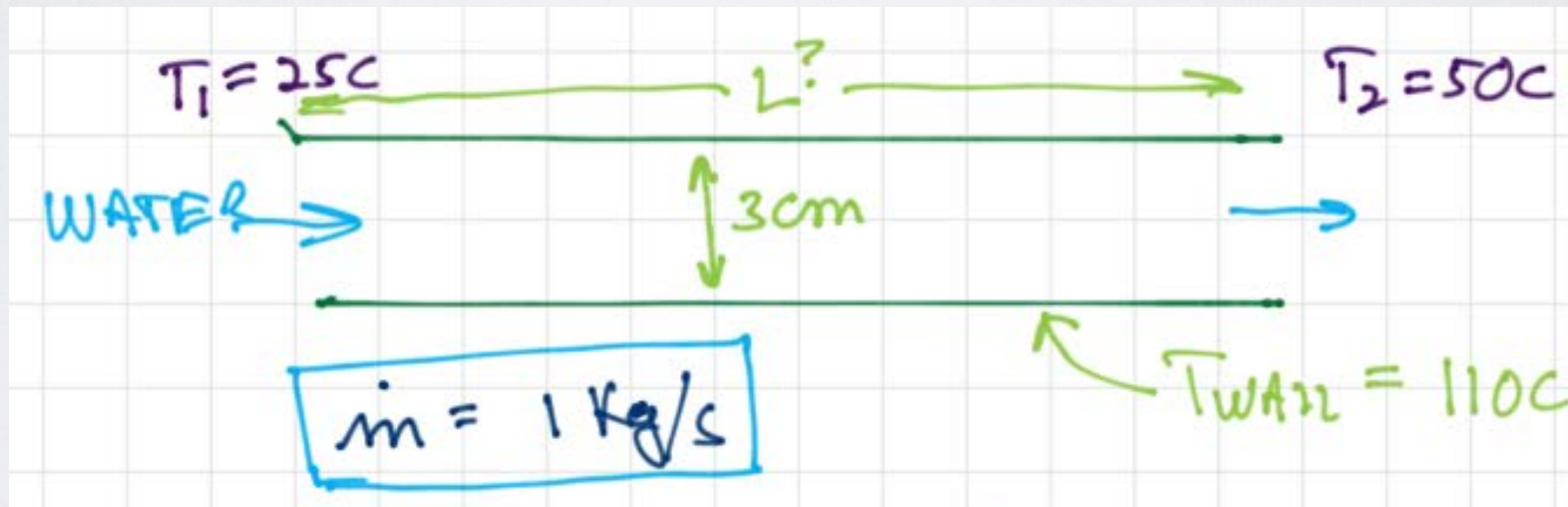
# HEAT TRANSFER CALCULATIONS

- With this short introduction to some fundamentals of heat transfer we now turn to example situations of interest.
- The device we will consider is a “double-pipe” heat exchanger in which one liquid is being heated by either a second liquid or condensing steam.
- The two fluids are not mixed.
- Countercurrent usually will provide a larger overall  $\Delta T$  for a given length



# HEAT TRANSFER PROBLEM

- The 1 kg/s water stream is flowing in a 3 cm pipe in a countercurrent, double pipe heat exchanger. The water temperature must be raised from 25 C to 50C. Condensing steam, saturated at 110C will provide the heat.
  - What flow rate of steam is needed?
  - How long should the pipe be?
- The heat transfer coefficient for condensing steam will be much larger than for the water flow so we can assume that the wall temperature will be constant at 110C.



# STEADY STATE HEATING OF A LIQUID FLOWING IN A PIPE: HEAT LOAD

$$Q = \dot{m} (\hat{H}_{out} - \hat{H}_{in}) + q$$

$$q = \dot{m} c_p (T_{out} - T_{in})$$

$$= (1 \text{ kg/s}) \left( 4180 \frac{\text{J}}{\text{kgK}} \right) (50 - 25) \text{ K}$$

