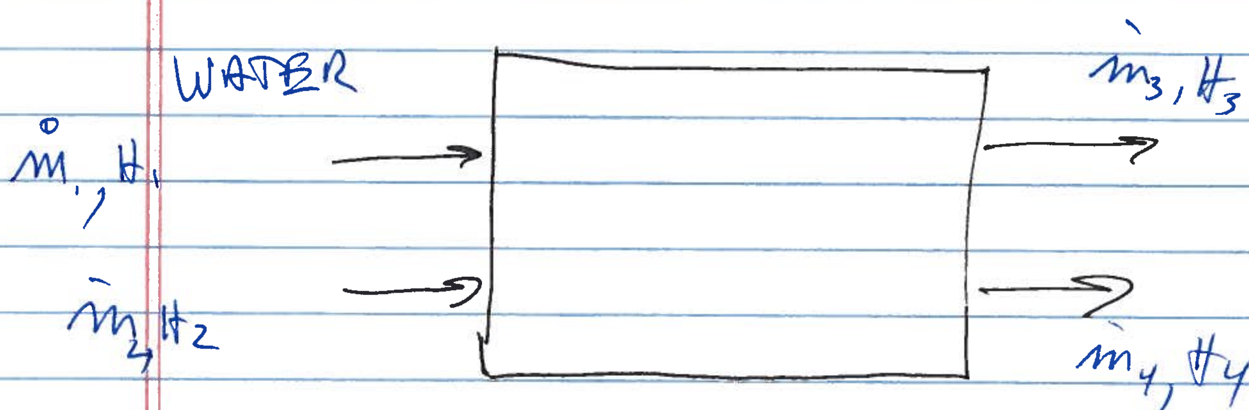
LECTURE 10   2/22/11

BASED ON WHAT WE HAVE DONE SO FAR, THERE ARE SOME UNEXPLAINED OBSERVATIONS AND, IT WOULD APPEAR THAT THE ENERGY BALANCE DOES NOT PREVENT SOME IMPLAUSIBLE PROCESSES.:

FOR EXAMPLE:



$$0 = \dot{m}_1 + \dot{m}_2 - \dot{m}_3 - \dot{m}_4$$

$$0 = \dot{m}_1 H_1 + \dot{m}_2 H_2 - \dot{m}_3 H_3 - \dot{m}_4 H_4$$

SUPPOSE $m_1 = m_2 = m_3 = m_4 = 100 \, kg/s$

$H_1 = 419$, $H_2 = 2676$

$T = 100\ C$, $P = \cancel{100}\ 101\ kPa$

COOLER LOWER P

$$\left\{ \begin{array}{l} H_3 = 0, \\ T = 0C \\ P = .6\, kPa \end{array} \right\} \left\{ \begin{array}{l} H_4 = 3095 \\ T = 325\ C \\ P = 1\ MPa \end{array} \right\}$$

HIGHER T, ~~HIGHER~~ HIGHER P

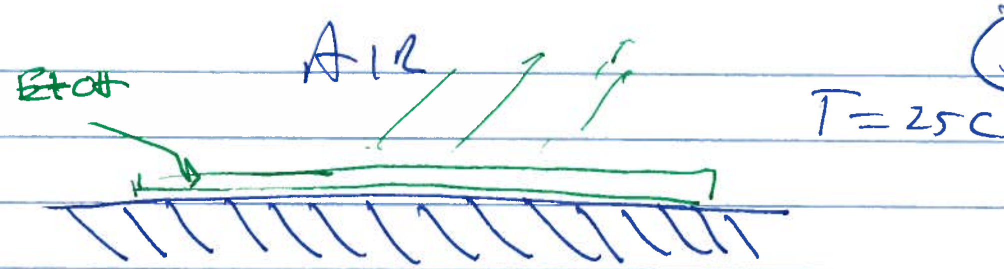
$$0 = 100\,(\overset{419}{\cancel{}}) + 100\,(2676) - 100\,(0) - 100\,(3095)$$

OBSERVATIONS, INTUITION, ETC ...