$$= 104,500 \text{ J/s}$$

$$q = 100 \text{ kW}$$

## STEAM FLOW RATE

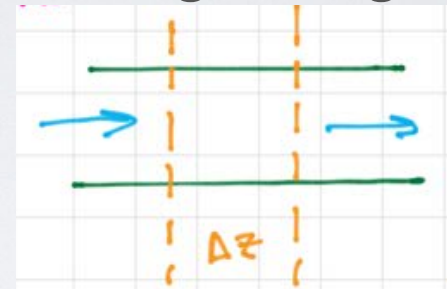
$$\hat{H}_{VAP} = 2691.3 \frac{\text{kJ}}{\text{kg}}, \hat{H}_{LIQ} = 461.3 \frac{\text{kJ}}{\text{kg}}$$

$$\dot{m}_{STEAM} = \frac{q}{(\hat{H}_{VAP} - \hat{H}_{LIQ})}$$

$$\dot{m}_{STEAM} = 0.05 \text{ kg/s}$$

# DIFFERENTIAL ANALYSIS TO GET LENGTH OF HEAT EXCHANGER

- The steam keeps the pipe wall at a constant temperature, but the temperature of the water in the pipe is changing. We will need a differential formulation of the temperature change along the pipe and then need to integrate to get the answer.
- Consider a differential slice of pipe.



STEADY STATE, DIFFERENTIAL SLICE OF PIPE

CHANGE IN ENTHALPY OF FLUID = RATE OF HEAT CROSSING PIPE WALL

Newton's Law of cooling

$$\dot{m}(\hat{H}_{out} - \hat{H}_{in}) = q$$

$$\dot{m}c_p(T_{out} - T_{in}) = -h\pi D L(T - T_{wall})$$

SHRINK TO  $\Delta z$

$$\dot{m}c_p \Delta T = h\pi D \Delta z (T - T_{wall})$$

$$\frac{\Delta T}{\Delta z} = -\frac{h\pi D}{\dot{m}c_p} (T - T_{wall})$$

$$\Delta z \rightarrow 0$$

$$\frac{dT}{dz} = -\frac{h\pi D}{\dot{m}c_p} (T - T_{wall})$$

NOW INTEGRATE FROM  $T = 25$   
TO  $T = 50$  C

$$\int_{T=25}^{T=50} \frac{dT}{T - T_{wall}} = -\frac{h\pi D}{\dot{m}c_p} \int dz$$

$$\ln \left( \frac{T_{out} - T_{wall}}{T_{in} - T_{wall}} \right) = -\frac{h\pi D}{\dot{m}c_p} L$$

## Logarithmic mean temperature difference

- Log mean temperature difference arises from this differential analysis when both streams are changing temperature

$$LMTD = \frac{\Delta T_A - \Delta T_B}{\ln \left( \frac{\Delta T_A}{\Delta T_B} \right)} = \frac{\Delta T_A - \Delta T_B}{\ln \Delta T_A - \ln \Delta T_B}$$

$$Q = U \times Ar \times LMTD$$

# CORRELATIONS FOR HEAT TRANSFER COEFFICIENT

- As with the “friction factor”, we look for the appropriate correlation that uses the correct dimensionless groups.
- For heat transfer we need to find a value for the Nusselt number,  $Nu$ , in terms of the Reynolds number,  $Re$ , and the Prandtl number,

$Pr$ .

$$Nu \equiv \frac{hD}{k}$$

$$Pr \equiv \frac{\nu}{\alpha} = \frac{M/s}{k/sCp} = \frac{Mc_p}{k}$$

THE APPROPRIATE CORRELATION IS:

$$Nu = 0.023 Re^{.8} Pr^{.4}$$

# SOME CORRELATIONS FROM BRODKEY & HERSHEY

wall roughness conditions. The modern form [M3, S6] of the Dittus-Boelter correlation [D3], which is based on Eq. (11.65), is

$$N_{Nu,mb} = \bar{h}d_i/k_{mb} = 0.023(N_{Re,mb})^{0.8}(N_{Pr,mb})^n \quad (11.66)$$

$$0.7 \leq N_{Pr,mb} \leq 100$$

$$10\,000 \leq N_{Re,mb} \leq 120\,000$$

$$L/d_i \geq 60 \quad (\text{smooth tubes})$$

where  $n$  is 0.4 for heating ( $T_w > T_b$ ) and 0.3 for cooling. Note that conditions listed below Eq. (11.66) are the range of data used in

For large  $\Delta T$ , another equation by Sieder and Tate [S4] is recommended:<sup>1</sup>

$$N_{Nu,mb} = 0.027(N_{Re,mb})^{0.8}(N_{Pr,mb})^{1/3}(\mu_{mb}/\mu_w)^{0.14} \quad (11.67)$$

$$0.7 < N_{Pr,mb} \leq 160$$

$$N_{Re,mb} \geq 10\,000$$

$$L/d_i \geq 60 \quad (\text{smooth tubes})$$

**Friend-Metzner analogy.** The Friend-Metzner analogy uses an equation of substantially different form in order to correlate data over wide ranges of  $N_{Pr}$  and  $N_{Sc}$  [F3]. Their correlation for heat transfer is

$$N_{Nu,mb} = \frac{N_{Re,mb}N_{Pr,mb}(f/2)(\mu_{mb}/\mu_w)^{0.14}}{1.20 + (11.8)(f/2)^{1/2}(N_{Pr,mb} - 1)(N_{Pr,mb})^{-1/3}} \quad (11.83)$$

$$0.5 \leq N_{Pr,mb} \leq 600 \quad N_{Re,mb} \geq 10\,000$$

# USEFUL INFORMATION

**TABLE 11.4**  
**Approximate magnitudes of heat transfer coefficients\***

Application	Range of values	
	$h, \text{W m}^{-2} \text{K}^{-1}$	$h, \text{Btu ft}^{-2} \text{h}^{-1} \text{°F}^{-1}$
Steam (dropwise condensation)	$3 \times 10^4 - 1 \times 10^5$	$5 \times 10^3 - 2 \times 10^4$
Steam (film-type condensation)	$5 \times 10^3 - 2 \times 10^4$	$1 \times 10^3 - 3 \times 10^3$
Boiling water	$2 \times 10^3 - 5 \times 10^4$	$300 - 9 \times 10^4$
Condensing organic vapors	$1 \times 10^3 - 2 \times 10^3$	200-400
Water (heating)	$300 - 2 \times 10^4$	$50 - 3 \times 10^3$
Oils (heating or cooling)	$60 - 2 \times 10^3$	10-300
Steam (superheating)	30-100	5-20
Air (heating or cooling)	1-60	0.2-10

\* From McAdams, *Heat Transmission*, 3d ed., p. 5, McGraw-Hill, New York, 1954. By permission.



# FOR OUR PROBLEM

- Check these numbers..
  - $Re = 66000, Pr = 5 \implies Nu = 290$
  - $h = 6060 \text{ W}/(\text{ m}^2 \text{ K})$
  - $L = 2.6 \text{ m}$

# CHECKS

$$Re = \frac{V D S}{\mu}$$

$$V = \frac{(1 \text{ kg/s})}{(1000 \text{ kg/m}^3)} \cdot \frac{4}{\pi (0.03 \text{ m})^2}$$
$$= 1.4 \text{ m/s}$$

$$Re = \frac{(1.4)(0.03)(1000)}{0.0007} = 60630$$

$$Nu = 0.023 (60630)^{0.8} (5)^{0.4}$$
$$= 293$$

$$h = \frac{293}{0.03 \text{ m}} (.62 \text{ W/mK})$$
$$= 6063 \text{ W/(m}^2 \text{K)}$$

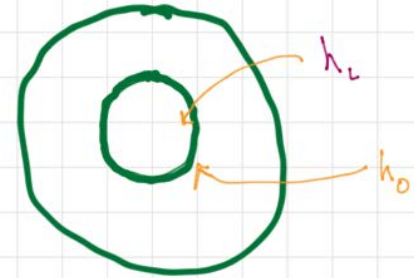
$$\therefore L = - \frac{(1 \text{ kg/s}) \left( \frac{4200 \text{ J}}{\text{kgK}} \right)}{\left( \frac{6063 \text{ J}}{\text{s m}^2 \text{K}} \right) \pi (0.03 \text{ m})} \ln \left( \frac{50 - 110}{25 - 110} \right)$$

$$L = 2.6 \text{ m}$$

# SUPPOSE THAT HOT WATER IS BEING USED TO HEAT THE COLD WATER

- In this case, both streams will be changing temperature
- We now will need to set up energy balances for the two streams and then integrate along the length of the heat exchanger