TELL US THAT THIS CANNOT OCCUR!!!

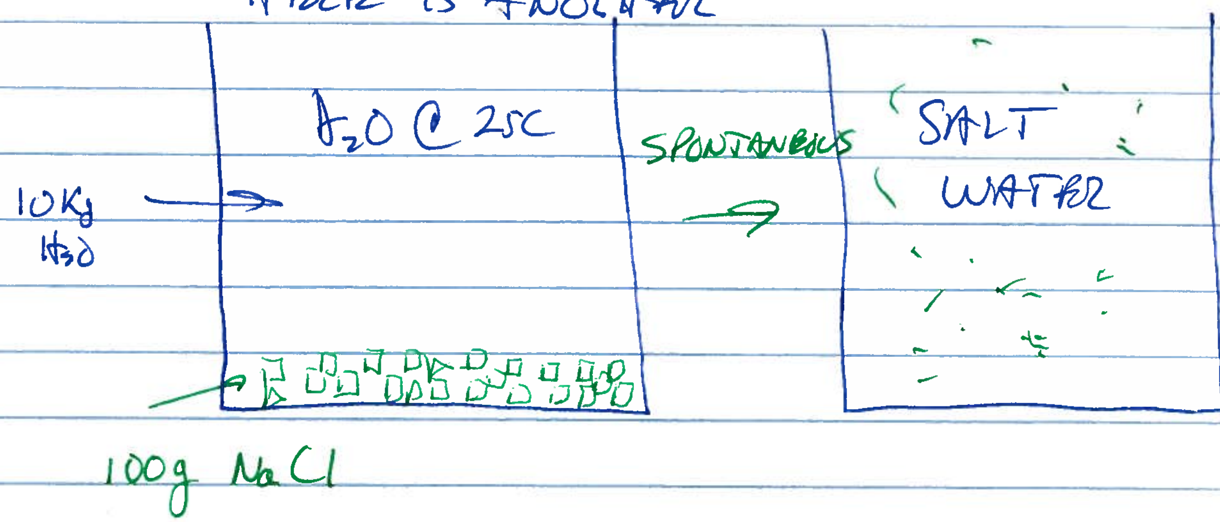
WE WOULD LIKE A FORMALISM

TO SUPPORT OUR INTUITION

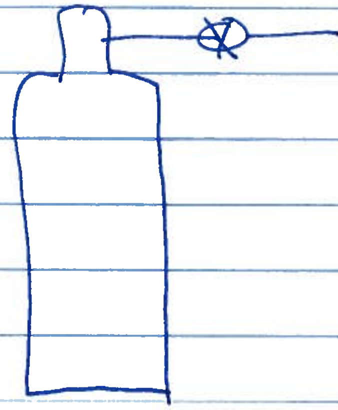


- ETHANOL WILL EVAPORATE, SURFACE WILL BE COOLED
- PROCESS OCCURS SPONTANEOUSLY !!

HERE IS ANOTHER



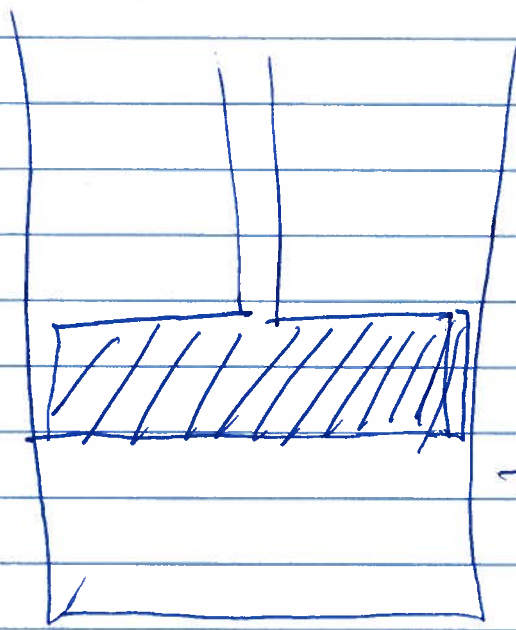
WE CAN REVERSE EITHER OF THESE AND MANY OTHER PROCESSES BY ADDING HEAT, OR DOING WORK (OR DOING SOME OTHER POSITIVE ACTION)



WE DID SOME PROBLEMS ...