Cross-section

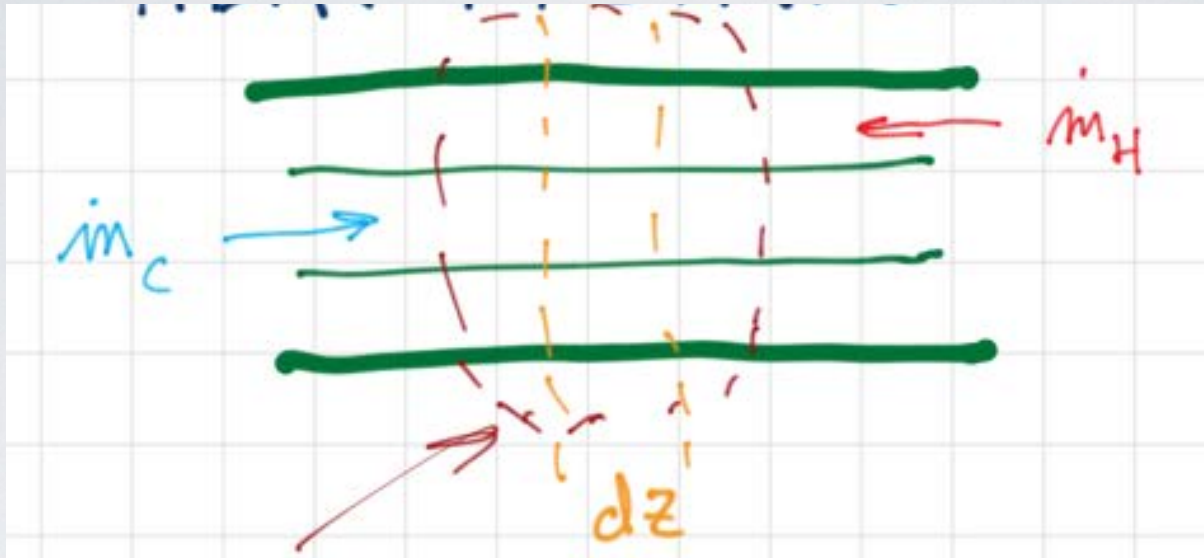


Heat transfer resistance occurs at both inside and outside of inner pipe, hence there is a heat transfer coefficient for each side.

Sum of resistances used to get an overall heat transfer coefficient, U

$$U_i \equiv \frac{q}{A_i \Delta T}$$
$$\frac{l}{A_i U_i} = \frac{l}{h_i A_i} + \frac{l}{k \Delta r} + \frac{l}{h_o A_o}$$
$$\frac{l}{U_i} = \frac{l}{h_i} + \frac{A_i}{k \Delta r} + \frac{A_i}{h_o A_i}$$

# ANALYSIS OF DOUBLE PIPE HEAT EXCHANGER



OVERALL ENERGY BALANCE

$$0 = \dot{m}_c d\hat{h}_c + \dot{m}_H d\hat{h}_H$$

COLD STREAM

$$\dot{m}_c d\hat{h}_c = \dot{m}_c C_p dT_c = dq_c$$

HOT STREAM

$$\dot{m}_H d\hat{h}_H = \dot{m}_H C_p dT_H = dq_H$$

Since heat just leaves the hot stream and enters the cold stream,  
 $dq_c = -dq_H$

- Recall that  $dq$ , the rate of heat transfer into or out of a stream, is modeled with Newton's law of cooling,

$$dq = U_i (T_H - T_c) dA_i = U_o (T_H - T_c) dA_o$$

USING THE TWO  $dT$ 'S

$$dT_c = \frac{dq}{\dot{m}_c c_p}, \quad dT_H = -\frac{dq}{\dot{m}_H c_p}$$

$$d(T_H - T_c) = -dq \left( \frac{1}{\dot{m}_H c_p} + \frac{1}{\dot{m}_c c_p} \right)$$

$$\frac{d(T_H - T_c)}{(T_H - T_c)} = - \left( \frac{1}{\dot{m}_H c_p} + \frac{1}{\dot{m}_c c_p} \right) U_i dA_i$$