FILLING + VENTING WERE NOT NECESSARILY THE REVERSE OF EACH OTHER

SAME WITH PISTON COMPRESSION OR EXPANSION



EXPANSION AGAINST CONSTANT LOAD IS REALISTIC AND ACHIEVABLE,

CAN'T REVERSE THIS PROCESS

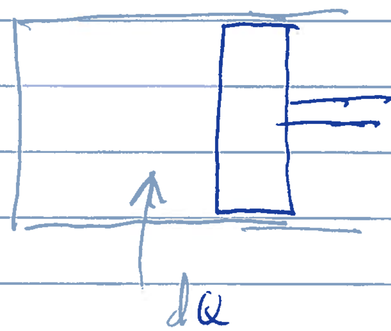
WE CAN'T HELP BUT THINK THERE IS SOME UNDERLYING PRINCIPLE THAT GIVES SOME INSIGHT INTO THESE PROBLEMS...

OF COURSE THERE IS: <u>ENTROPY</u>

- ENTROPY, "S" IS A STATE FUNCTION
- WE WILL HYPOTHESIZE A BALANCE EQUATION FOR S THAT IS CONSISTENT WITH THE UNIDIRECTIONAL SPONTINEITY

HOW DOES S HAVE TO BEHAVE?

PERHAPS, "Q" IS THE KEY.



← CONST FORCE

P = CONST

$Q \sim \Delta T$ SO A DIRECTIONALITY CAN BE ESTABLISHED

$$d\underline{S} = \frac{dQ_{\text{REVERSIBLE}}}{T}$$

NO FRICTION, NO ACCELERATION, MINIMAL GRADIENTS IN T OR VELOCITY OR CONCENTRATION

$$d\underline{S} \geq \frac{dQ}{T}$$

SO THE CHANGE IN S COULD BE GREATER THAN THIS...

SO, STRICTLY SPEAKING WE WON'T BE DEVELOPING A "CONSERVATION" EQUATION FOR ENTROPY!!

WE REALLY CAN'T DO MUCH MORE THAN MAKE A CONJECTURE, THAT WE CAN TEST FOR CONSISTENCY

$$\frac{dS}{dt} = \sum_{\text{INLET}} s^{IN}\,\dot{m}^{IN} - \sum_{\text{OUTLET}} s^{OUT}\,\dot{m}^{OUT} + \frac{\dot{Q}}{T_{SYSTEM}}$$

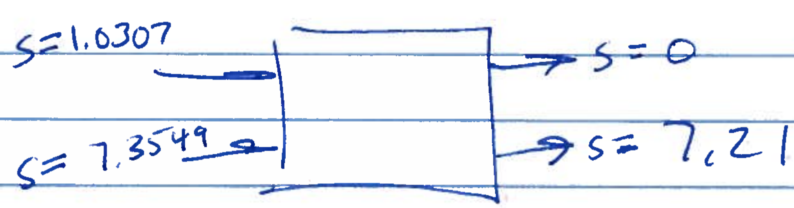
$$+\ \dot{S}_{GEN}$$

WITH A "GENERATION" TERM, THAT ≥ 0 WE HAVE AN EQUALITY IN OUR EQUATION

WE KNOW WHAT THIS TERM IS

USE THIS EQUATION TO LOOK AT

FIRST PROBLEM OF THE DAY ...



$$0 = \dot{m}_1(1.031) + \dot{m}_2(7.355) - \dot{m}_3(0)$$
$$- \dot{m}_4(7.21)$$

$$0 \neq 100(-.9)$$

SO THE ENTROPY BALANCE NEEDS WHAT IN!!

WE CAN'T SHOW RIGHT NOW WHY

YOU CAN'T ASSUME $\dot{S}_{GEN}$ TAKES A

VALUE TO MAKE THIS WORK

REVERSIBLE PROCESS : $\dot{S}_{GEN} = 0$

$\Delta S_{GEN} = 0$

PROCESSES WILL NOT BE PERFECTLY REVERSIBLE, BUT WE WON'T BE ABLE TO JUST "PICK" THIS AT OUR CONVENIENCE ...

LET'S CONSIDER AN IDEALIZED

HEAT ENGINE : HEAT GOES IN

WORK COMES OUT

RECALL THAT WE ESTABLISHED

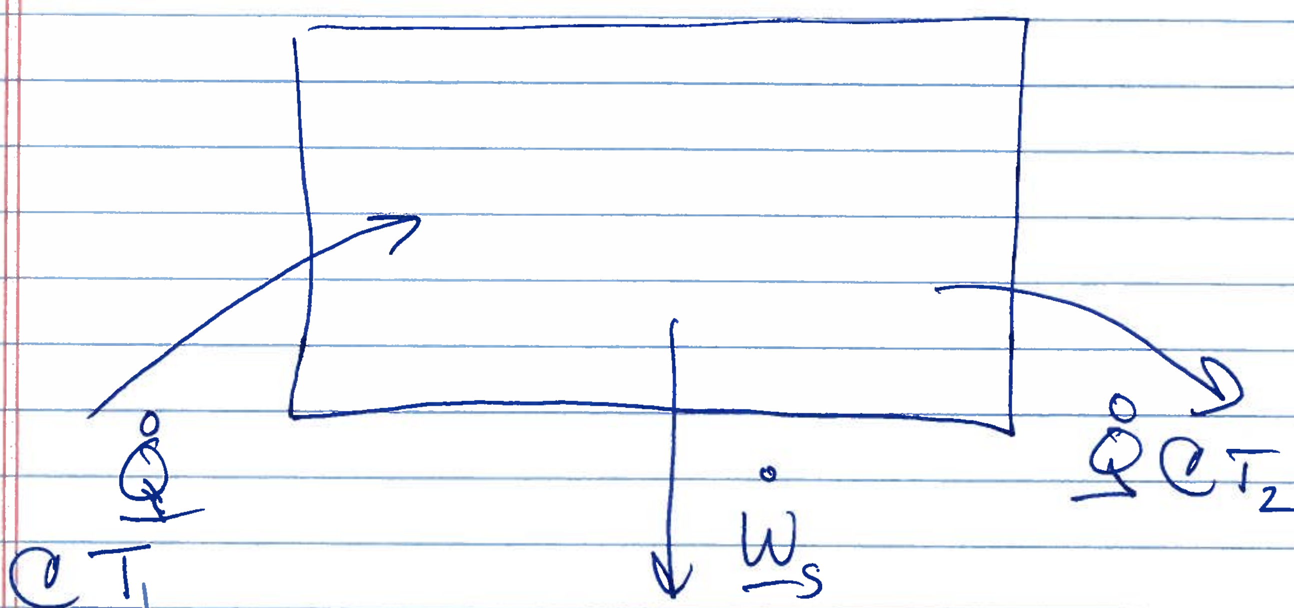
THE EXISTENCE OF SUCH DEVICES !!

S.S CONTINUOUS HEAT ENGINE.

WE WON'T SPECIFIC HOW IT WORKS

JUST GIVE REQUIREMENTS AND

DO THE ANALYSIS.



HEAT IN AT T_1, HEAT OUT AT T_2

WORK OUT.

$$T_1 > T_2$$

WE COULD HAVE $T_2 > T_1$ BUT
THEN WORK WOULD HAVE TO GO IN
AND WE WOULD HAVE A HEAT PUMP.