- Let's make a " $\Delta T$ "

$$d\Delta T \equiv d(T_H - T_C)$$

$$dA_i = \pi D_i dz$$

THUS WE CAN INTEGRATE

$$\int_{\Delta T(z=0)}^{\Delta T(z=L)} \frac{d\Delta T}{\Delta T} = - \frac{U_i \pi D_i}{\dot{m}_H c_p + \dot{m}_C c_p} \int_0^L dz$$

(ASSUMING THAT  $c_p, U_i = \text{CONST}$ )

$$\ln \left( \frac{\Delta T_L}{\Delta T_0} \right) = - \frac{U_i \pi D_i L}{(\dot{m}_H c_p + \dot{m}_C c_p)}$$

RECALL

$$\begin{aligned} \bar{q} &= \dot{m}_C c_p (T_{Cout} - T_{Cin}) \\ \bar{q} &= -\dot{m}_H c_p (T_{Hot} - T_{Hin}) \end{aligned}$$

REPLACING THE  $\dot{m} c_p$ 'S  
WE HAVE

$$\frac{\bar{q}}{U_i A_i} = \frac{(T_{Hot} - T_{Cin}) - (T_{Hin} - T_{Cout})}{\ln \left( \frac{T_{Hot} - T_{Cin}}{T_{Hin} - T_{Cout}} \right)} \quad (1)$$

$$= \frac{\Delta T_2 - \Delta T_1}{\ln \left( \frac{\Delta T_2}{\Delta T_1} \right)}$$

$$= \Delta T_{LM} \quad LM = \text{"LOG-MEAN"}$$

# ANOTHER WATER HEATING PROBLEM

- We wish to heat the 1 kg/s water from 25-50C with the inside pipe diameter the same 3 cm, inside a double pipe heat exchanger where instead of steam, a 75C water stream of 2 kg/s is available.
- For simplicity we will neglect the resistance of the pipe wall, which is 0.2cm thick. Also, we know that the outside heat transfer coefficient is 3000 W/(m<sup>2</sup> K).
- Consider both concurrent and countercurrent configurations.

FIRST WE NEED THE  
EXIT WATER TEMP

$$\dot{m} c_p \Delta T = q$$

$$\Delta T = \frac{100000 \text{ W}}{(2 \text{ kg/s}) (4180) \frac{\text{J}}{\text{K kg}}}$$

$$\Delta T = 12 \text{ K}$$

WATER EXIT T IS 63C

LETS GET  $U_i$

$$\frac{1}{U_i A_i} = \frac{1}{h_i A_i} + \frac{1}{h_o A_o}$$

$$\frac{1}{U_i} = \frac{1}{6000} + \frac{.03}{3000 (.032)}$$

$$U_i = 2100 \frac{\text{W}}{\text{m}^2 \text{K}}$$



$$A_i = \frac{q}{U_i \Delta T_{LM}}$$

COCURRENT:

$$= \frac{100000 \text{ W}}{2100 \frac{\text{W}}{\text{m}^2\text{K}} \left( \frac{(75-25) - (63-50)}{\ln \frac{(75-25)(63-50)}{(63-50)}} \right)}$$