FOR OUR HEAT ENGINE

ENERGY BALANCE

$$0 = \dot{Q}_1 + \dot{Q}_2 + \dot{W}_s$$

ENTROPY BALANCE

$$\underbrace{\cancel{\frac{dS}{dt}}}_{=0,\ s.s.} = \underbrace{\cancel{\sum_{in} S^{in}\dot{m}^{in}}}_{\text{NO FLOW}} - \underbrace{\cancel{\sum_{out} S^{out}\dot{m}^{out}}}_{\text{NO FLOW}} + \frac{\dot{Q}}{T_{sys}} + \cancel{\dot{S}_{GEN}}$$

START WITH THIS = 0.

$$0 = \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2}$$

DO SOME MANIPULATION:

$$\dot{Q}_2 = -\frac{\dot{Q}_1}{T_1} T_2$$

THEN SUB INTO E BALANCE...

$$0 = \dot{Q}_1 - \frac{\dot{Q}_1 T_2}{T_1} + \dot{W}_s$$

$$-\dot{W}_s = \dot{Q}_1 \left(1 - \frac{T_2}{T_1} \right)$$

IF WE CONSIDER AN EFFICIENCY

$$\varepsilon \equiv \left(\frac{\dot{W}_s}{\dot{Q}_1} \right) = \frac{\text{NET WORK OUT}}{\text{HEAT IN}}$$

$$\varepsilon = \left(1 - \frac{T_2}{T_1} \right) \leq 1$$

SO WE CANT TURN ALL OF OUR

HEAT INTO WORK !!

(UNLESS $T_2 = 0$)

- WE HAVE NOT SHOWN THAT WE CAN MAKE SUCH A DEVICE,

BUT IF WE CAN IT WILL BE THE MAXIMUM EFFICIENCY FOR A HEAT ENGINE

MEANING IF WE EITHER START WITH "HEAT" → SAY INTERNAL ENERGY OF A GAS.

OR WE START OUR PROCESS WITH A COMBUSTION REACTION, SO THAT THE INTERNAL ENERGY IS UNCORRELATED, RANDOM MOTION...

PROFOUND OBSERVATIONS

- THIS IS THE BEST WE CAN DO
- INCREASING T_1 OR DROPPING T_2 WILL BE THE WAY TO IMPROVE EFFICIENCY

CAN WE FIGURE OUT A CYCLE
THAT COULD GIVE THIS

"HIGH" EFFICIENCY ?

OF COURSE, IT IS A CARNOT CYCLE
AND HAS BEEN KNOWN FOR A LONG TIME

BUT IT IS NOT PARTICULARLY PRACTICAL
NOR SHOULD IT BE CONSIDERED
SOMETHING OBVIOUS

- WE JUST INTRODUCED ENTROPY
SO LET'S MAKE AN OBSERVATION

$$d\underline{S} = \frac{d\underline{Q}}{T}$$

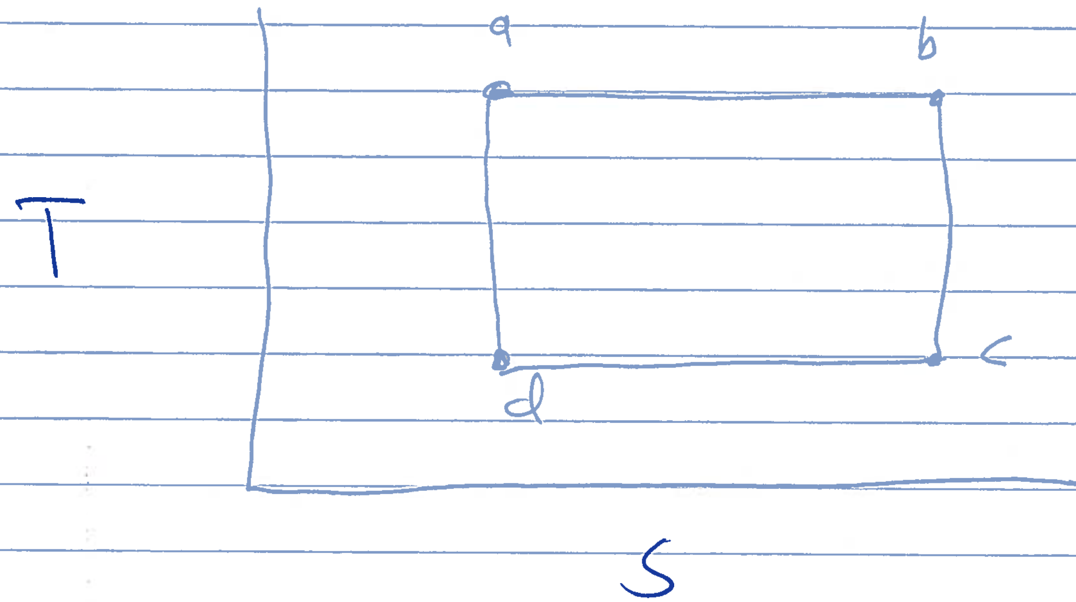
SO IF $d\underline{Q} = 0$ (ADIABATIC)

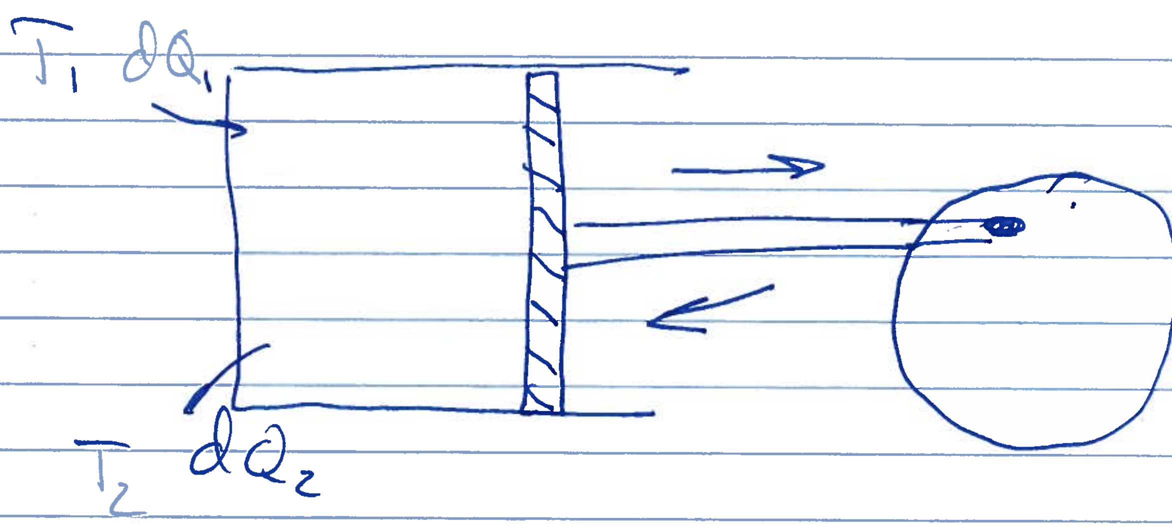
THEN $d\underline{S} = 0$

ALSO NOTE: WE SAID HEAT
ENGINE ADDS HEAT AT T_1 AND
REMOVES HEAT AT T_2

ISOTHERMAL STEPS

THUS HOW ABOUT THE FOLLOWING PROCESS





a → b $\boxed{\text{CONST } T}$

1) STEP-1 — HEAT INTO PISTON FROM T_1 SOURCE, EXPANSION OF GAS, WORK OUT

2) STEP-2 b → c ADIABATIC EXPANSION
FLUID COOLS, WORK OUT

3) STEP-3 c → d $\boxed{\text{CONST } T}$
COMPRESSION HEAT OUT

4) STEP-4 d → a ADIABATIC COMPRESSION
BACK TO START
WORK IN

STEP-1
$$d\underline{U}_1 = d\underline{Q}_1 - \overbrace{(P\,d\underline{V})_1}^{=\,dW_1}$$
$$d\underline{S}_1 = \frac{d\underline{Q}_1}{T_1}$$