$$\Delta T_{LM} = 27.5 \text{ K}$$

$$A_i = 1.7 \text{ m}^2 \quad L = 19 \text{ m}$$

COUNTERCURRENT

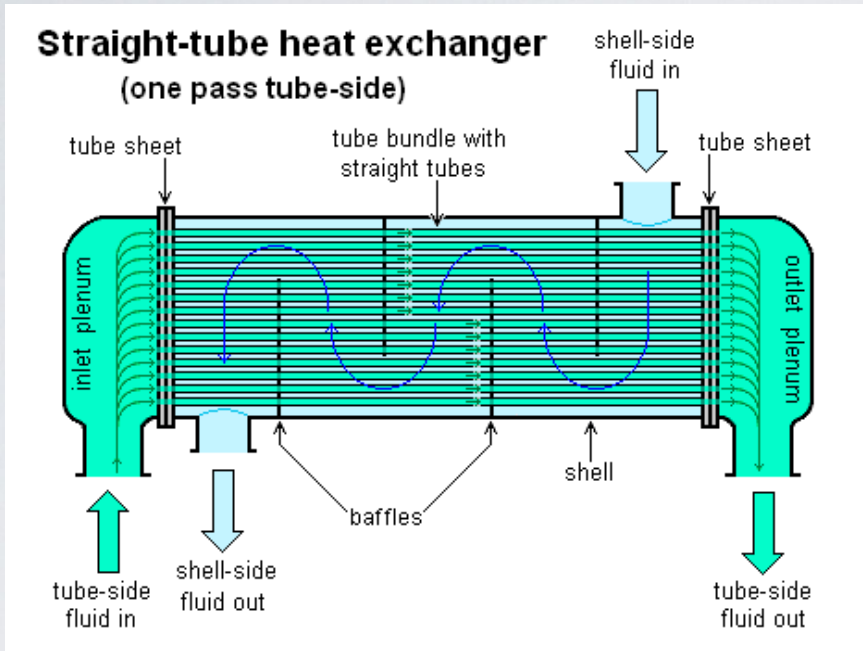
$$A_i = \frac{100000 \text{ J/s}}{2100 \frac{\text{W}}{\text{m}^2\text{K}} \left( \frac{(75-50) - (63-25)}{\ln \frac{(75-50)(63-25)}{(63-25)}} \right)}$$