STEP-2
$$d\underline{U}_2 = -\overbrace{(P\,d\underline{V})_2}^{=\,dW_2}$$
$$d\underline{S}_2 = 0$$

STEP-3
$$d\underline{U}_3 = d\underline{Q}_3 - \overbrace{(P\,d\underline{V})_3}^{=\,dW_3}$$
$$d\underline{S}_3 = \frac{d\underline{Q}_3}{T_3}$$

STEP-4
$$d\underline{U}_4 = -\overbrace{(P\,d\underline{V})_4}^{=\,dW_4}$$
$$d\underline{S}_4 = 0$$

SINCE IT IS A CYCLE

$$\sum d\underline{U} = 0$$

$$\sum d\underline{S} = 0$$

ENERGY BALANCE FOR CYCLE

$$0 = d\underline{Q}_1 + d\underline{Q}_3 + d\underline{W}_1 + d\underline{W}_2 + d\underline{W}_3 + d\underline{W}_4$$

ENTROPY FOR CYCLE

$$0 = \frac{d\underline{Q}_1}{T_1} + \frac{d\underline{Q}_3}{T_3}$$

$$-\sum_i d\underline{W}_i = d\underline{Q}_1 - \frac{T_3}{T_1} d\underline{Q}_1$$

$$-\sum_i d\underline{W}_i = d\underline{Q}_1 \left(1 - \frac{T_3}{T_1} \right)$$

THIS IS THE SAME RESULT AS BEFORE!

WE KNOW HOW TO MAKE A MAXIMALLY EFFICIENT ENGINE!

$$\text{NET WORK} = \text{HEAT IN}\left(1 - \frac{T_3}{T_1}\right)$$

IF YOU WANT TO MAKE SUCH A DEVICE LOOK LIKE CONTINUOUS S.S. ENGINE: CONSIDER 4 CYLINDERS.

(RESULT TELLS THE BEST WE CAN DO)

WE WILL NEED TO COMPARE OUR REAL CYCLES AGAINST IT.

- CAN WE UNDERSTAND THIS LIMITED EFFICIENCY?

K.E. FOR GAS

$$U_1 \sim \frac{3}{2}\langle u_1^2 \rangle$$

THE MOTION IS RANDOM!

IF WE LET IT EXPAND TO $P_2 < P_1$

WE STILL HAVE SOME RANDOM MOTION

$P_2, T_2 \sim \frac{3}{2}\langle u_2^2 \rangle$, NOW u, U ARE SMALLER

SINCE MOTION IS INITIALLY RANDOM,

WE CAN'T GET IT ALL BACK.

WE GET MORE IF ΔT IS HIGHER.

SO ~ $\varepsilon \sim \dfrac{T_1 - T_3}{T_1}$

DOES NOT SEEM UNREASONABLE.

SO OUR CARNOT CYCLE ANALYSIS

INFORMS SIGNIFICANT FUNDAMENTAL

UNDERSTANDING !!

SUMMARY

ENTROPY

- NEW (TO US) STATE FUNCTION
- IT COMES WITH A SIMPLE DEFINITION

$$dS = \frac{dQ}{T}$$

AND A BALANCE EQ.

$$\frac{dS}{dt} = \sum \dot{m}\,\bar{S}_{IN} - \sum \dot{m}\,\bar{S}_{OUT} + \frac{\dot{Q}}{T} + \dot{S}_{GEN}$$

MAXIMUM EFFICIENCY FOR HEAT ENGINE:

$$\varepsilon = \left(1 - \frac{T_2}{T_1}\right)$$

$T_1 > T_2$

- "S" IS WHAT THE EQUATIONS SAY IT IS!!
- IF YOU WANT TO THINK OF SOME MECHANISTICS FOR AN IDEAL GAS...

S IS A (RELATIVE) MEASURE OF DISORDER

READ E+L