$$A_i = 1.5 \text{ m}^2 \quad L = 16 \text{ m}$$

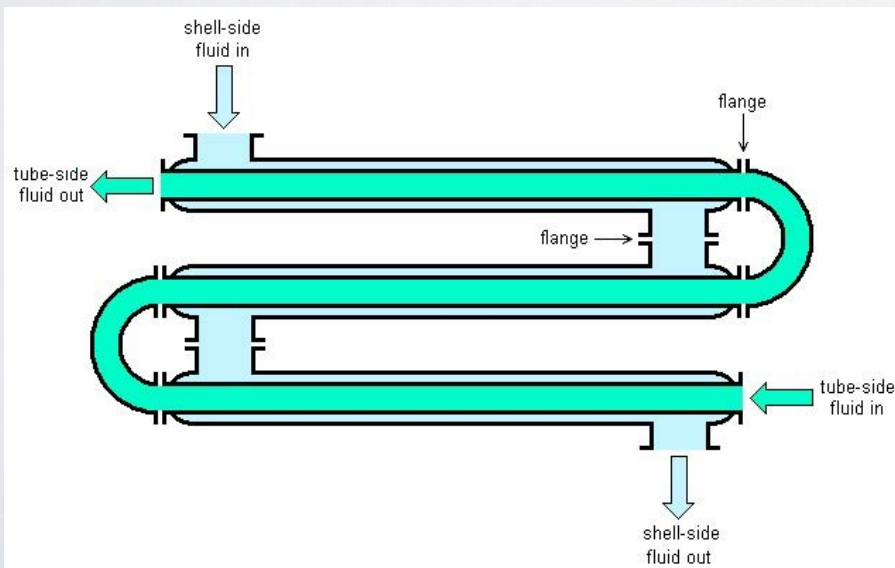
$$\Delta T_{LM} = 31 \text{ K}$$

A larger driving force for countercurrent gives a shorter length

# HEAT EXCHANGERS

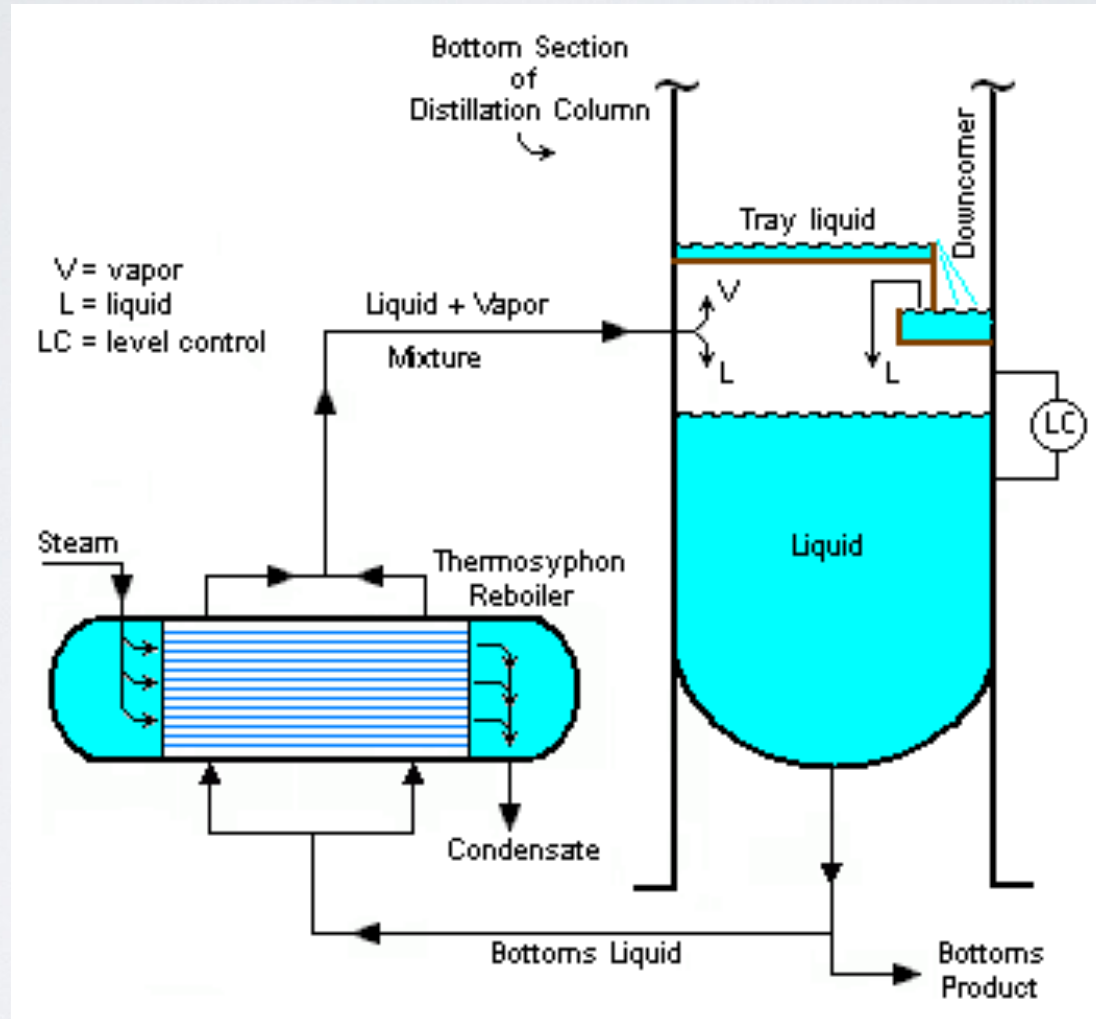


- Shell and Tube
- a combination of counter current and cross flow

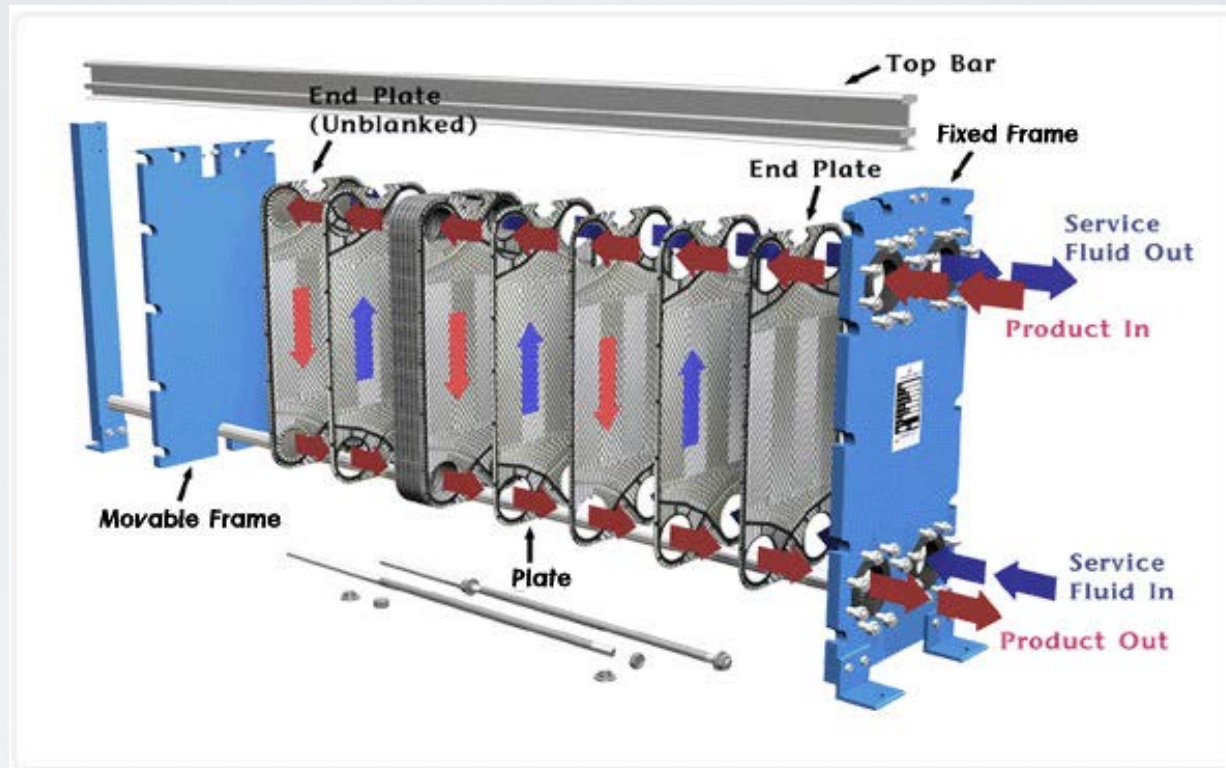


- Double pipe
- true counter current

# THERMOSIPHON



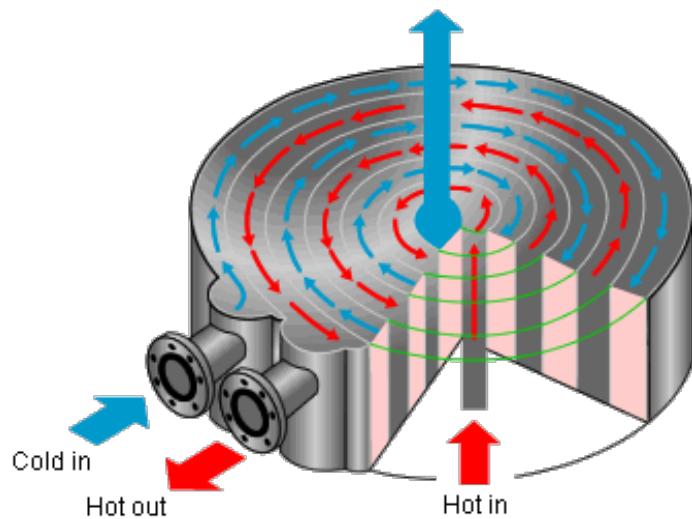
# “PLATE” HEAT EXCHANGERS



- Air-Air

- More plates could be added

# SPIRAL HEAT EXCHANGER



effectively a high surface area, “double-pipe” heat exchanger in a compact space

# SUMMARY OF HEAT TRANSFER FUNDAMENTALS

- Three modes of heat transfer can occur:
  - Radiation (electromagnetic radiation)  $q \sim \epsilon \sigma (T^4 - T_0^4)$
  - Conduction (random motion of molecules, atoms and electrons)  $q \sim k_A \frac{\Delta T_A}{l}$
  - Convection (heat transfer that is aided by bulk fluid motion)

# HEAT EXCHANGER SUMMARY

- Heat exchangers are first analyzed using an energy balance

$$\dot{m}_c d\hat{T}_c = \dot{m}_c c_p dT_c = dq_c$$

- The rate of transfer across the walls is modeled using Newton's Law of cooling  $q' = hA(T_w - T_\infty)$

- We get individual  $h$ 's from correlations

$$Nu = 0.023 Re^{.8} Pr^{.4}$$

- We get a  $U$ 's from a sum of resistances

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{A_i}{k\Delta r} + \frac{A_i}{h_o A_o}$$

- Because the temperature difference between the two sides of the heat exchanger is changing along the pipe, we formulate the problem as a differential slice of pipe and integrate. This gives the temperature driving forces as a "Log-Mean delta T"

$$\frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$